Graph Sketching

Ahn-Guha-McGregor '12

$O(n \log^c n)$ linear sketch of graph edges
(✓ supports edge insert/delete)

- Sample random edge from cut
- Find spanning tree
- $(1+\varepsilon)$-approx. min in spanning tree
- test bipartiteness
- estimate size of cut to $1 \pm \varepsilon$

Idea: apply linear sketch matrix A to adjacency of each node.

M: $\binom{n}{2} \times n$ matrix

Orient each edge

$$M_{e,u} = \begin{cases} 1 & \text{if } e = (u,v) \\ -1 & \text{if } e = (v,u) \end{cases}$$

D_E: diagonal, $(D_E)_e = 1 \iff e \in E$

$A \in \mathbb{R}^{s \times \binom{n}{2}}$ linear sketch $(s = \log^c n)$

Store $A \cdot D_E \cdot M \in \mathbb{R}^{s \times n}$

$= Y_1, Y_2, \ldots, Y_n \in \mathbb{R}^s$

$Y_u = A \cdot X_u$, where

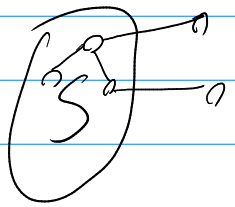
$X_u \in \mathbb{R}^{\binom{n}{2}}$ has $(X_u)_e = \begin{cases} \pm 1 & \text{if } e \in E \text{ and } u \in e \\ 0 & \text{otherwise} \end{cases}$

$e = (u,v)$ has $(X_u)_e = 1$, $(X_v)_e = -1$

So, to sample edge adjacent to a vertex:

set $A = \ell_0$ Sampler.
run alg on $Y_u = A X_u$
get random edge $e \in E$ s.t. $u \in e$.

Magic: can also sample edge over cut



Run alg on $Y_S := \sum_{u \in S} Y_u = A \cdot \left(\sum_{u \in S} X_u\right) =: A X_S$.

For $e = (u,v)$, $(X_S)_e = \begin{cases} 0 & \text{if } u, v \notin S \\ 1 & \text{if } u \in S, v \notin S \\ -1 & \text{if } u \notin S, v \in S \\ 0 & \text{if } u, v \in S \end{cases}$