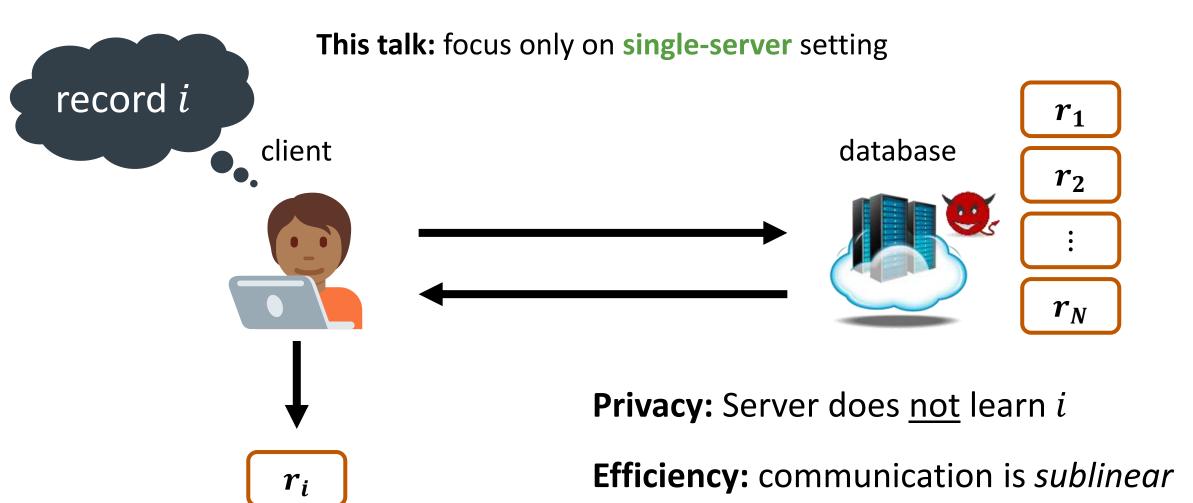
# Private Information Retrieval: Opportunities and Challenges

#### David Wu June 2025

based on joint work with Samir Menon

#### **Private Information Retrieval (PIR)**

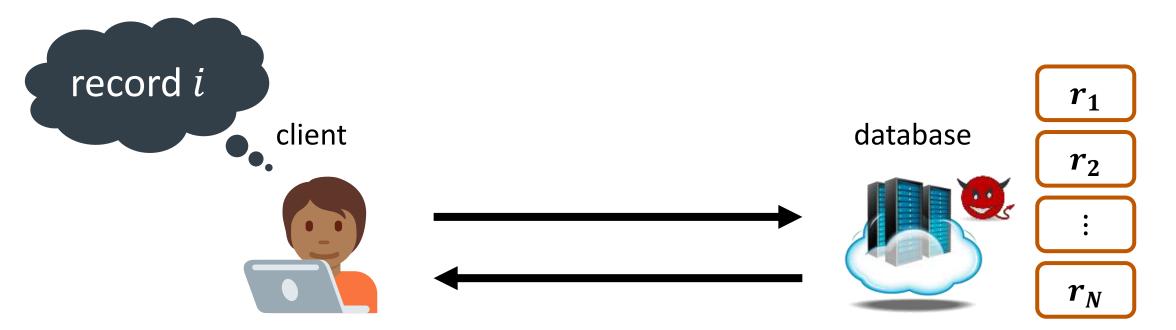
[CGKS95]



in database size (ideally: polylog(N))

**Correctness:** Client learns r<sub>i</sub>

# **Private Information Retrieval (PIR)**



Basic building block in many privacy-preserving protocols

- Metadata-private messaging
- Certificate transparency auditing =
- Private content delivery

- Contact discovery
- Private contact tracing

**Private DNS** 



Reassword breach checking

[CGKS95]



Private navigation

Private web search

# **Application to Certificate Transparency**

[LLK13, Lau14]

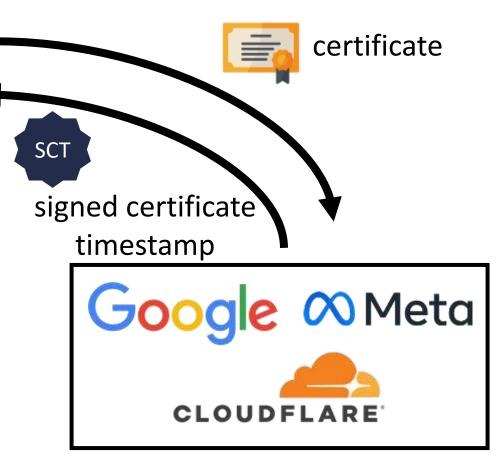


certificate authorities

**Goal:** monitor issuance of certificates and detect rogue certificates

#### Approach:

- When certificate authority (CA) issues certificate, it deposits it into a log server and receives a signed certificate timestamp (SCT)
- Servers can check log server to see all certificates issued for their domain name



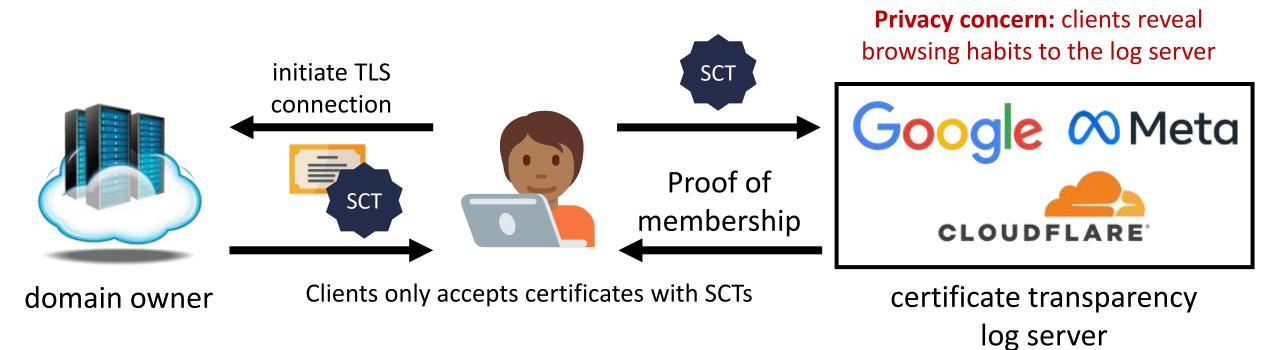
certificate transparency log server

# **Application to Certificate Transparency**

[LLK13, Lau14]

A valid SCT means that the certificate was deposited into a log server

But is the log server honest? Clients will periodically audit log server to check that SCT is actually present



**Goal:** monitor issuance of certificates and detect rogue certificates

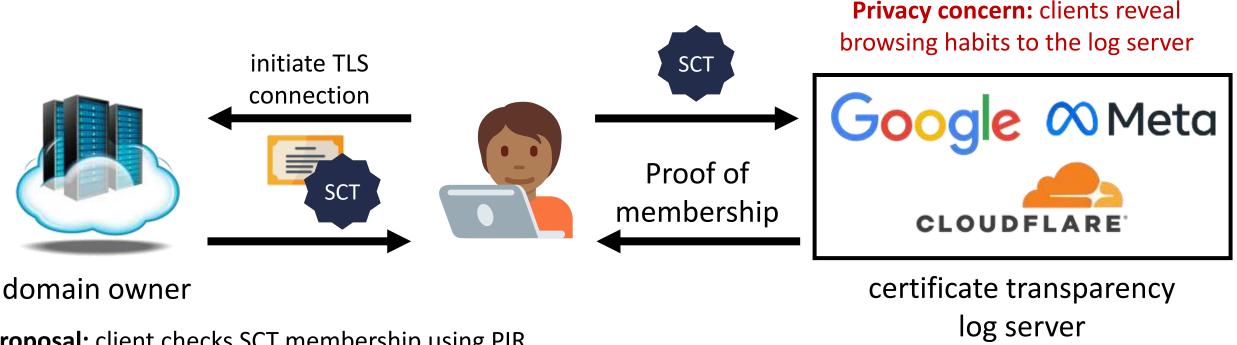
**Today:** Chrome reveals a few bits of the hash of the SCT (*k*-anonymity-based solution)

#### **PIR for Certificate Transparency**

[LG15, KOR19, HHCMV23]

A valid SCT means that the certificate was deposited into a log server

But is the log server honest? Clients will periodically **audit** log server to check that SCT is actually present



**Proposal:** client checks SCT membership using PIR

[HHCMV23]: Server maintains a Bloom filter of the set of SCTs deposited

To audit, client reads one bit of the Bloom filter (high false positive rate, but handled by relying on many clients)

#### **How Efficient is PIR?**

#### **On the Computational Practicality of Private Information Retrieval**

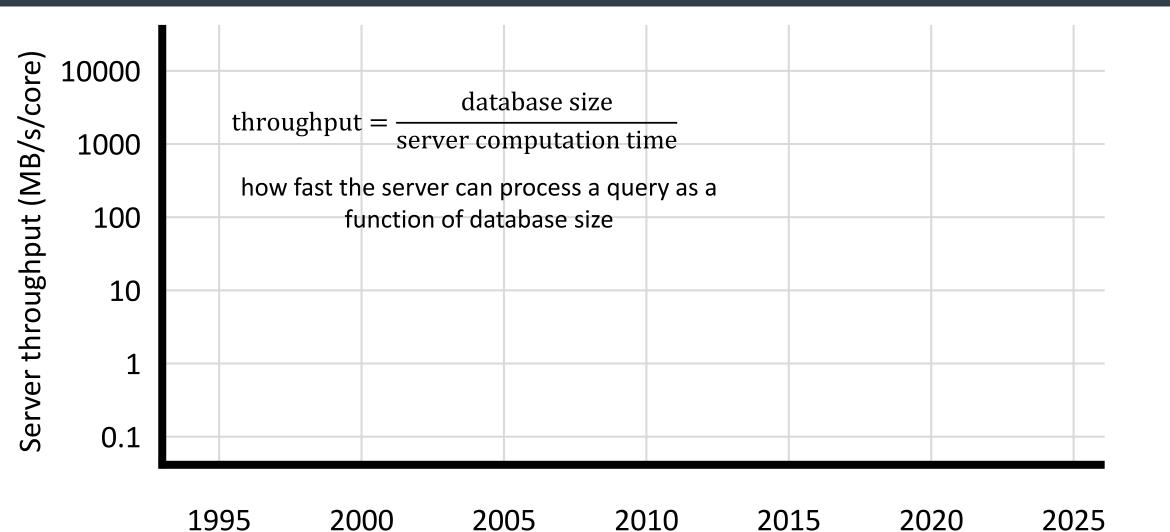
Radu Sion \* Network Security and Applied Cryptography Lab Computer Sciences, Stony Brook University sion@cs.stonybrook.edu Bogdan Carbunar Pervasive Platforms and Architectures Motorola Labs carbunar@motorola.com

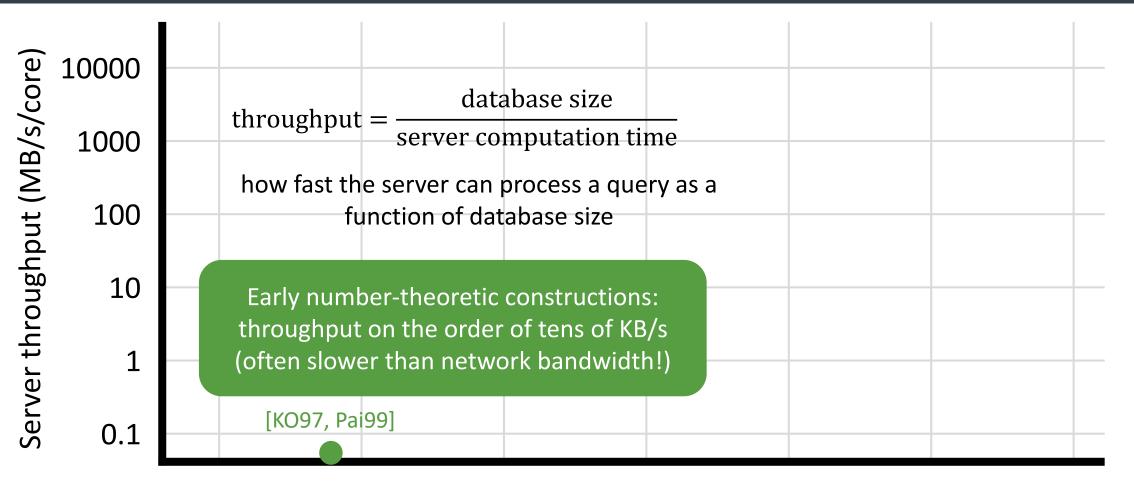
#### Abstract

We explore the limits of single-server computational private information retrieval (PIR) for the purpose of preserving client access patterns leakage. We show that deployment of non-trivial single server PIR protocols on real hardware of the recent past would have been orders of magnitude less time-efficient than trivially transferring the entire database. We stress that these results are beyond existing knowledge of mere "impracticality" under unfavorable assumptions. They rather reflect an inherent limitation with respect to modern hardware, likely the result of a communication-cost centric protocol design. We argue that this is likely to hold on non-specialized traditional hardware in the foreseeable future. We validate our reasoning in an experimental setup on modern off-the-shelf hardware. Ultimately, we hope our results will stimulate practical designs. Here we discuss single-server computational PIR for the purpose of preserving client access patterns leakage. We show that deployment of non-trivial single server private information retrieval protocols on real hardware of the recent past would have been orders of magnitude more time-consuming than trivially transferring the entire database. The deployment of computational PIR would in fact *increase* overall execution time, as well as the probability of *forward* leakage, when the deployed present trapdoors become eventually vulnerable – e.g., today's queries will be revealed once factoring of today's values will become possible in the future.

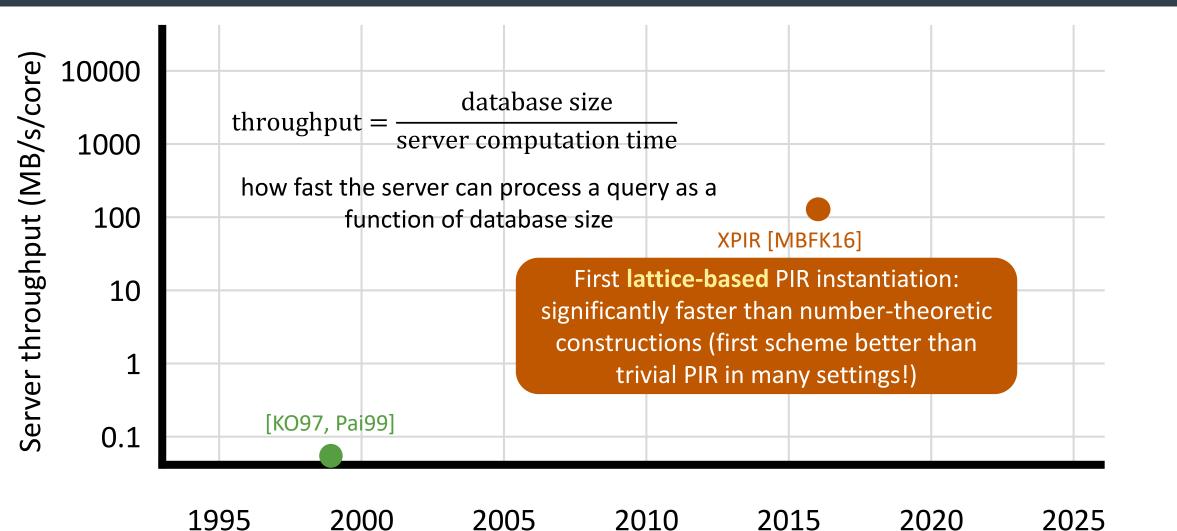
We stress that this is beyond existing knowledge of mere "impracticality" under unfavorable assumptions. On real hardware, *no* existing non-trivial single server PIR protocol could have possibly had outperformed the trivial client-toserver transfer of records in the past, and is likely not to do so in the future either. This is due to the fact that on any Take-away (2007): PIR schemes are too expensive and better to just have client download the database; need new constructions

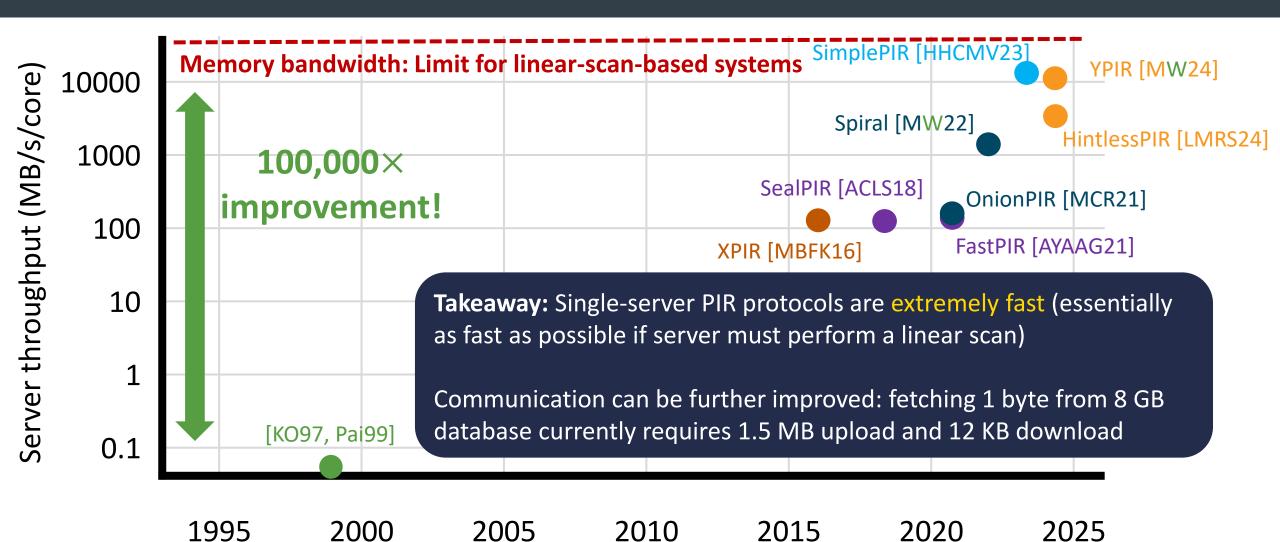
Recurring theme in cryptography: powerful tools, but often (concretely) expensive





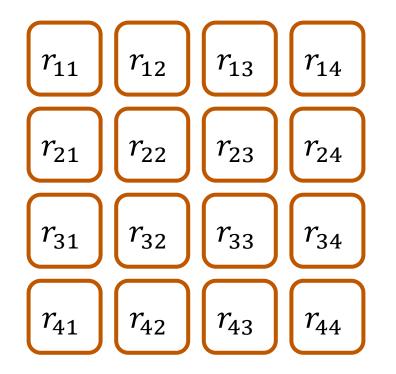
1995 2000 2005 2010 2015 2020 2025







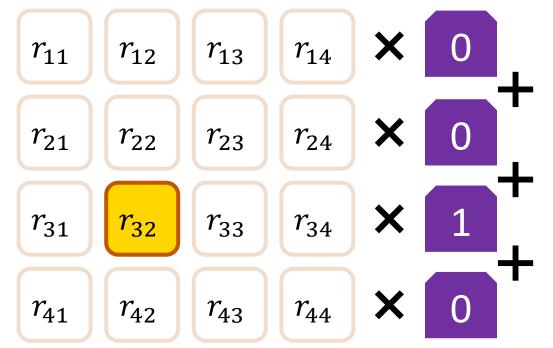
#### **Starting point:** a $\sqrt{N}$ construction (N = number of records)



Arrange the database as a  $\sqrt{N}$ -by- $\sqrt{N}$  matrix



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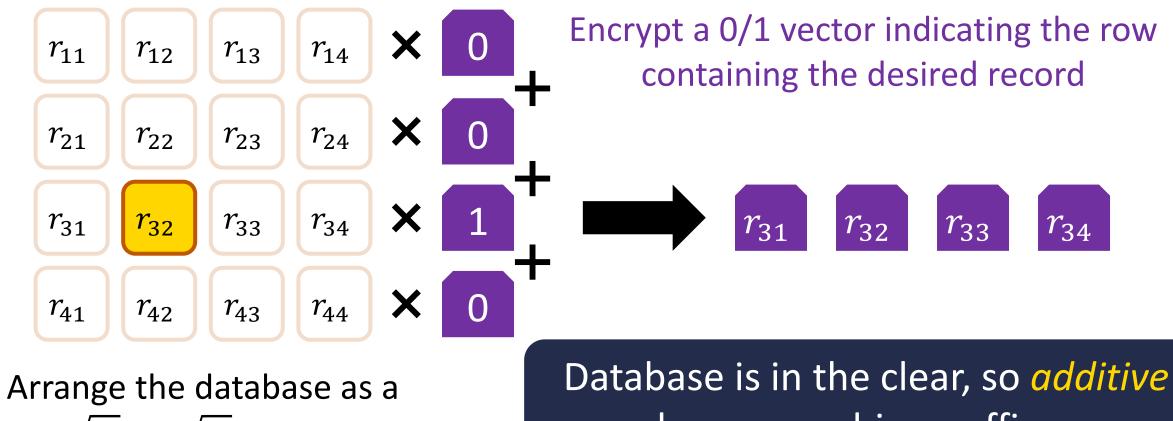
Encrypt a 0/1 vector indicating the row containing the desired record

Arrange the database as a  $\sqrt{N}$ -by- $\sqrt{N}$  matrix

*Homomorphically* compute product between query vector and database matrix



#### **Starting point:** a $\sqrt{N}$ construction (N = number of records)



 $\sqrt{N}$ -by- $\sqrt{N}$  matrix

# homomorphism suffices

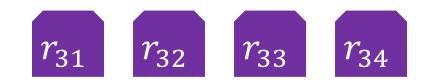


#### **Starting point:** a $\sqrt{N}$ construction (N = number of records)

Client decrypts to learn records

Encrypt a 0/1 vector indicating the row containing the desired record

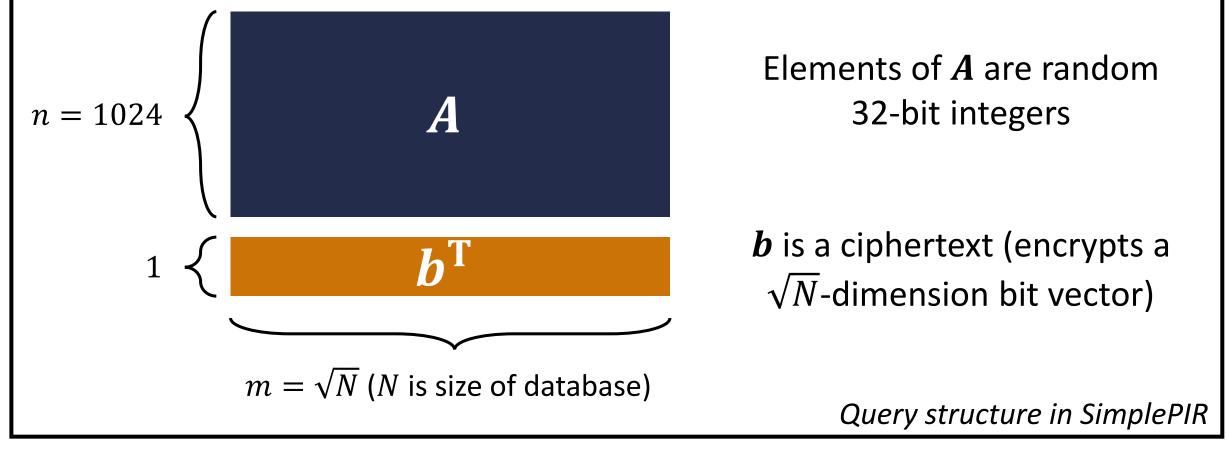




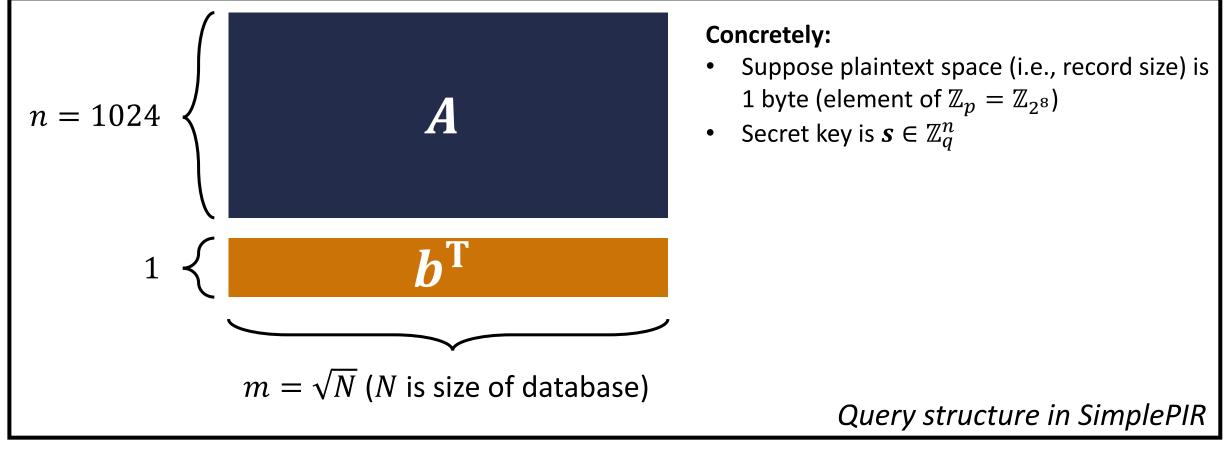
**Response size:**  $O_{\lambda}(\sqrt{N})$ 

*Homomorphically* compute product between query vector and database matrix

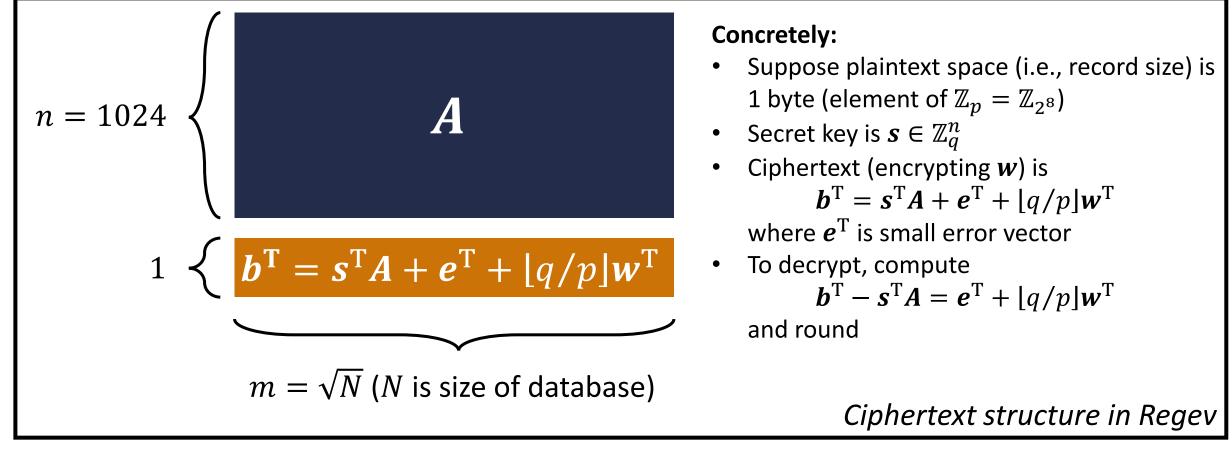
**Building block:** Regev's linearly homomorphic encryption from LWE [Reg05]



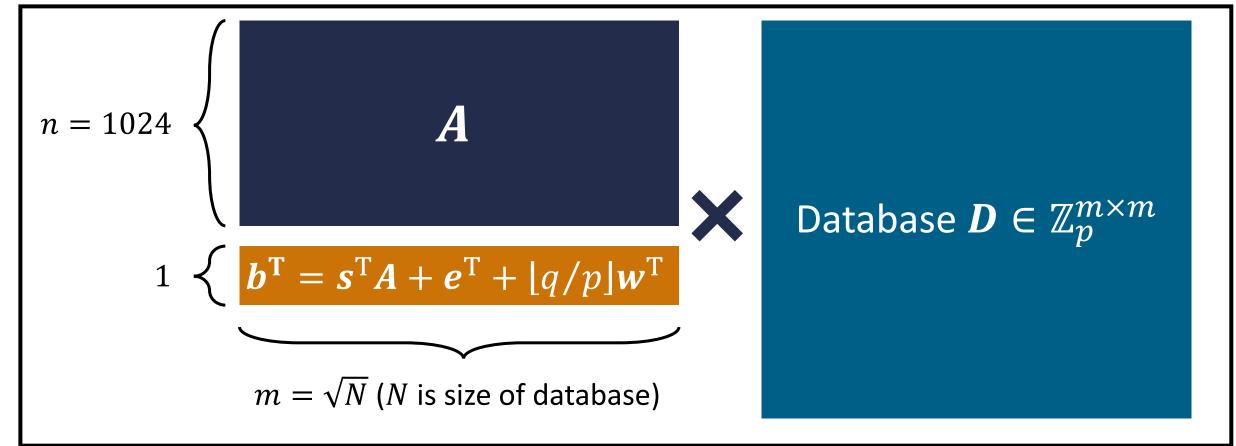
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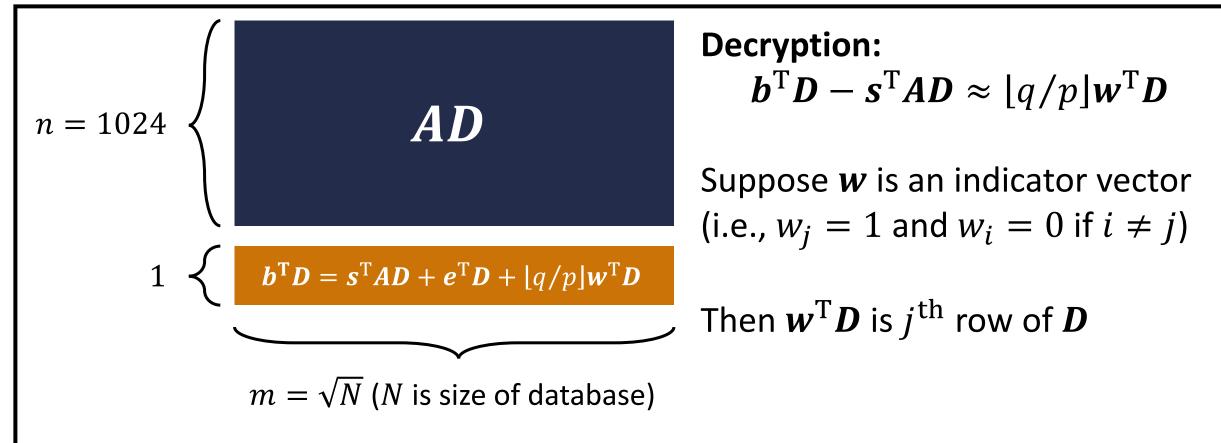
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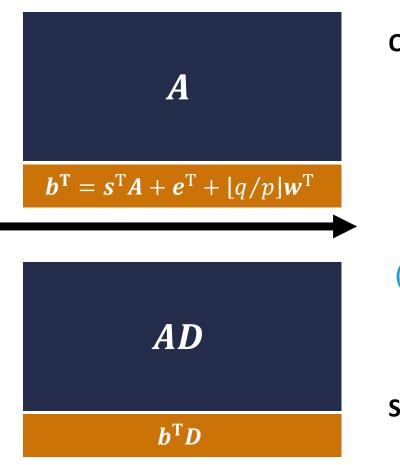


#### [HHCMV23]

#### To recover record in row *j*:



Response size:  $(n+1)\sqrt{N}$  elements over  $\mathbb{Z}_q$ 



$$q = 2^{32}$$
,  $p = 2^8$ ,  $n = 2^{10}$ 

Query size:  $(n+1)\sqrt{N}$  elements over  $\mathbb{Z}_q$ 

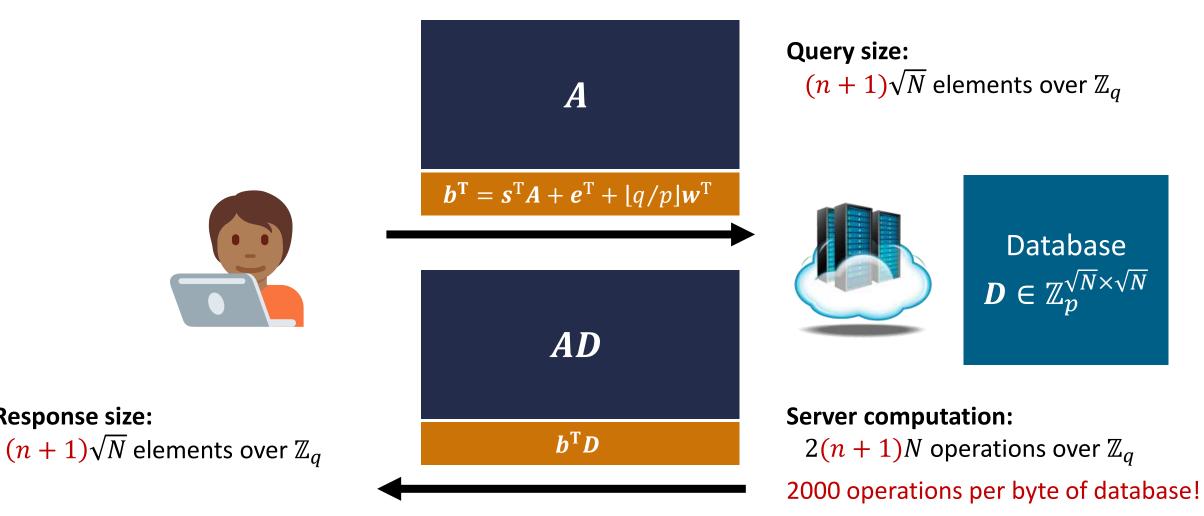


#### Server computation: 2(n + 1)N operations over $\mathbb{Z}_q$ 2000 operations per byte of database!

[HHCMV23]

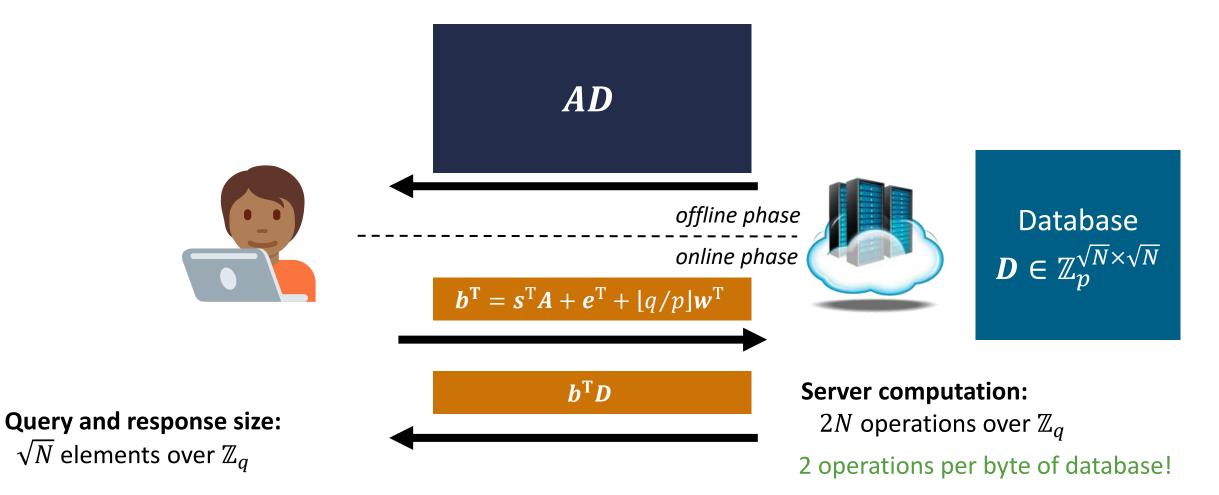
Key insight in SimplePIR: A is query-independent so move computation of AD offline

**Response size:** 



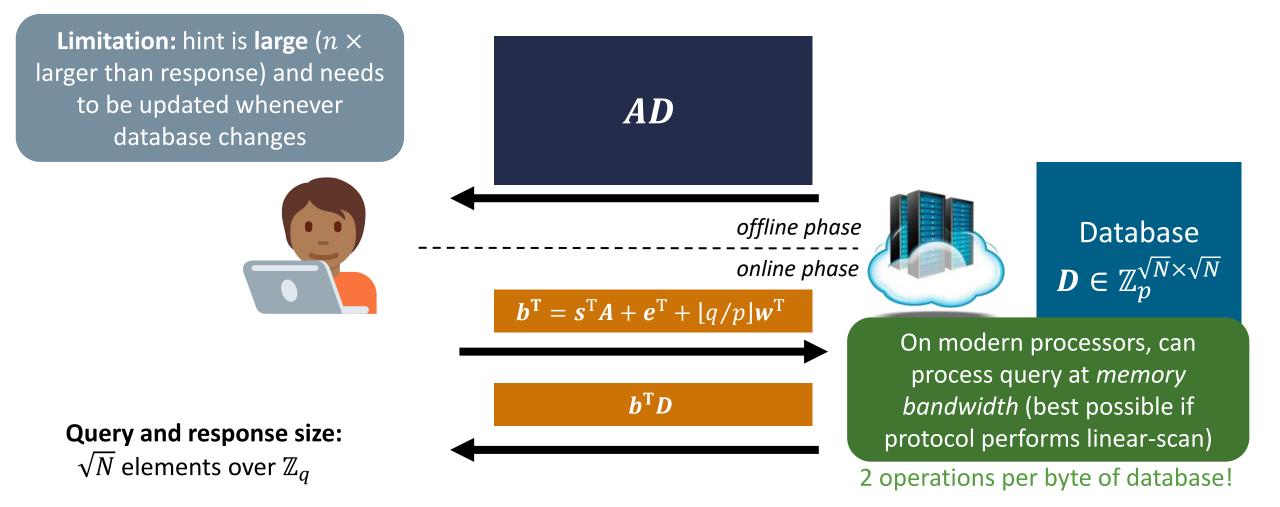
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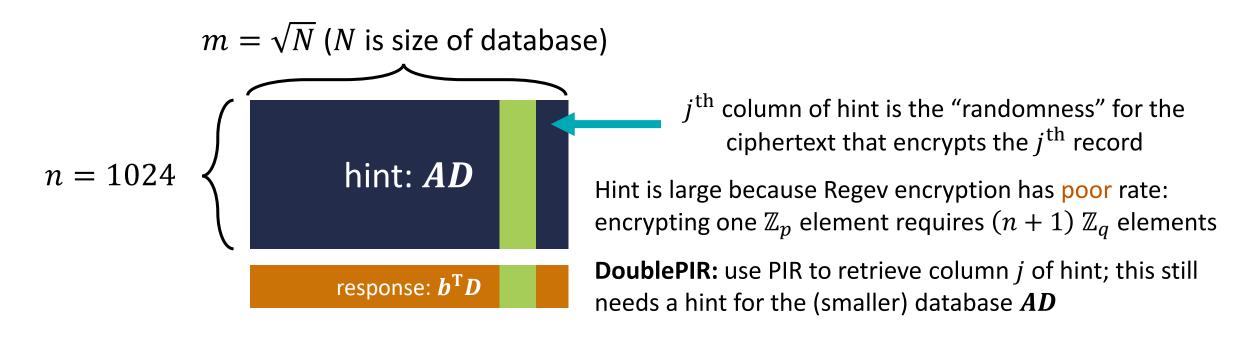


#### PIR with *Silent* Preprocessing

#### Silent preprocessing: allow offline preprocessing, but no communication

No need to manage hints; better suited for dynamic databases

**Approach:** *compress* the hint and include as part of the response [HintlessPIR; LMRS24] [YPIR; MW24] Why is the SimplePIR hint so large?



#### SimplePIR using Polynomial Rings

Common technique to get better rate: use polynomial rings

Plaintext space: 
$$\mathbb{Z}_p \longrightarrow R_p = \mathbb{Z}_p[x]/(x^d + 1)$$

Ciphertext space:

degree-d polynomials with coefficients in 
$$\mathbb{Z}_p$$

$$\mathbb{Z}_q^{n+1} \longrightarrow R_q^2 = \left( \mathbb{Z}_q[x] / (x^d + 1) \right)^2$$

security relies on ring learning with errors (RLWE)

rate = 
$$\frac{\text{size of plaintext}}{\text{size of ciphertext}} = \frac{\log p}{(n+1)\log q} \longrightarrow \frac{\log p}{2\log q}$$

rings allow packing more data into each ciphertext

Recall  $n = 2^{10} = 1024$ , so using rings gives over  $500 \times$  reduction in size

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SimplePIR over polynomial rings incurs  $\approx \log q / \log p \approx 4$  overhead (due to blow-up in size of database representation)

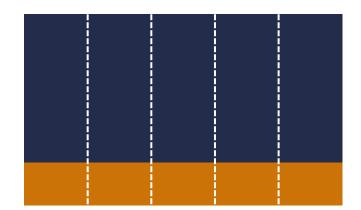
$$-\frac{\log p}{2\log q}$$

rings allow packing more data into each ciphertext

Recall  $n = 2^{10} = 1024$ , so using rings gives over  $500 \times$  reduction in size



YPIR approach: pack LWE ciphertexts into an RLWE ciphertext



Each column is an LWE ciphertext encrypting a record  $z_i \in \mathbb{Z}_p$ 

**Limitation:** LWE ciphertext has **poor** rate

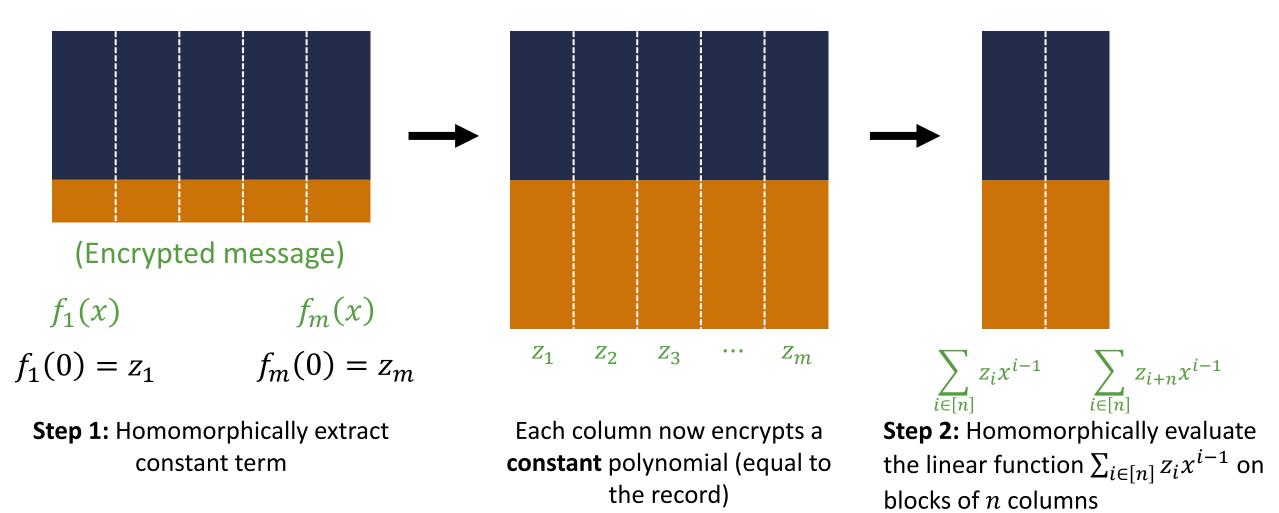
**Observation:** when A is a structured matrix then each column is also an RLWE ciphertext that encrypts a polynomial whose **constant** coefficient is  $z_i$ 

For example,  $i^{\text{th}}$  column might decrypt to the polynomial  $z_i + r_1 x + r_2 x^2 + \dots + r_{n-1} x^{n-1}$ where  $r_1, \dots, r_{n-1}$  are (arbitrary) coefficients and n = 1024

**Observation [CDKS21]:** When  $n = 2^d$ , there is a simple homomorphic procedure that maps encryption of polynomial f to an encryption of f(0)

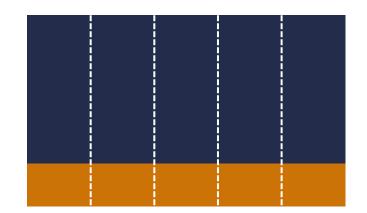


YPIR approach: replace nested encryption with an LWE-to-RLWE packing technique



[MW24]

#### **YPIR approach:** replace nested encryption with an LWE-to-RLWE packing technique



(Encrypted message)

 $f_1(x) \qquad f_m(x)$  $f_1(0) = z_1 \qquad f_m(0) = z_m$ 

**Step 1:** Homomorphically extract constant term

SimplePIR response size: (n + 1)m elements of  $\mathbb{Z}_q$ Packed RLWE response size:  $\frac{m}{n} \cdot 2n = 2m$  elements of  $\mathbb{Z}_q$ 

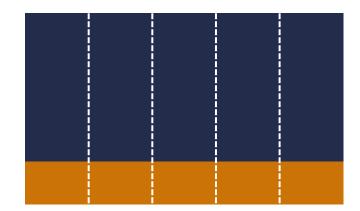
**Recall:**  $n \ge 2^{10}$ , so substantial reduction in practice

Increases server compute, but observe that packing is applied to the SimplePIR **response**, which has size  $O_{\lambda}(\sqrt{N})$   $\sum a a^{i-1}$ 

$$\sum_{i \in [n]}^{Z_i x^{i-1}} \sum_{i \in [n]}^{Z_{i+n} x^{i-1}}$$
  
Step 2: Homomorphically evaluate  
the linear function  $\sum_{i \in [n]} z_i x^{i-1}$  on  
blocks of *n* columns



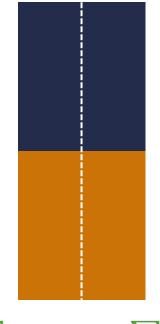
#### **YPIR approach:** replace nested encryption with an LWE-to-RLWE packing technique



If we apply packing to DoublePIR (SimplePIR with 1 recursive step), then packing is applied to  $O_{\lambda}(1)$  size response SimplePIR response size: (n + 1)m elements of  $\mathbb{Z}_q$ Packed RLWE response size:  $\frac{m}{n} \cdot 2n = 2m$  elements of  $\mathbb{Z}_q$ 

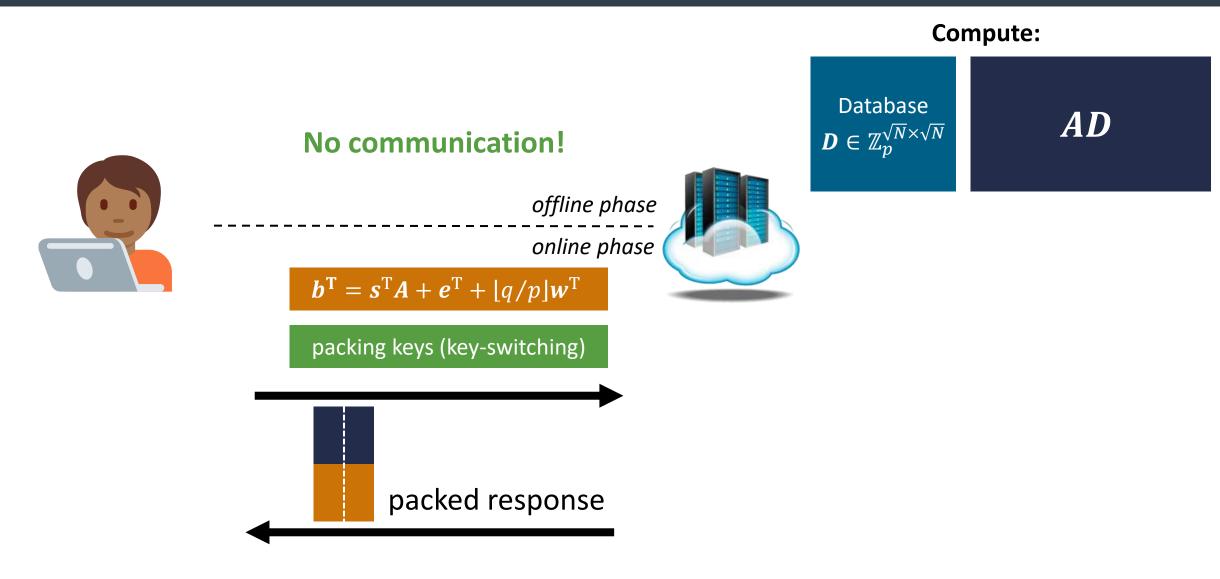
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 $\sum_{i=1}^{n} z_i x^{i-1} \qquad \sum_{i=1}^{n} z_{i+n} x^{i-1}$ 

**Step 2:** Homomorphically evaluate the linear function  $\sum_{i \in [n]} z_i x^{i-1}$  on blocks of n columns



### **PIR for Certificate Transparency Auditing**

**Setup:** 5 billion SCTs, encoded as Bloom filter with 2<sup>36</sup> bits (8 GB)

SCT audit consists of a single PIR query

[HHCMV23] uses DoublePIR

		DoublePIR	HintlessPIR	YPIR
Offline	Download	14 MB	_	_
Online (	Upload	960 KB	1.4 MB	1.5 MB
	Download	12 KB	1.7 MB	12 KB
	Throughput	12.5 GB/s	4.9 GB/s	11.6 GB/s

1.6× larger queries and 93% of the throughput of fastest scheme

### **PIR for Certificate Transparency Auditing**

#### **Setup:** 5 billion SCTs, encoded as Bloom filter with 2<sup>36</sup> bits (8 GB)

SCT audit consists of a single PIR query

Assuming each client performs 20 SCT audits each week (based on client making  $10^4$  TLS connections and auditing 1/500 fraction of connections) – achieves detection rate of 1/1000 (Chrome's current approach)

Weekly server costs based on AWS cost model (free inbound communication) to support 1000 clients:

		DoublePIR	DoublePIR	HintlessPIR	YPIR	
_	SCT Update Frequency	Weekly	Daily	Daily	Daily	_
Server Costs	Communication	\$1.25	\$8.63	\$3.22	\$0.02	
	Computation	\$0.19	\$0.25	\$0.49	\$0.21	
	Total	\$1.44	\$8.88	\$3.71	\$0.23	84% cheaper!

### **PIR for Certificate Transparency Auditing**

#### **Setup:** 5 billion SCTs, encoded as Bloom filter with 2<sup>36</sup> bits (8 GB)

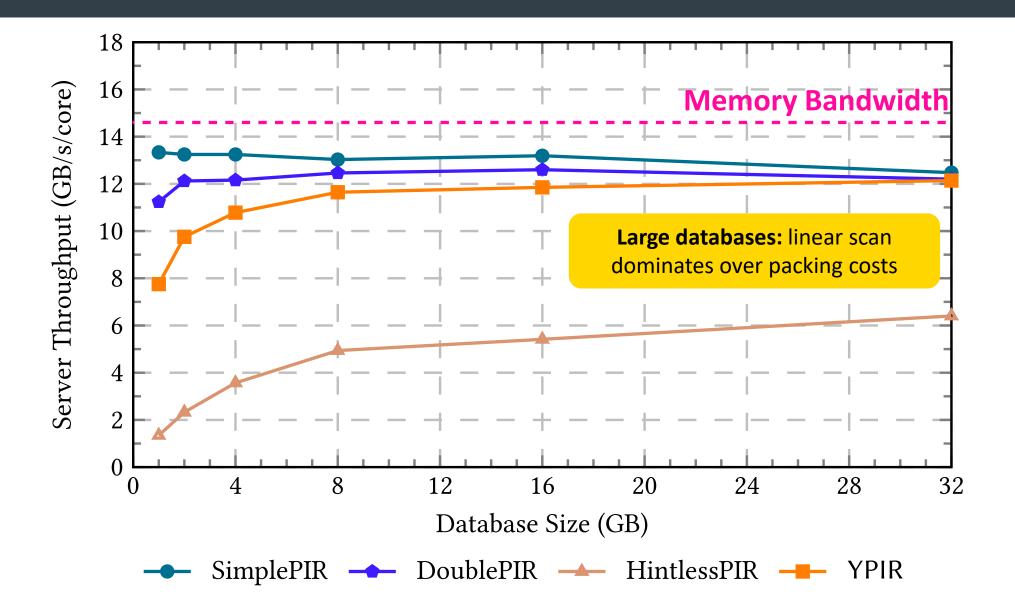
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		DoublePIR	DoublePIR	HintlessPIR	YPIR	
Server Costs	SCT Update Frequency	Weekly	Daily	Daily	Daily	_
	Communication	<i>k</i> -anonymity (Chrome): 2.3 MB total communication per client per week		\$0.02		
	Computation				\$0.21	
	Total		YPIR: 29 MB total communication per client per week (12.6× higher)		\$0.23	84% cheaper!

#### **PIR Throughput**



#### **Retrieving Larger Records**

#### Application: Password breach monitoring

Consider database with SHA-256 hashes of 250 million compromised passwords

Database size is 8 GB, partition into 250,000 records of size 32 KB

Technically, this is keyword PIR, but can be reduced to PIR via hashing

		SimplePIR	HintlessPIR	YPIR
Offline	Download	362 MB		
Online (	Upload	362 KB	1.4 MB	1.3 MB
	Download	362 KB	1.7 MB	228 KB
	Throughput	11 GB/s	5 GB/s	5 GB/s

Extra computational overhead from packing transformation

# **Recent Improvements (for Private SCT Auditing)**

**Distributional PIR** [LHC25]: Better performance if we have prior information on query distribution (e.g., private SCT auditing)

- Distributional PIR + YPIR for private SCT auditing: reduces computational costs by 12× and communication by 3×
- **Cryptographic privacy** at 4× communication cost over Chrome's *k*-anonymity-based approach

**System architecture for private SCT auditing** [HPW25]: Consider better data structures and system architecture to support private SCT auditing – "seem to bring wide SCT auditing to the brink of practicality"

Existing single-server PIR protocols are fast enough to support some privacy-preserving applications at scale

Recent progress: Apple's use of PIR for private caller ID lookup and private image search

#### **Recent Developments in PIR**

The bottleneck for linear-scan-based PIR is the memory bandwidth, and recent schemes essentially hit this limit – will **not** cut it when database is 100 GB or 1 TB or even larger

Piano [ZPSZ23], QuarterPIR [GZS24], [RMS24], Plinko [HPPY25], [WR25]

Sublinear server computational costs (can scale better to databases that are >100 GB) Preprocessing phase requires *streaming* the entire database (and client storing some state) Can avoid streaming the database in the two-server model (but rely on non-collusion assumption)

# **Doubly-Efficient PIR**

[CHR17, BIPW17, LMW23]

- Server performs one-time encoding of the database
- In online phase, server can then answer queries by reading polylog(N) bits of the encoded value (no client-specific state needed)

Implication: private data access is essentially free (in an asymptotic sense)

#### **Communication:**

```
Without privacy: \log N bits
```

With privacy:  $\log N + \tilde{O}(\lambda)$  bits (where  $\lambda$  is a security parameter)

[BV11]

**Computation:** 

```
Without privacy: 1 probe
```

```
With privacy: poly(\lambda, \log N) probes
```

[LMW23]

# **Doubly-Efficient PIR**

[CHR17, BIPW17, LMW23]

- Server performs one-time encoding of the database
- In online phase, server can then answer queries by reading polylog(N) bits of the encoded value (no client-specific state needed)

#### Limitation: Still far from practical

#### For 350 MB database [OPPW24]:

- Size of server encoding is 412 PB (2<sup>59</sup> bytes)
- Encoding time: > 8 million core-years (> \$4 billion)
- Communication: 573 MB (larger than database)
- Computation: 4.3 core-weeks (\$39/query)

**Open problem:** Practical doubly-efficient PIR (interesting even in multi-server setting)

#### **Open Problems**

Reduce concrete communication costs of PIR with silent preprocessing

Current approaches all have  $O(\sqrt{N})$  communication – can we get to polylog(N) with similar (concrete) computational overhead

Practical doubly-efficient PIR (single-server *or* multi-server)

What will it take to deploy PIR in practical systems?

Recent progress: Apple's use of PIR for private caller ID lookup and private image search

#### Thank you!