

Succinct Matrix Commitments and Their Applications

David Wu

Full disclosure: this talk is primarily about computational objects...

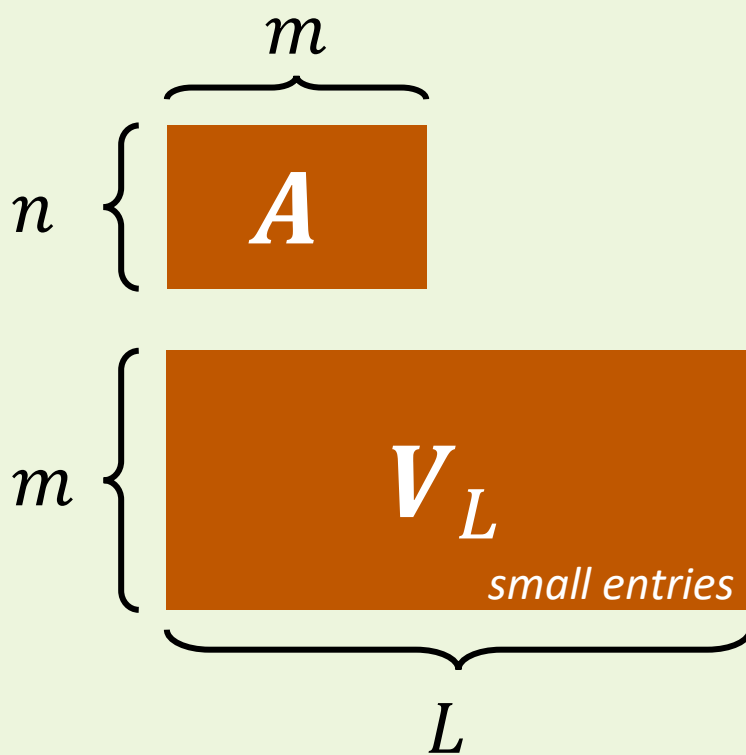
Matrix Commitments

[Wee24, Wee25]

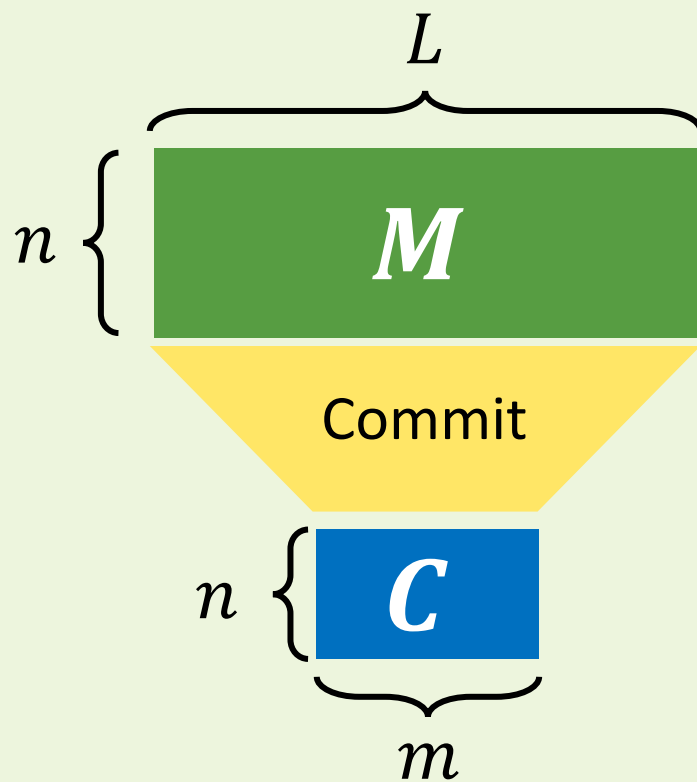
Succinct commitment to a matrix $M \in \mathbb{Z}_q^{n \times L}$

$$\begin{aligned} m &> n \log q \\ L &\gg n \end{aligned}$$

Public parameters pp

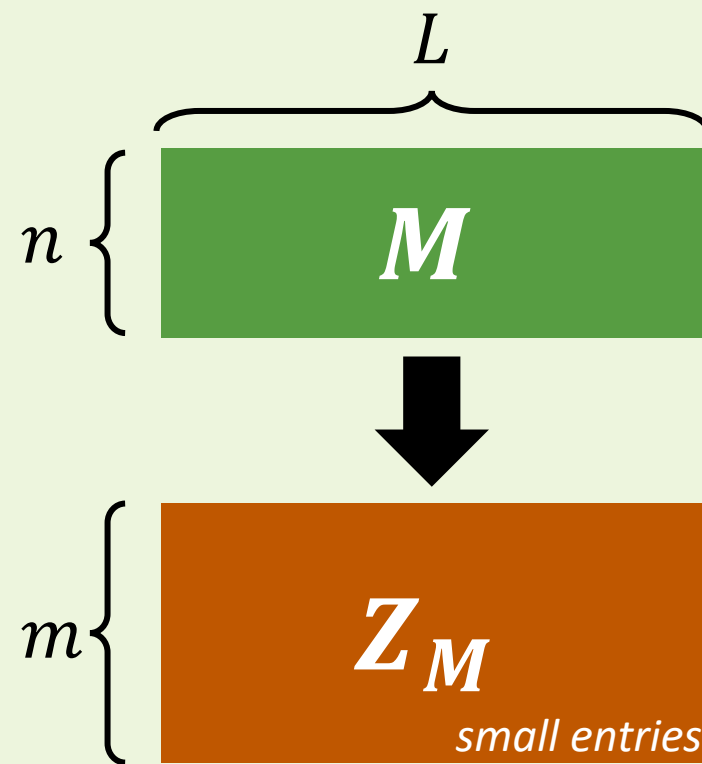


Commit(pp, M)



compress the matrix M

Open(pp, M)



Matrix Commitments

[Wee24, Wee25]

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$$m > n \log q$$

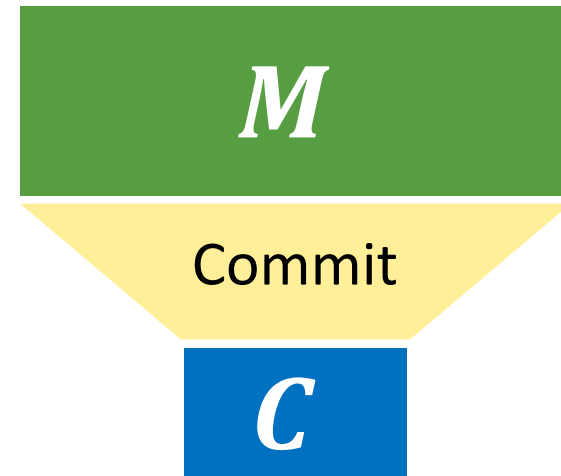
$$\text{Commit}(\text{pp}, M) \rightarrow C \in \mathbb{Z}_q^{n \times m}$$

$$\text{Open}(\text{pp}, M) \rightarrow Z_M \in \mathbb{Z}^{m \times L} \text{ low-norm}$$

public parameters $\text{pp} = (A, V_L)$

$$A \in \mathbb{Z}_q^{n \times m}$$

$$V_L \in \mathbb{Z}^{m \times L} \text{ low-norm}$$



Key opening relation:

$$C \cdot V_L = M - A \cdot Z_M$$

C is a compressed representation of M

Can expand C to translation of M by multiplying by a fixed low-norm matrix V_L

So far... this object is information-theoretic...

Matrix Commitments

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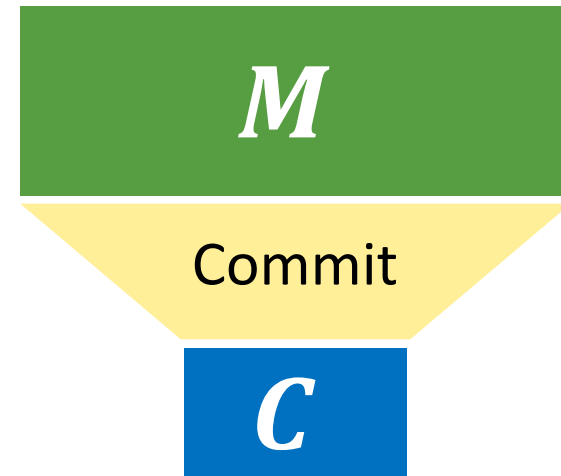
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Key opening relation:

$$C \cdot V_L = M - A \cdot Z_M$$

Security assumption: SIS or LWE problem is hard with respect to A given pp

SIS problem: given A , find low-norm $\mathbf{0} \neq \mathbf{x} \in \mathbb{Z}^m$ where $A\mathbf{x} = \mathbf{0}$

LWE problem: distinguish $\mathbf{s}^T A + \mathbf{e}^T$ from random when $\mathbf{s} \leftarrow \mathbb{Z}_q^n$ and $\mathbf{e} \in \mathbb{Z}^m$ is low-norm

Matrix Commitments

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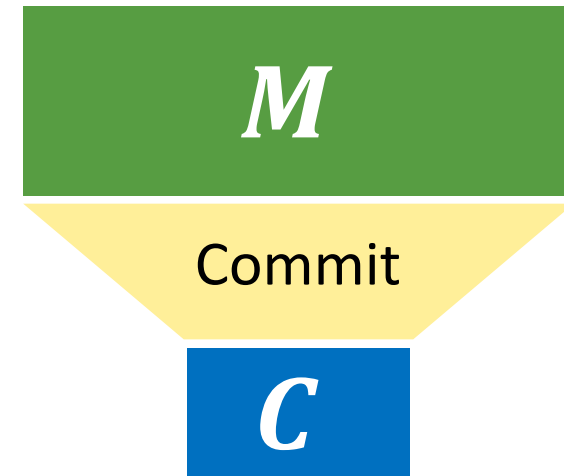
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Security assumptions hold under
the succinct/decomposed
SIS/LWE assumptions

Key opening relation:

$$C \cdot V_L = M - A \cdot Z_M$$



Security assumption: SIS or LWE problem is hard with respect to A given pp

SIS problem: given A , find low-norm $0 \neq x \in \mathbb{Z}^m$ where $Ax = 0$

LWE problem: distinguish $s^T A + e^T$ from random when $s \leftarrow \mathbb{Z}_q^n$ and $e \in \mathbb{Z}^m$ is low-norm

So What is it Good For?

Succinct commitment to a matrix $M \in \mathbb{Z}_q^{n \times L}$

$$\text{Commit}(\text{pp}, M) \rightarrow C \in \mathbb{Z}_q^{n \times m}$$

$$\text{pp} = (A \in \mathbb{Z}^{n \times m}, V_L \in \mathbb{Z}^{m \times L})$$

$$\text{Open}(\text{pp}, M) \rightarrow Z_M \in \mathbb{Z}^{m \times L}$$

$$C \cdot V_L = M - A \cdot Z_M$$

Functional commitments for all circuits + SNARGs for P/poly [WW23a, WW23b, Wee24, Wee25]

Optimal broadcast encryption [Wee25, AMR26, GY26a]

Distributed broadcast encryption [CW24, CHW25, WW25, AMR26, CW26, GY26a, GY26b]

Nearly optimal continuous group key agreement [BBDGW26]

Silent threshold encryption [GY26b, GY26c, CWY26]

Nearly-optimal KP-ABE and CP-ABE for circuits [Wee24, Wee25]

Registered ABE for circuits [CHW25, AMR25, WW25]

Succinct computational secret sharing for DNFs [CW26, GY26c]

Compressing [BGG⁺] Encodings

[W23, Wee24, Wee25, AMR25]

[BGG⁺] encoding of $\mathbf{x} \in \{0,1\}^\ell$: $\mathbf{B} - \mathbf{x}^\top \otimes \mathbf{G}$

$$\boxed{\mathbf{B}_1 - x_1 \mathbf{G}} \quad \boxed{\mathbf{B}_2 - x_2 \mathbf{G}} \quad \cdots \quad \boxed{\mathbf{B}_k - x_k \mathbf{G}}$$

Useful property: supports homomorphic operations

In lattice-based ABE: ciphertext contains LWE encoding $\mathbf{s}^\top (\mathbf{B} - \mathbf{x}^\top \otimes \mathbf{G})$

Ciphertext size scales with $|\mathbf{x}|$

Compressing [BGG⁺] encoding

$$\mathbf{C} \cdot \mathbf{V}_L = (\mathbf{x}^\top \otimes \mathbf{G}) - \mathbf{A} \cdot \mathbf{Z}_x$$

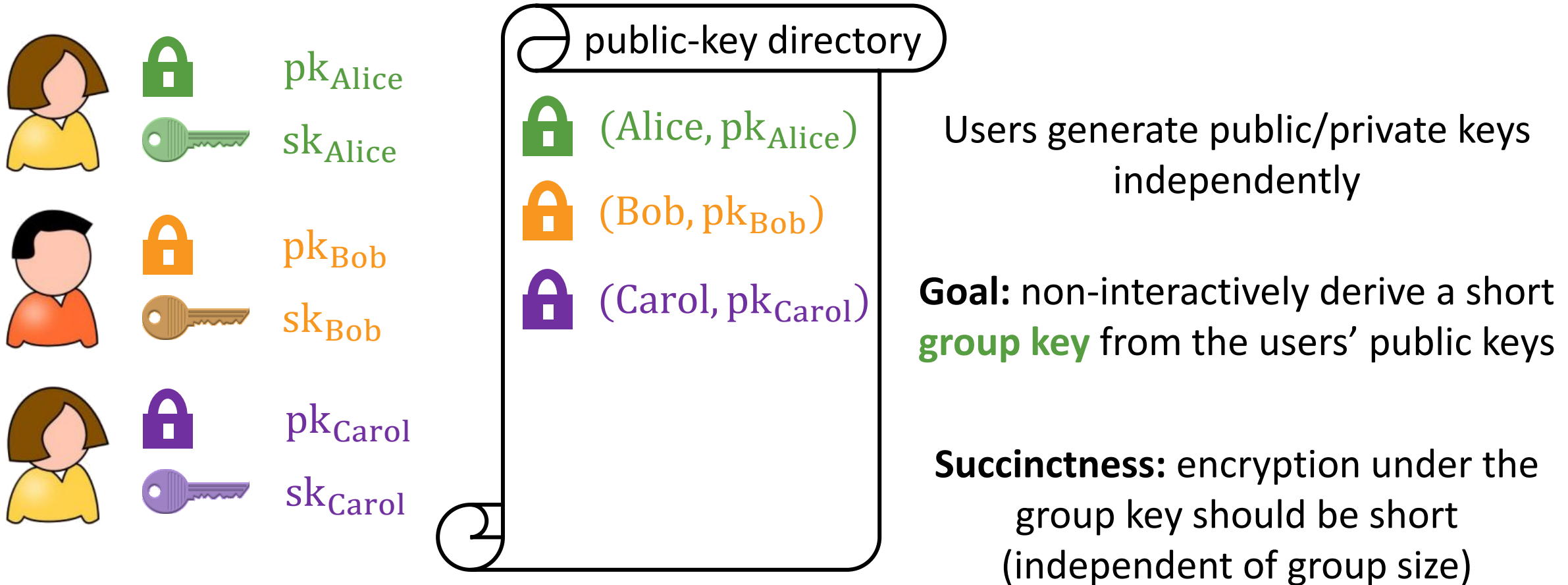
Replace $\mathbf{s}^\top (\mathbf{B} - \mathbf{x}^\top \otimes \mathbf{G})$ with $(\mathbf{s}^\top \mathbf{A}, \mathbf{s}^\top (\mathbf{B}_0 - \mathbf{C}))$ where $\mathbf{C}$ is a commitment to $\mathbf{x}^\top \otimes \mathbf{G}$

Can **expand** by computing

$$(\mathbf{s}^\top (\mathbf{B}_0 - \mathbf{C})) \cdot \mathbf{V}_L - (\mathbf{s}^\top \mathbf{A}) \cdot \mathbf{Z}_x = \mathbf{s}^\top (\overbrace{\mathbf{B}_0 \mathbf{V}_L}^{\mathbf{B}} - \mathbf{x}^\top \otimes \mathbf{G}) \quad \text{dropping noise terms}$$

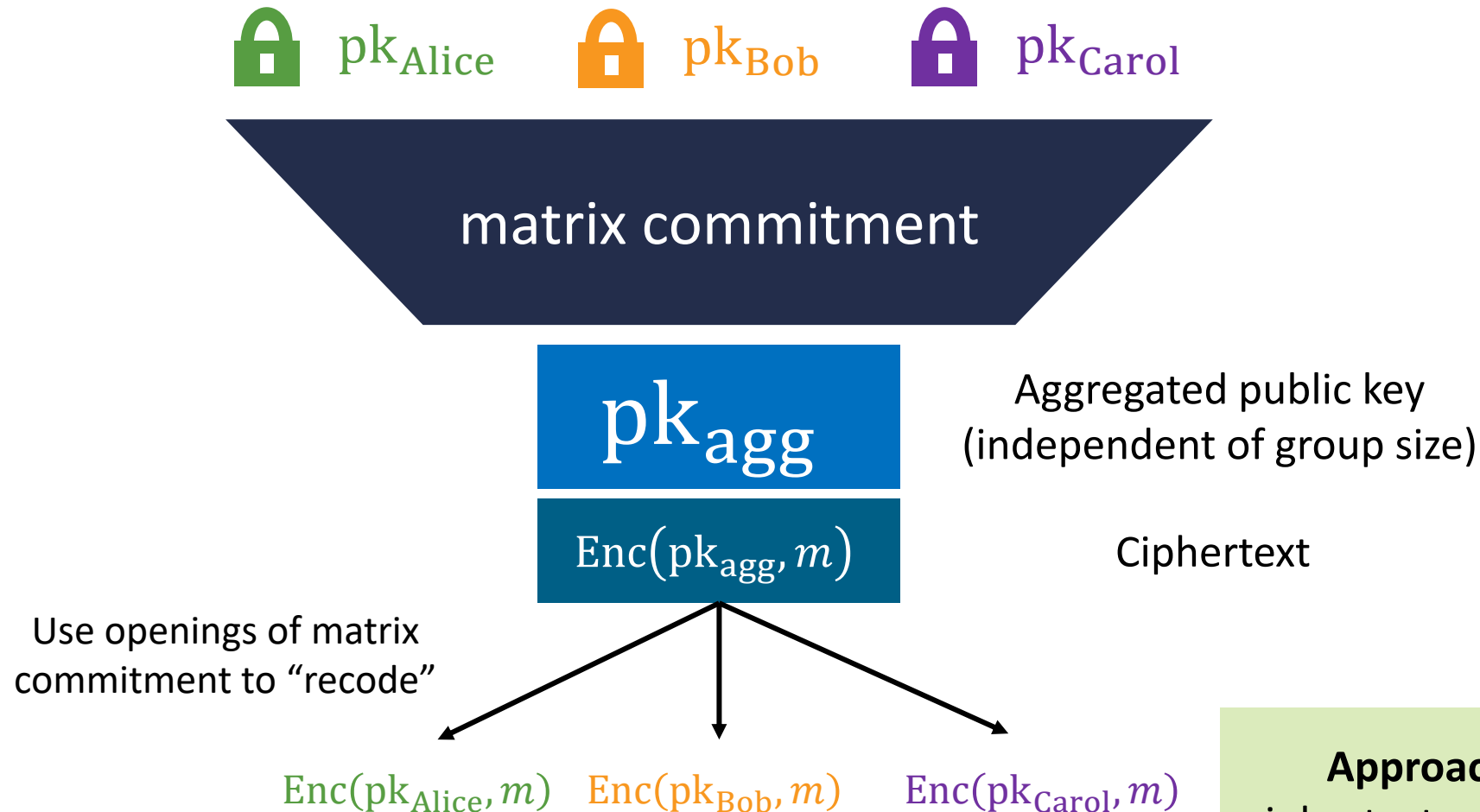
Group Messaging and Broadcast Encryption

[WW25, BBDGW26]



Group Messaging and Broadcast Encryption

[WW25, BBDGW26]



Approach: "recode" an LWE ciphertext with respect to pk_{agg} to a ciphertext with respect to pk_{user}

Group Messaging and Broadcast Encryption

[WW25, BBDGW26]

Recall: dual Regev encryption

pk: $A \in \mathbb{Z}_q^{n \times m}$, $p \in \mathbb{Z}_q^n$

sk: $r \leftarrow A^{-1}(p)$

$$A \cdot r = p$$

Ciphertext:

$s \leftarrow \mathbb{Z}_q^n$

$s^T A + e_1^T$, $s^T p + e_2 + \mu \cdot \lfloor q/2 \rfloor$

Multiply by r

$s^T p + e_1^T r$

$\mu \cdot \lfloor q/2 \rfloor + (e_2 - e_1^T r)$

Group Messaging and Broadcast Encryption

[WW25, BBDGW26]

Public parameters: $A \in \mathbb{Z}_q^{n \times m}$, $p \in \mathbb{Z}_q^n$



$$pk_1 = t_1 = Ar_1 + p \in \mathbb{Z}_q^n$$



$$sk_1 = r_1 \leftarrow \{0,1\}^m$$



$$pk_2 = t_2 = Ar_2 + p \in \mathbb{Z}_q^n$$



$$sk_2 = r_2 \leftarrow \{0,1\}^m$$



$$pk_3 = t_3 = Ar_3 + p \in \mathbb{Z}_q^n$$



$$sk_3 = r_3 \leftarrow \{0,1\}^m$$

Aggregated group key:



Commit($[t_1 \mid t_2 \mid t_3]$)

$$C \in \mathbb{Z}_q^{n \times m}$$

Invariant: $Cv_i = t_i - Az_i$ where v_i, z_i are low-norm

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$$\text{pk}_1 = t_1 = Ar_1 + p \in \mathbb{Z}_q^n$$

$$\text{sk}_1 = r_1 \leftarrow \{0,1\}^m$$

Ciphertext:

(suppressing error terms)

$$s \leftarrow \mathbb{Z}_q^n$$

$$s^T A, s^T p + \mu \cdot \lfloor q/2 \rfloor, s^T C$$

Use v_i, z_i to recode to encryption under t_i :

Aggregated group key:

$$t_1$$

$$t_2$$

$$t_3$$

$$\text{Commit}([t_1 \mid t_2 \mid t_3])$$

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$$s \leftarrow \mathbb{Z}_q^n$$

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Use v_i, z_i to recode to encryption under t_i :

$$s^T C \cdot v_i + s^T A \cdot z_i \approx s^T t_i - s^T A z_i + s^T A z_i$$

$$= s^T t_i = s^T A r_i + s^T p$$

$$s^T C \cdot v_i + s^T A \cdot z_i - s^T A \cdot r_i \approx s^T p$$

Aggregated group key:

$$C \in \mathbb{Z}_q^{n \times m}$$

v_i

$$t_1$$

$$t_i = Ar_i + p$$

$$s^T C$$

recode

$$s^T t_i$$

Invariant: $Cv_i = t_i - Az_i$ where v_i, z_i are low-norm

Group Messaging and Broadcast Encryption

[WW25, BBDGW26]

Public parameters: $A \in \mathbb{Z}_q^{n \times m}$, $p \in \mathbb{Z}_q^n$



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Ciphertext:

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To prove security, need to introduce an additional offset (i.e., so the reduction can simulate $s^T C$)

Aggregated group key:

$$C \in \mathbb{Z}_q^{n \times m}$$

v_i

$$t_1$$

$$t_i = Ar_i + p$$

$$s^T C$$

decode

$$s^T t_i$$

Invariant: $Cv_i = t_i - Az_i$ where v_i, z_i are low-norm

Group Messaging and Broadcast Encryption

[WW25, BBDGW26]

Public parameters: $A \in \mathbb{Z}_q^{n \times m}$, $p \in \mathbb{Z}_q^n$



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Ciphertext:

(suppressing error terms)

$$s \leftarrow \mathbb{Z}_q^n$$

$$s^T A, s^T p + \mu \cdot \lfloor q/2 \rfloor, s^T C$$

Overall ciphertext size:
 $|\text{ct}| = \text{poly}(\lambda)$

(independent of group size)

Aggregated group key:

$$C \in \mathbb{Z}_q^{n \times m}$$

v_i

$$t_1$$

$$t_i = Ar_i + p$$

$$s^T C$$

decode

$$s^T t_i$$

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Can augment scheme to support incremental updates and key refreshes $\Rightarrow$ continuous group key-agreement and secure group messaging

Aggregated group key:

$$C \in \mathbb{Z}_q^{n \times m}$$

v_i

$$t_1$$

$$t_i = Ar_i + p$$

$$s^T C$$

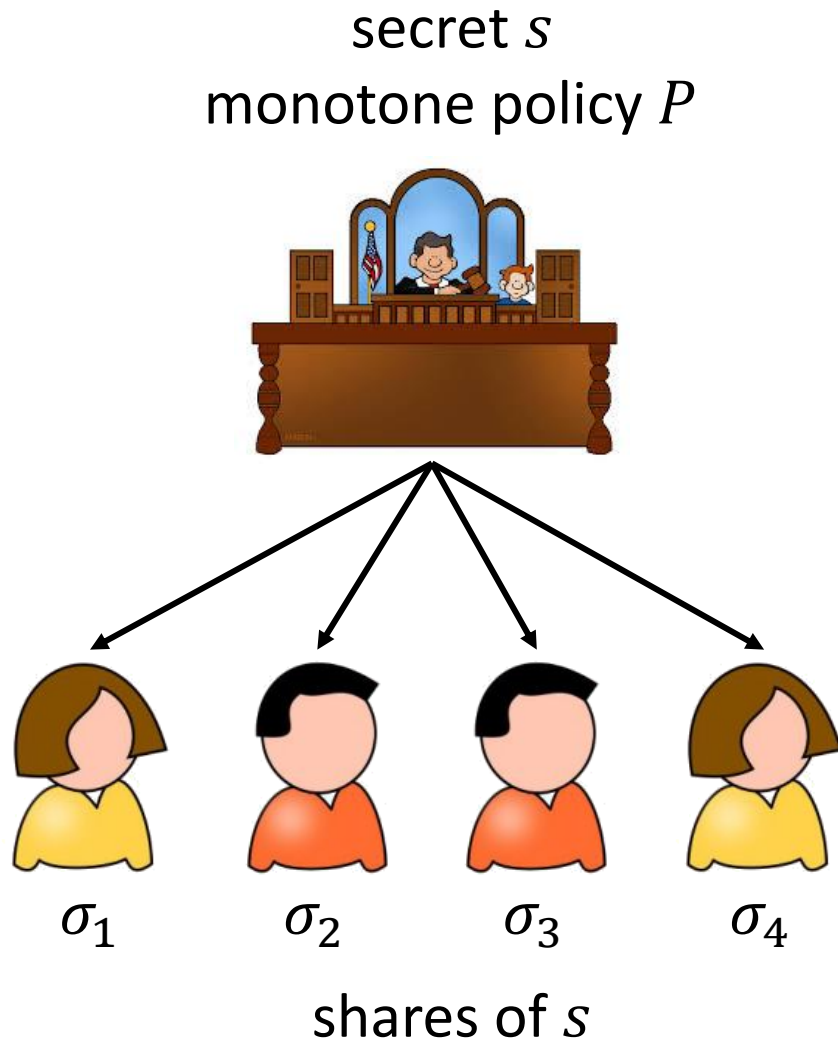
decode

$$s^T t_i$$

Invariant: $Cv_i = t_i - Az_i$ where v_i, z_i are low-norm

Succinct Computational Secret Sharing

[ABIKLV25]



Correctness: any set of users T where $P(T) = 1$ can reconstruct s

Security: shares for users T where $P(T) = 0$ should hide s

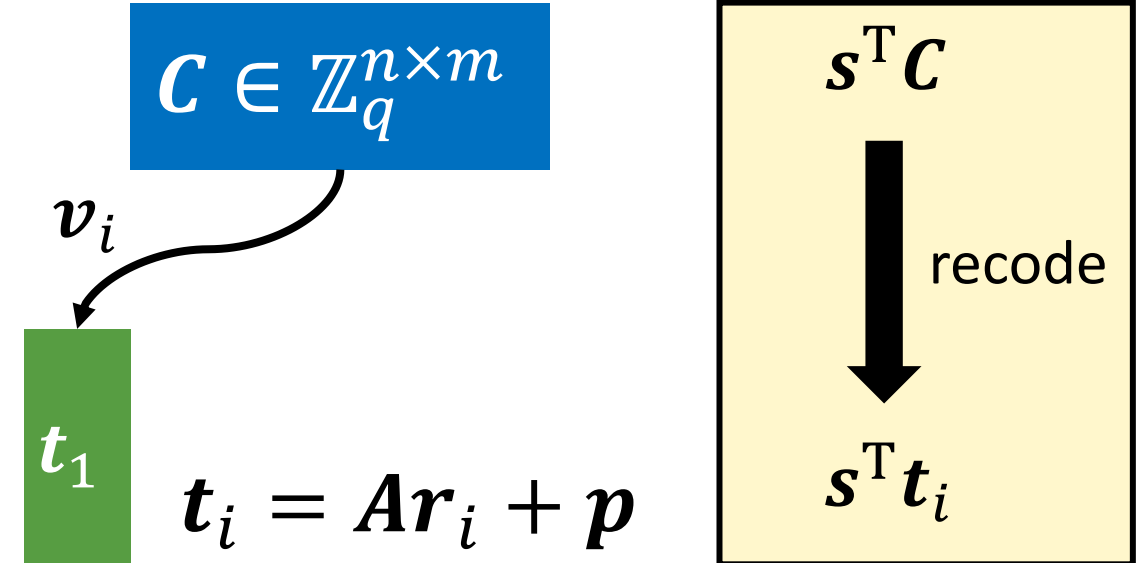
Succinctness: share size σ_i should be polylogarithmic in size of P

Succinct Computational Secret Sharing for DNFs

[CW26]

Recall approach to group messaging:

$$s \leftarrow \mathbb{Z}_q^n \quad s^T A, s^T p + \mu \cdot \lfloor q/2 \rfloor, s^T C$$



r_i is user i 's secret key

Invariant: if party i knows r_i , can recover $s^T p$ and decrypt

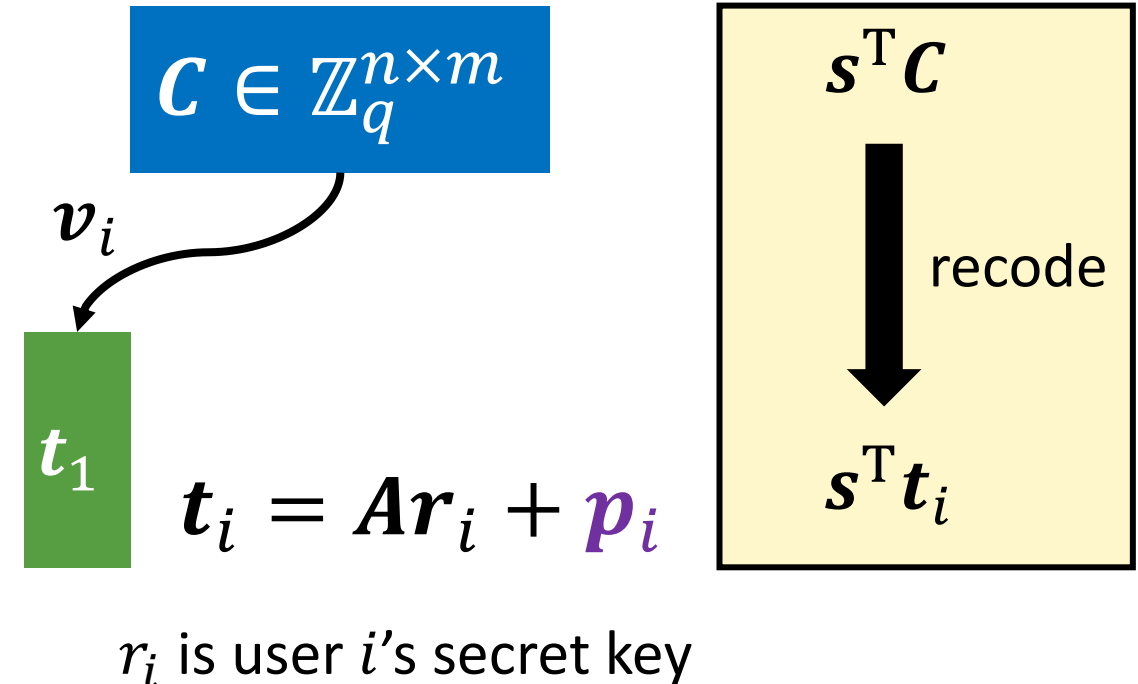
Succinct Computational Secret Sharing for DNFs

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$$s \leftarrow \mathbb{Z}_q^n \quad s^T A, s^T p + \mu \cdot \lfloor q/2 \rfloor, s^T C$$

In more detail: secret share p according to a **non-succinct** linear secret sharing scheme, attach a public key to each share, and commit to the public keys



Invariant: if party i knows r_i , can recover $s^T p$ and decrypt

Secret sharing: if party i knows r_i , can recover $s^T p_i$ where p_i is a **share** of p

Approach: commit to t_i that now recodes to p_i for each user (each party can have multiple t_i)

Conclusion: succinct CSS for DNFs with $\text{poly}(\lambda)$ -size shares

[some details omitted]

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Your favorite application here