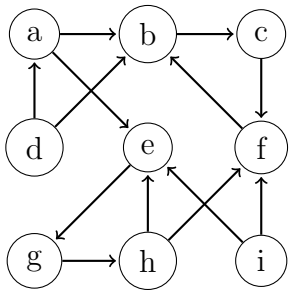
Problem Set 6

CS 331

Due Wednesday, March 23

[Note: the first two problems are short-answer only.]

1. Consider running DFSAll on the following graph, where every iteration (the outer one over all vertices, and the inner ones over edges) is in alphabetical order by vertex name:



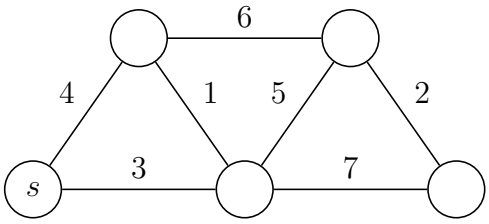
   (a) What is the preorder of the vertices? What is the postorder?

   (b) Shade in the "tree" edges.

   (c) Circle the strongly connected components.

   (d) Give a topological ordering of the strong component graph.

2. Consider minimum spanning tree algorithms for the following graph:



   (a) In what order would Prim's algorithm, starting at s, add edges to the minimum spanning tree? Give the sequence of edge weights, in order.

   (b) In what order would Kruskal's algorithm add edges to the minimum spanning tree? Give the sequence of edge weights, in order.

   (c) In what order would Boruvka's algorithm add edges to the minimum spanning tree? Give the set of edge weights added in the first round, the second round, etc.

3. You are building out internet for a collection of rural houses. For each house, you need to either purchase satellite internet at that house, or connect it via a series of fiber links to a house that has purchased satellite internet.

   There are n houses, and buying satellite internet costs P dollars at any house. There are m pairs of houses that can be directly connected by fiber; this is given as a list of triples (u_i, v_i, c_i), denoting that houses u_i and v_i can be connected at a cost of c_i dollars.

   Give an $O(m \log n)$ time algorithm to determine the minimum cost of hooking everyone up to internet.