

Problem Set 8

Randomized Algorithms

Due Friday, October 25

1. Let S be an unknown set of n items (with n known). Suppose that you receive a sample T of k items chosen from S uniformly at random *without replacement*. Show how to construct a sample T' of k items from S , whose distribution is identical to a uniform sample of k items from S drawn *with replacement*.
2. Consider the example given in class for how online bipartite matching using random edges achieves a competitive ratio of $R = 1/2$: each arriving vertex x_i has an edge to y_i as well as all of $y_{n/2}, \dots, y_n$. Show that the algorithm that the algorithm given in class, which randomly ranks the right vertices y_i , has $R \leq 3/4$ on this example.
3. [Moshkovitz] Suppose that you have a giant (i.e., infinite) bag of coins. You know that 90% of the coins are highly biased, and come up heads 90% of the time. The other 10% of coins are unbiased, and come up heads 50% of the time. You do not know which coins are which, and you would like to find *one* of the biased coins.

You are allowed to flip coins n times – each coin you flip can be either a fresh random coin from the bag, or a coin that you have flipped before. At the end of n coin flips, you must output a coin. You succeed if the coin is biased, and fail if the coin is unbiased. What is the minimum probability of failure, and how can you achieve this?

- (a) Show that the failure probability must be at least $\exp(-O(n))$.
- (b) Suppose that the biased coins were actually 100% biased. Show how to achieve $\exp(-\Omega(n))$ failure probability.
- (c) Show how to achieve $\exp(-\Omega(n))$ failure probability in the setting described, where the biased coins are 90% biased.

Hint (rot13): Gur nytbevguz vf fvzvyne gb gur bar sbe ebohfg ovanel frnepu ba ceboyrrz frg 2.