Libflame – no more "0 users, 0 complaints"

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It takes a village

- Many have contributed to the FLAME libflame projects. My apologies for not listing all
- Early support for FLAME came from NSF and a number of corporate gifts (Microsoft, Intel, and others)
- The purpose of this talk is to quickly get to talking about the future

FLAME is ...

- A notation
- A methodology for deriving families of algorithms
- A family of APIs
- A library (libflame)
- A productivity multiplier
- A future

FLAME is a notation

Algorithm: $A := \text{CHOL\_BLK\_VAR3}(A)$

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$

where A_{TL} is 0×0

while $m(A_{TL}) < m(A)$ **do**

Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$$

where A_{11} is $b \times b$

$$A_{11} := L_{11} = \text{Chol}(A_{11})$$
$$A_{21} := L_{21} = A_{21} L_{11}^{-T} \quad \text{(TRSM)}$$
$$A_{22} := A_{22} - L_{21} L_{21}^{T} \quad \text{(SYRK)}$$

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$$

endwhile

FLAME is a methodology

Step	Annotated Algorithm: $A := \text{CHOL}(A)$
1a	$\left\{ A = \widehat{A} \right\}$
4	**Partition** $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$, $L \rightarrow \left(\begin{array}{c\|c} L_{TL} & 0 \\ \hline L_{BL} & L_{BR} \end{array}\right)$, $U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline 0 & U_{BR} \end{array}\right)$ **where** A_{TL}, L_{TL}, and U_{TL} are 0×0
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & \star \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L_{TL} & \star \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \begin{array}{l\|l} L_{TL}L_{TL}^T = \widehat{A}_{TL} & \\ \hline L_{BL}L_{TL}^T = \widehat{A}_{BL} & \end{array} \right\}$
3	**while** $m(A_{TL}) < m(A)$ **do**
2,3	$\left\{ \left(\left(\begin{array}{c\|c} A_{TL} & \star \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L_{TL} & \star \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \begin{array}{l\|l} L_{TL}L_{TL}^T = \widehat{A}_{TL} & \\ \hline L_{BL}L_{TL}^T = \widehat{A}_{BL} & \end{array} \right) \wedge m(A_{TL}) < m(A) \right\}$
5a	**Repartition** $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} L_{TL} & 0 \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} L_{00} & 0 & 0 \\ \hline L_{10} & L_{11} & 0 \\ \hline L_{20} & L_{21} & L_{22} \end{array}\right)$ **where** A_{11} and L_{11} are $b \times b$
6	$\left\{ \left(\begin{array}{c\|c\|c} A_{00} & \star & \star \\ \hline A_{10} & A_{11} & \star \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right) = \left(\begin{array}{c\|c\|c} L_{00} & \star & \star \\ \hline L_{10} & \widehat{A}_{11} - L_{10}L_{10}^T & \star \\ \hline L_{20} & \widehat{A}_{21} - L_{20}L_{10}^T & \widehat{A}_{22} - L_{20}L_{20}^T \end{array}\right) \wedge \begin{array}{l\|ll} L_{00}L_{00}^T = \widehat{A}_{00} & \star & \star \\ \hline L_{10}L_{00}^T = \widehat{A}_{10} & & \\ \hline L_{20}L_{00}^T = \widehat{A}_{20} & & \end{array} \right\}$
8	$A_{11} := L_{11} = \text{Chol}(A_{11})$ $A_{21} := L_{21} = A_{21}L_{11}^{-T}$ (TRSM) $A_{22} := A_{22} - L_{21}L_{21}^T$ (SYRK)
5b	**Continue with** $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} L_{TL} & 0 \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} L_{00} & 0 & 0 \\ \hline L_{10} & L_{11} & 0 \\ \hline L_{20} & L_{21} & L_{22} \end{array}\right)$
7	$\left\{ !! \left(\begin{array}{c\|c\|c} A_{00} & \star & \star \\ \hline A_{10} & A_{11} & \star \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right) = \left(\begin{array}{c\|c\|c} L_{00} & \star & \star \\ \hline L_{10} & L_{11} & \star \\ \hline L_{20} & L_{21} & \widehat{A}_{22} - L_{20}L_{20}^T - L_{21}L_{21}^T \end{array}\right) \wedge \begin{array}{l\|ll} L_{00}L_{00}^T = \widehat{A}_{00} & & \\ \hline L_{10}U_{00}^T = \widehat{A}_{10} & L_{10}L_{10}^T + L_{11}L_{11}^T = \widehat{A}_{11} & \\ \hline L_{20}L_{00}^T = \widehat{A}_{20} & L_{20}L_{10}^T + L_{21}L_{11}^T = \widehat{A}_{21} & \end{array} \right\}$
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & \star \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L_{TL} & \star \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \begin{array}{l\|l} L_{TL}L_{TL}^T = \widehat{A}_{TL} & \\ \hline L_{BL}L_{TL}^T = \widehat{A}_{BL} & \end{array} \right\}$
	endwhile
2,3	$\left\{ \left(\left(\begin{array}{c\|c} A_{TL} & \star \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L_{TL} & \star \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \begin{array}{l\|l} L_{TL}L_{TL}^T = \widehat{A}_{TL} & \\ \hline L_{BL}L_{TL}^T = \widehat{A}_{BL} & \end{array} \right) \wedge \neg(m(A_{TL}) < m(A)) \right\}$
1b	$\left\{ A = L \wedge LL^T = \widehat{A} \right\}$

FLAME is a family of APIs

Algorithm: $A := \text{CHOL\_BLK\_VAR3}(A)$

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$

where A_{TL} **is** 0×0

while $m(A_{TL}) < m(A)$ **do**

Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$$

where A_{11} **is** $b \times b$

$$A_{11} := L_{11} = \text{Chol}(A_{11})$$
$$A_{21} := L_{21} = A_{21} L_{11}^{-T} \quad \text{(TRSM)}$$
$$A_{22} := A_{22} - L_{21} L_{21}^{T} \quad \text{(SYRK)}$$

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$$

endwhile

```
FLA_Part_2x2( A,    &ATL, &ATR,
                    &ABL, &ABR,     0, 0, FLA_TL );

while ( FLA_Obj_length( ATL ) < FLA_Obj_length( A ) ){

  b = FLA_Determine_blocksize( ABR, FLA_BR, FLA_Cntl_blocksize( cntl ) );

  FLA_Repart_2x2_to_3x3( ATL, /**/ ATR,       &A00, /**/ &A01, &A02,
                      /* ************* */   /* ********************* */
                                              &A10, /**/ &A11, &A12,
                         ABL, /**/ ABR,       &A20, /**/ &A21, &A22,
                         b, b, FLA_BR );

  /*---------------------------------------------------------*/

  // A11 = chol( A11 )
  r_val = FLA_Chol_internal( FLA_LOWER_TRIANGULAR, A11,
                             FLA_Cntl_sub_chol( cntl ) );

  if ( r_val != FLA_SUCCESS )
    return ( FLA_Obj_length( A00 ) + r_val );

  // A21 = A21 * inv( tril( A11 )' )
  FLA_Trsm_internal( FLA_RIGHT, FLA_LOWER_TRIANGULAR,
                     FLA_CONJ_TRANSPOSE, FLA_NONUNIT_DIAG,
                     FLA_ONE, A11, A21,
                     FLA_Cntl_sub_trsm( cntl ) );

  // A22 = A22 - A21 * A21'
  FLA_Herk_internal( FLA_LOWER_TRIANGULAR, FLA_NO_TRANSPOSE,
                     FLA_MINUS_ONE, A21, FLA_ONE, A22,
                     FLA_Cntl_sub_herk( cntl ) );

  /*---------------------------------------------------------*/

  FLA_Cont_with_3x3_to_2x2( &ATL, /**/ &ATR,       A00, A01, /**/ A02,
                                                   A10, A11, /**/ A12,
                         /* ************** */   /* ****************** */
                           &ABL, /**/ &ABR,       A20, A21, /**/ A22,
                           FLA_TL );
}
```

Algorithm: $A := \text{CHOL\_BLK\_VAR3}(A)$

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$

where A_{TL} is 0×0

while $m(A_{TL}) < m(A)$ **do**

Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$$

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$$

endwhile

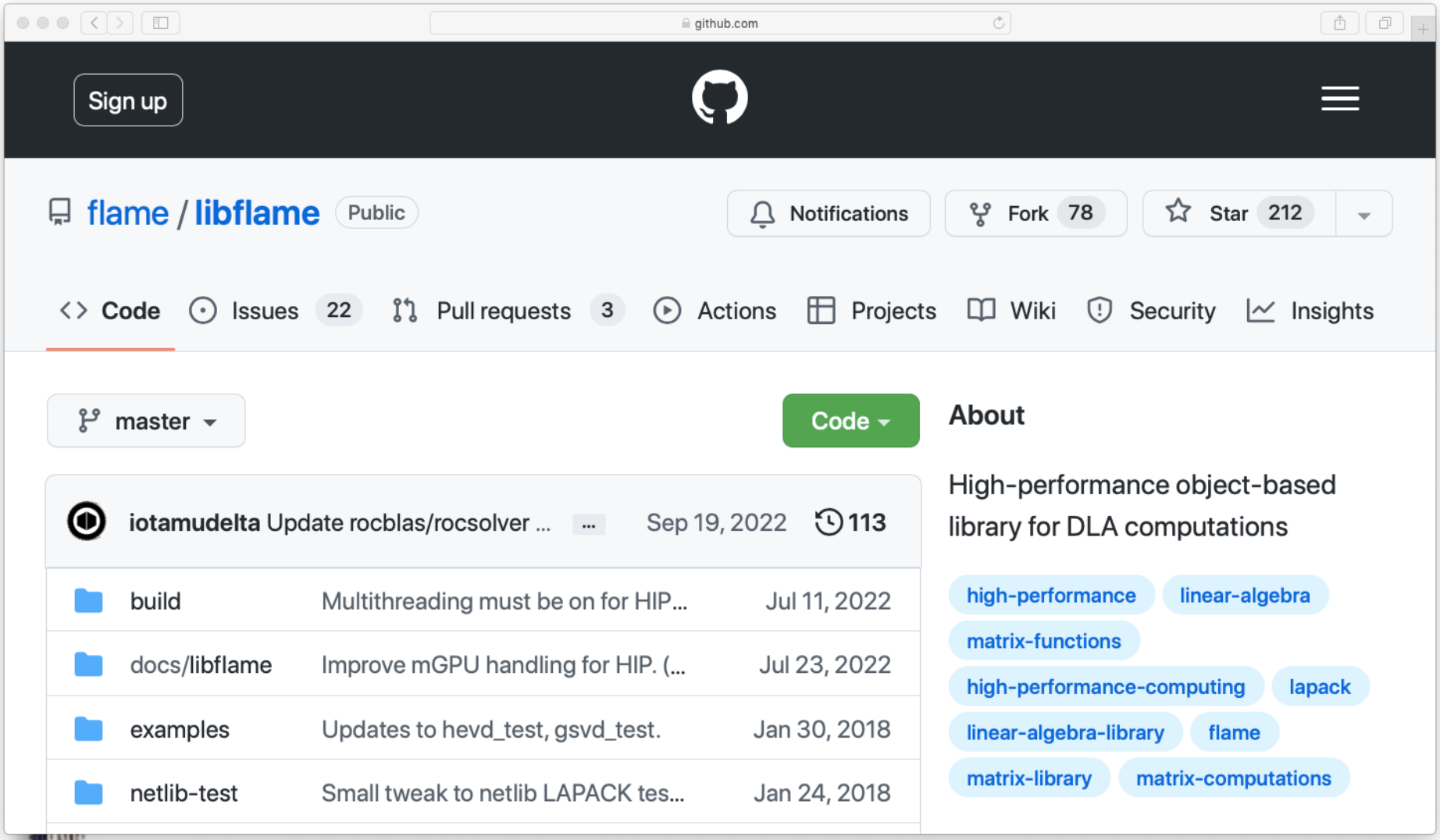
```
FLA_Trsm_internal( FLA_RIGHT, FLA_LOWER_TRIANGULAR,
                   FLA_CONJ_TRANSPOSE, FLA_NONUNIT_DIAG,
                   FLA_ONE, A11, A21,
                   FLA_Cntl_sub_trsm( cntl ) );
```

```
FLA_Part_2x2( A,    &ATL, &ATR,
                    &ABL, &ABR,     0, 0, FLA_TL );

while ( FLA_Obj_length( ATL ) < FLA_Obj_length( A ) ){

  b = FLA_Determine_blocksize( ABR, FLA_BR, FLA_Cntl_blocksize( cntl ) );

  FLA_Repart_2x2_to_3x3( ATL, /**/ ATR,       &A00, /**/ &A01, &A02,
                      /* ************* */   /* ********************* */
                                              &A10, /**/ &A11, &A12,
                         ABL, /**/ ABR,       &A20, /**/ &A21, &A22,
                         b, b, FLA_BR );

  /*--------------------------------------------------------------*/

  // A11 = chol( A11 )

                      FLA_ONE, A11, A21,
                      FLA_Cntl_sub_trsm( cntl ) );

  // A22 = A22 - A21 * A21'
  FLA_Herk_internal( FLA_LOWER_TRIANGULAR, FLA_NO_TRANSPOSE,
                     FLA_MINUS_ONE, A21, FLA_ONE, A22,
                     FLA_Cntl_sub_herk( cntl ) );

  /*--------------------------------------------------------------*/

  FLA_Cont_with_3x3_to_2x2( &ATL, /**/ &ATR,       A00, A01, /**/ A02,
                                                   A10, A11, /**/ A12,
                         /* *************** */   /* ******************* */
                           &ABL, /**/ &ABR,       A20, A21, /**/ A22,
                           FLA_TL );
}
```

```
FLA_Part_2x2( A,    &ATL, &ATR,
                    &ABL, &ABR,     0, 0, FLA_TL );

while ( FLA_Obj_length( ATL ) < FLA_Obj_length( A ) ){

  FLA_Repart_2x2_to_3x3( ATL, /**/ ATR,       &A00,  /**/ &a01,     &A02,
                      /* ************* */   /* ************************** */
                                              &a10t, /**/ &alpha11, &a12t,
                         ABL, /**/ ABR,       &A20,  /**/ &a21,     &A22,
                         1, 1, FLA_BR );

  /*------------------------------------------------------------*/

  // alpha11 = sqrt( alpha11 )
  r_val = FLA_Sqrt( alpha11 );

  if ( r_val != FLA_SUCCESS )
    return ( FLA_Obj_length( A00 ) );

  // a21 = a21 / alpha11
  FLA_Inv_scal_external( alpha11, a21 );

  // A22 = A22 - a21 * a21'
  FLA_Her_external( FLA_LOWER_TRIANGULAR, FLA_MINUS_ONE, a21, A22 );

  /*------------------------------------------------------------*/

  FLA_Cont_with_3x3_to_2x2( &ATL, /**/ &ATR,       A00,  a01,     /**/ A02,
                                                   a10t, alpha11, /**/ a12t,
                         /* ************** */  /* ************************ */
                           &ABL, /**/ &ABR,       A20,  a21,     /**/ A22,
                           FLA_TL );
}
```

```
for ( i = 0; i < mn_A; ++i )
{
  float*    alpha11   = buff_A + (i  )*cs_A + (i  )*rs_A;
  float*    a21       = buff_A + (i  )*cs_A + (i+1)*rs_A;
  float*    A22       = buff_A + (i+1)*cs_A + (i+1)*rs_A;

  int       mn_ahead  = mn_A - i - 1;
  int       mn_behind = i;

  /*------------------------------------------------------------*/

  // r_val = FLA_Sqrt( alpha11 );
  // if ( r_val != FLA_SUCCESS )
  //   return ( FLA_Obj_length( A00 ) + 1 );
  bl1_ssqrte( alpha11, &e_val );
  if ( e_val != FLA_SUCCESS ) return mn_behind;

  // FLA_Inv_scal_external( alpha11, a21 );
  bl1_sinvscalv( BLIS1_NO_CONJUGATE,
                 mn_ahead,
                 alpha11,
                 a21, rs_A );

  // FLA_Her_external( FLA_LOWER_TRIANGULAR, FLA_MINUS_ONE, a21, A22 );
  bl1_ssyr( BLIS1_LOWER_TRIANGULAR,
            mn_ahead,
            buff_m1,
            a21, rs_A,
            A22, rs_A, cs_A );

  /*------------------------------------------------------------*/

}
```

FLAME is a library (libflame)



FLAME is a library (libflame)

- Core libflame
  - Writing with the FLAMEC API
  - Overlaps with much of LAPACK functionality
- flapack
  - Adds functionality of LAPACK not in core libflame
  - LAPACK run through f2c
- libflame = core libflame + flapack
  - Does not require a fortran compiler
  - All of lapack functionality (circa 2014?)

FLAME is a productivity multiplier

- …
- So many algorithms! When to pick which?
  - Control trees (that also appear in BLIS)
- SuperMatrix (algorithms by blocks)
  - Unit of data: submatrix
  - Unit of computation: operation with submatrices
  - Execute the algorithm to build DAG of tasks
  - Runtime schedules tasks to resources (multiple CPU and/or GPUs)
  - Runtime can, for example, incorporate a software cache

SuperMatrix code

```
FLA_Part_2x2( A,    &ATL, &ATR,
                    &ABL, &ABR,     0, 0, FLA_TL );

while ( FLA_Obj_length( ATL ) < FLA_Obj_length( A ) ){

  b = FLA_Determine_blocksize( ABR, FLA_BR, FLA_Cntl_blocksize( cntl ) );

  FLA_Repart_2x2_to_3x3( ATL, /**/ ATR,       &A00, /**/ &A01, &A02,
                      /* ************* */   /* ********************* */
                                              &A10, /**/ &A11, &A12,
                         ABL, /**/ ABR,       &A20, /**/ &A21, &A22,
                         b, b, FLA_BR );

  /*-----------------------------------------------------------*/

  // A11 = chol( A11 )
  r_val = FLA_Chol_internal( FLA_LOWER_TRIANGULAR, A11,
                             FLA_Cntl_sub_chol( cntl ) );

  if ( r_val != FLA_SUCCESS )
    return ( FLA_Obj_length( A00 ) + r_val );

  // A21 = A21 * inv( tril( A11 )' )
  FLA_Trsm_internal( FLA_RIGHT, FLA_LOWER_TRIANGULAR,
                     FLA_CONJ_TRANSPOSE, FLA_NONUNIT_DIAG,
                     FLA_ONE, A11, A21,
                     FLA_Cntl_sub_trsm( cntl ) );

  // A22 = A22 - A21 * A21'
  FLA_Herk_internal( FLA_LOWER_TRIANGULAR, FLA_NO_TRANSPOSE,
                     FLA_MINUS_ONE, A21, FLA_ONE, A22,
                     FLA_Cntl_sub_herk( cntl ) );

  /*-----------------------------------------------------------*/

  FLA_Cont_with_3x3_to_2x2( &ATL, /**/ &ATR,       A00, A01, /**/ A02,
                                                   A10, A11, /**/ A12,
                         /* ************** */   /* ****************** */
                            &ABL, /**/ &ABR,       A20, A21, /**/ A22,
                            FLA_TL );
}
```

FLAME is a future

There are a number of efforts to leverage libflame

- Oracle:
  - Vertical integration of libflame and BLIS
  - Incorporation of key functionality of libflame into BLIS
- AMD:
  -
  -
- …

How do we coordinate efforts?