Homework 3

CS 331

Due Thursday, February 8

1. [Book exercise 3.37] You have mined a large slab of marble from a quarry. For simplicity, suppose the marble slab is a rectangle measuring \( m \) inches in height and \( n \) inches in width. You want to cut the slab into smaller rectangles of various sizes—some for kitchen counter tops, some for large sculpture projects, others for memorial headstones. You have a marble saw that can make either horizontal or vertical cuts across any rectangular slab. (Note: when the saw is used, it must cut all the way through the piece it is used on.) At any time, you can query the spot price \( P[x, y] \) of an \( x \)-inch by \( y \)-inch marble rectangle, for any positive integers \( x \) and \( y \). These prices depend on customer demand, and people who buy marble counter tops are weird, so don’t make any assumptions about them; in particular, larger rectangles may have significantly smaller spot prices. You may assume \( P[x, y] = P[y, x] \). Given the array of spot prices and the integers \( m \) and \( n \) as input, describe a dynamic programming algorithm to compute how to subdivide an \( m \times n \) marble slab to maximize your profit.

Note that, to present a dynamic programming algorithm, you should give:

- A description of the subproblems you solve, in an English sentence or two. (“\( f(i) \) is 1 if \( S[j : i] \) can be segmented into words and 0 otherwise.”)
- A mathematical description of the recurrence involved. (“Base case: \( f(0) = 0 \). Recurrence: \( f(i) = 1 \) if \( \exists j < i \) with \( f(j) = 1 \) and \( S[j : i] \) is a word.”)
- How to compute the final answer using this recurrence (“Answer is \( f(n) \).”)
- A description of how to solve all the subproblems (for example, if you build a table, in what order do you fill it in?), and analysis of the runtime.

2. There’s a Jupyter Notebook linked from the class webpage.