Problem Set 9

CS 331H

Due Monday, April 22

1. Show NP-completeness for each of the following problems. They are all simple, direct reductions from one of the problems we have shown to be NP-complete in class.

   (a) **Minimum set cover**. You are given a set S, a collection of subsets $S_1, \ldots, S_n \subseteq S$, and an integer k. Do there exist a set of k subsets $T \subseteq [n]$ such that

   $$\bigcup_{i \in T} S_i = S?$$

   Hint, encoded as ROT-13: `iregrkpbire`.

   (b) **Subgraph Isomorphism**. You are given two graphs, G and H. Does $G = (V_G, E_G)$ contain a subgraph isomorphic to $H = (V_H, E_H)$? That is, is there an injection $f : V_H \to V_G$ such that for every $u, v \in V_H$, $(u, v) \in E_H$ if, and only if, $(f(u), f(v)) \in E_G$?

   (c) **Partition**. You are given a set of n positive integers $x_1, \ldots, x_n \in \mathbb{Z}^+$. Does there exist a subset $S \subseteq [n]$ such that

   $$\sum_{i \in S} x_i = \sum_{i \in [n] \setminus S} x_i?$$

   Hint, encoded as ROT-13: `fhofrgfhz`. Extra hint: `lbh jvyy jnag gb nqq n fvatyr rkgen vagrtre gb gur vachg`.

2. Consider an interval scheduling problem where we have multiple machines, and each interval can specify which machines it can run on. That is, you have n jobs, and each job j is described by (s_j, f_j, M_j) where $M_j \subseteq [m]$ is a subset of machines, and s_j, f_j are integers describing the start and end time of the job. You would like to schedule each job on at most one machine in its subset, and the set of jobs scheduled on each machine must have disjoint intervals.

   The multiple-machine interval scheduling problem asks for the maximum number of jobs that can be scheduled.

   (a) For any graph $G = (V, E)$, consider the following instance of multiple machine scheduling. Create one machine for each vertex $v \in V$. Create $|E| + |V|$ jobs: for the ith edge $e_i = (u_i, v_i)$ create the job $(i, i+1, \{u_i, v_i\})$, and for each vertex u create the job $(1, |E| + 1, \{u\})$.

   Let k be the size of the maximum independent set in G. Show that the maximum number of jobs that can be scheduled is exactly $|E| + k$.

   (b) Show that the multiple-machine interval scheduling problem is NP-complete.