Schnorr's protocol based on discrete log (no secrets, stateless, interactive)

client ($x \in \mathbb{Z}_p$) server ($h = g^x$)

"commitment" $u = g^r$

"challenge" $t \xleftarrow{R} \mathbb{Z}_p$

"response" $z = r + xt$

check $g^z = u \cdot h^t$

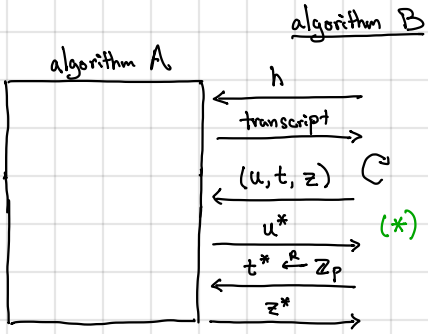
client will prove knowledge of x to the server

Protocols with this structure called a Σ-protocol (also require that verifier's challenge is a random string)

Correctness: $g^z = g^{r+xt} = g^r (g^x)^t = u \cdot h^t$

Security against passive adversaries: Suppose A can break security with probability 1: (will relax later)

- Algorithm A can request authentication transcripts, so reduction must simulate these



This is distributed identically to the real protocol

- in real protocol, $t \xleftarrow{R} \mathbb{Z}_p$, $z = r + tx$ where $r \xleftarrow{R} \mathbb{Z}_p$

given z and t, $u = g^z / h^t$ is fully determined

To extract, algorithm B will "reset" state of algorith A to (*)

- Namely, algorithm B runs A to get a transcript (u^*, t_1^*, z_1^*)
- Then algorithm B "rewinds" A to (*) but supplies a different $t_2^* \xleftarrow{R} \mathbb{Z}_p$
- Let (u^*, t_2^*, z_2^*) be the resulting transcript
- Algorithm B outputs $x = (z_1^* - z_2^*)(t_1^* - t_2^*)^{-1}$

if A succeeds w.p. 1, then

$g^{z_1^*} = u^* \cdot h^{t_1^*}$

$g^{z_2^*} = u^* \cdot h^{t_2^*}$

$\Downarrow$

$g^{z_1^* - z_2^*} = h^{t_1^* - t_2^*}$

$\Downarrow$

$z_1^* - z_2^* = x(t_1^* - t_2^*)$

In general, if A succeeds w.p. ε, then algorithm B succeeds w.p. $\varepsilon^2 - \varepsilon/p$

["rewinding lemma"]

We refer to this property as a "proof of knowledge"

↳ Any client that succeeds in this protocol with good probability must in fact know x.

Is this protocol secure against an active adversary? ["fake ATM machine" / "credit card skimmer"]

Active adversary is able to first impersonate the server (i.e., interact with the client in an arbitrary manner) and afterwards, it tries to authenticate to the server (without further assistance from the client)

It is not known whether Schnorr's identification protocol is secure against active adversaries!