Problem Set 6

CS 331H

Due Monday, March 25

1. You have n power plants which, being fancy modern renewable energy designs, do not work all the time (they depend on sun, wind, tides, etc). Each power plant runs from a start time s_i to a finish time f_i, and costs c_i to run (you either run it the whole time or no time). You would like to find a set of power plants to run such that you have power over the entire interval $[0, T]$. What is the minimum cost achievable?

   You may suppose that the s_i and f_i are integers between 0 and $T = O(n)$.

   (a) Observe that the answer corresponds to the shortest path on an appropriate graph, which can be solved in $O(n \log n)$ time using Dijkstra's algorithm. [You may have already done something very similar on a previous problem set.]

   (b) Now suppose that you can sell off extra power if you have more than one power plant running at a time. At each time step $t \in [T]$, each power plant beyond the first that you run gives you value $v_t \geq 0$, decreasing your costs. Show how to modify the part (a) graph to handle this case. Your new graph is likely to have negative edge weights.

   (c) Suppose that the cost c_i to run a power plant is larger than the value of the electricity it produces, $\sum_{t=s_i}^{f_i} v_t$. Show that Dijkstra's algorithm can still be used to find the solution to the part (b) graph in $O(n \log n)$ time, by constructing an appropriate potential function so the edge costs become nonnegative.

   (d) [Optional] Now solve the previous part without assuming that the cost to run a power plant is larger than the value of the electricity it produces.

2. Given an undirected graph with positive edge weights, a source s, and a sink t, find the shortest path from s to t and back to s that uses each edge at most once. Aim for $O(m + n \log n)$ time.

   Hints: Look for an "augmenting path," inspired by Ford-Fulkerson but slightly different. And to get the desired runtime, you may need to use a potential function.