1. Suppose we are given a sequence of $n$ linear inequalities of the form

$$a_i x + b_i y \leq c_i.$$ 

Collectively, these $n$ inequalities describe a convex polygon $P$ in the plane.

(a) Describe a linear program whose solution describes the largest axis-aligned square that lies entirely inside $P$.

(“Axis-aligned” means that the edges are horizontal and vertical.)

(b) Describe a linear program whose solution describes the maximum-perimeter axis-aligned rectangle that lies entirely inside $P$. 


(c) Describe a linear program whose solution describes the largest circle that lies entirely inside $P$.

(d) Describe a polynomial-time algorithm to compute two interior-disjoint axis-aligned squares with maximum total perimeter that lie entirely inside $P$. [Hint: There are exactly two interesting cases to consider; for each case, formulate a corresponding linear program.]