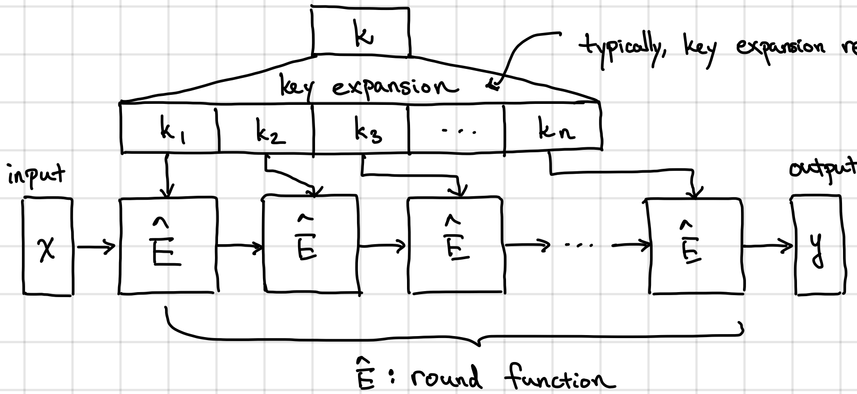
**Constructing block ciphers:** typically, relies on an "iterated cipher"



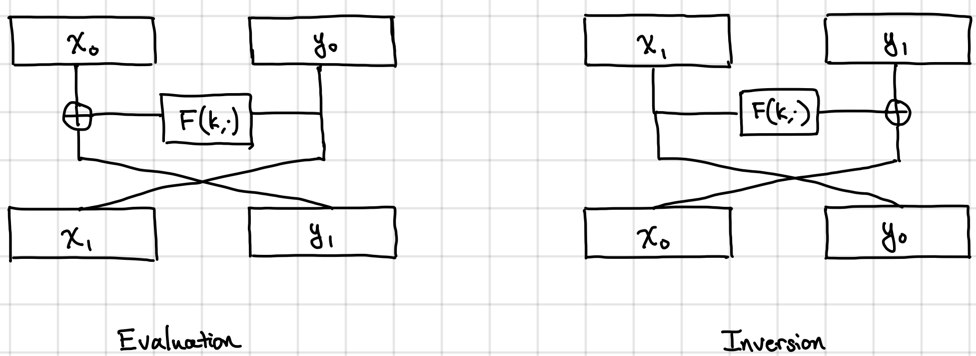

typically, key expansion relies on a PRG

Difficult to design! Never invent your own crypto – use well-studied, standardized constructions and implementations!

We will look at two classic designs:

- DES / 3DES (Data Encryption Standard) 1977 (developed at IBM)
- AES (Advanced Encryption Standard) 2002 [most widely used block cipher, implemented in hardware in Intel processors]
  - on modern Intel processors, (with AES-NI), ~4 cycles/round

**DES:** relies on the Feistel design:




Can stack multiple layers together

**Observe:** the function F does not have to be invertible $\Rightarrow$ Feistel network is still invertible!

**Theorem (Luby-Rackoff).** If F is a secure PRF, then a 3-round Feistel construction yields a secure PRP. Similarly, a 4-round Feistel construction implements a strong PRP.

↳ strong: a PRP where the adversary can also query the inversion oracle (i.e., $F^{-1}(k, \cdot)$ in the pseudorandom world and $f^{-1}(\cdot)$ in the random world)

→ Shows that Feistel construction is sound for constructing block cipher (but now need a good random-looking function F)

↳ called the round function → DES round function will not be a PRF, so overall construction relies on more rounds (but general design philosophy supported by theory)

**DES:** block size: 64 bits → round function operates on 32-bit blocks

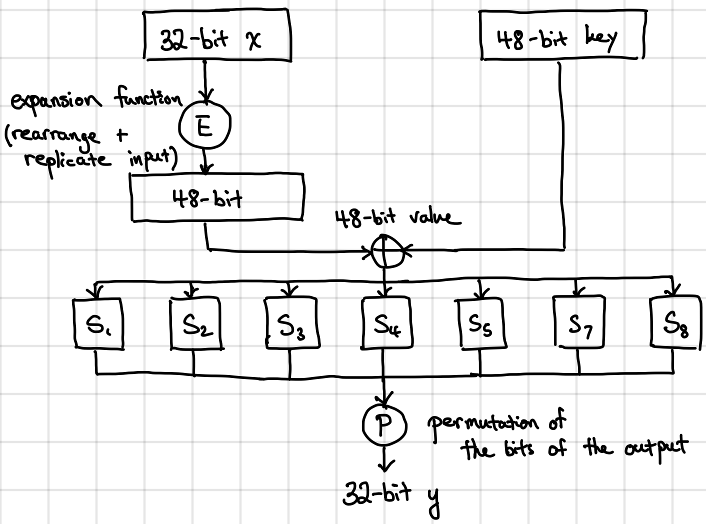
key size: 56 bits (to comply with export control regulations)

↓ used to derive 16 round keys (48 bits)

↳ DES overall is a 16-round Feistel network

→ simple approach: each 48-bit key is subset of the original 56-bit key

## DES round function $F(k, x)$:




S-boxes (substitution boxes)

- each S-box maps 6-bits to 4-bits (carefully designed to be non-linear)
- implemented as a truth table (hard-wired in the DES specification)
- only source of non-linearity in the design

S-box design extremely important for security

- ↳ NSA made recommendation to tweak some entries
- ↳ disclosed in 1994 after discovery of differential cryptanalysis that S-boxes were designed to be robust against these attacks
  - (NSA knew of these techniques in the late 70s!)

56-bit keys was a compromise between 40-bit keys (NIST/NSA) and 64-bit keys (cryptographers - notably Hellman)

- ↳ turned out to be insufficient
  - 1997: DES challenge solved in 96 days (massive distributed effort)
  - 1998: with dedicated hardware, DES can be broken in just 56 hours → not secure enough!
  - 2007: using off-the-shelf FPGAs (120), can break DES in just 12.8 days → anyone can now break DES!
- ↳ 2-DES: apply DES twice (keys now 112-bits)
  - ↳ meet-in-the-middle attack gives no advantage (though space usage is high)
- ↳ 3-DES: apply DES three times $[\text{3DES}((k_1, k_2, k_3), x) := \text{DES}(k_3, \text{DES}^{-1}(k_2, \text{DES}(k_1, x)))]$
  - ↳ 168-bit keys — standardized in 1998 after brute force attacks on DES shown to be feasible

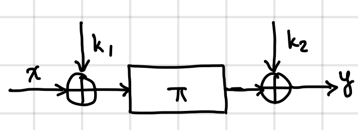
## AES (2002 - most common block cipher in use today):

- 3DES is slow (3x slower than DES)
- 64-bit block size not ideal (recall that block size determines adversary's advantage when block cipher used for encryption)

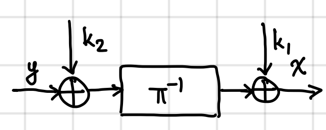
AES block cipher has 128-bit blocks (and 128-bit keys)
(← also have 192-bit and 256-bit variants (but block size always $2^{128}$))

- ↳ follows another classic design paradigm: iterated Even-Mansour (also called alternating key ciphers)

Even-Mansour block cipher: keys $(k_1, k_2)$, input $x$:



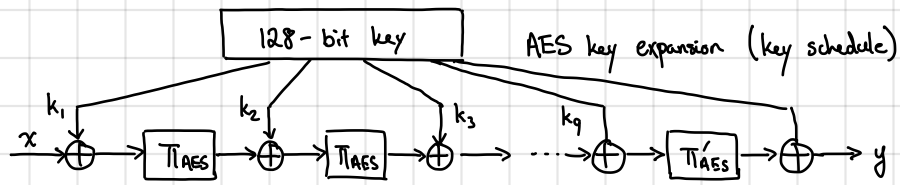

Evaluation




Inversion

**Theorem (Even-Mansour):** If $\pi$ is modeled as a random permutation, then the Even-Mansour block cipher is secure (i.e., it is a secure PRP).

The AES block cipher can be viewed as an iterated Even-Mansour cipher:




key-size ↓

- AES-128: 10 rounds
- AES-192: 12 rounds
- AES-256: 14 rounds

(block-size all 128 bits)

Permutations $\Pi_{AES}$ and $\Pi'_{AES}$ are fixed permutations and <u>cannot</u> be ideal permutations
↳ cannot write down random permutation over $\{0,1\}^{128}$

↳ Cannot appeal to security of Even-Mansour for security
↳ But still provides evidence that this design strategy is <u>viable</u> [similar to DES and Luby-Rackoff]

AES round permutation: composed of three invertible operations that each operate on a 128-bit block

| | | | |
|---|---|---|---|
| $a_0$ | $a_1$ | $a_2$ | $a_3$ |
| $a_4$ | $a_5$ | $a_6$ | $a_7$ |
| $a_8$ | $a_9$ | $a_{10}$ | $a_{11}$ |
| $a_{12}$ | $a_{13}$ | $a_{14}$ | $a_{15}$ |

128 bits arranged in 4-by-4 grid of bytes ($\{0,1\}^8$)

SubBytes: apply a <u>fixed</u> permutation $S: \{0,1\}^8 \to \{0,1\}^8$ to each cell
↳ hard coded in the AES standard (similar to S-box)
(chosen very carefully to resist attacks)

ShiftRows: cyclic shift the rows of the matrix
- 1st row unchanged
- 2nd row shifted left by 1
- 3rd row shifted left by 2
- 4th row shifted left by 3

MixColumns: the matrix is interpreted as a 4-by-4 matrix over $GF(2^8)$ and multiplied by a fixed <u>invertible</u> matrix (also carefully chosen and hard-coded into the standard)

elements are polynomials over $GF(2)$ ($\mathbb{F}_2$) modulo the irreducible polynomial $x^8 + x^4 + x^3 + x + 1$

<u>Observe</u>: Every operation is invertible, so composition is also invertible

$\Pi_{AES}$: SubBytes; ShiftRows; MixColumns

$\Pi'_{AES}$: SubBytes; ShiftRows    No MixColumns for the last round [done so AES decryption circuit better resembles AES encryption]

<u>Security of AES</u>: Brute-force attack: $2^{128}$
Best-known key recovery attack: $2^{126.1}$ time — only 4x better than brute force!

What does $2^{128}$-time look like?
- Suppose we can try $2^{40}$ keys a second.
  ↳ $2^{88}$ seconds to break 1 AES key $\sim 10^{19}$ years (710 million times larger than age of the universe!)
- Total computing power on Earth (circa 2015)
  ↳ estimated to be $\sim 2^{70}$ operations/second (currently, bitcoin mining computes $\sim 2^{66}$ hashes/second)

  Let's say we can do $2^{80}$ operations/second
  ↳ still require $2^{48}$ seconds to break AES $\sim$ 9 million years of compute

If we move to 256-bit keys, best brute force attack takes $2^{254.2}$ time (on AES-256)

In well-implemented systems, the cryptography is not the weak point — breaking the crypto requires new <u>algorithmic</u> techniques (e.g., quantum computers)
↳ But side channels / bad implementations can compromise crypto