Problem Set 7

CS 331H

Due Thursday, April 6

1. [Problem 372 of Brian Dean's book.] Given an undirected, unweighted graph, we would like to compute the subgraph of maximum edge density. The *edge density* of a subgraph is the number of edges divided by the number of vertices.

   Consider the following construction. For a given "guess" λ, construct a dummy source s and sink t. Draw an edge from s to each graph node u of capacity m; one from each graph node u to t of capacity $m + 2\lambda - d_u$, where d_u is the degree of u in the original graph; and give each edge (u, v) in the original graph capacity 1.

   (a) For a nonempty set S of vertices in the original graph, express the cost of cutting $S \cup \{s\}$ from the rest of the graph, in terms of the number of edges fully contained in S and the degrees in S.

   (b) Show that this value is less than mn if, and only if, the edge density of S is more than λ.

   (c) Show how a max-flow algorithm and binary search can narrow down on the maximum edge density of any subgraph. Show that after $O(\log n)$ steps of binary search, you can compute the maximum edge density exactly.

   (d) Show how to compute the set S^* of maximum edge density, not just its value.

2. Work through the Jupyter notebook on the website.