Problem Set 7

Randomized Algorithms

Due Friday, October 18

1. [Karger] Suppose you are given a graph whose edge lengths are all integers in the range from 0 to B. Suppose also that you are given the all-pairs distance matrix for this graph (it can be constructed by a variant of Seidel's deterministic distance algorithm). Prove that you can identify the (successor matrix representation of the) shortest paths in $O(B^2 MM(n) \log^2 n)$ time, where $MM(n)$ is the time to multiply $n \times n$ matrices.

2. In class we presented an efficient randomized algorithm for bipartite matching on d-regular graphs.

   (a) What goes wrong if the graph is not d-regular?

   (b) In class, we showed that the algorithm achieves $O(n \log n)$ time in expectation. Show a variant of the algorithm that achieves $O(n \log n)$ time with high probability. **Hint:** this is similar to why the coupon collector takes $O(n \log n)$ samples with high probability.