



BLIS
THE FINAL FRONTIER

Some History...

Formal Linear Algebra Methods Environment

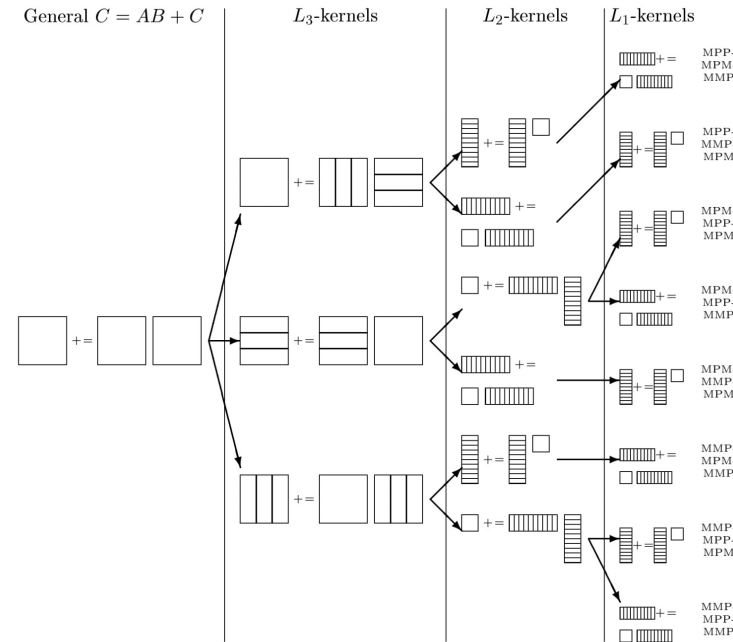


Figure 3: Possible algorithms for matrices in memory level L_3 given all L_2 -kernels.

ITX-GEMM



John Gunnel (UT, now NVIDIA)



Greg Henry (Intel)

High-performance implementation of the level-3 BLAS

Authors:  Kazushige Goto,  Robert Van De Geijn [Authors Info & Claims](#)

ACM Transactions on Mathematical Software, Volume 35, Issue 1 • Article No.: 4, pp 1–14 • <https://doi.org/10.1145/1377603.1377607>

Published: 25 July 2008 [Publication History](#)



 205  1,534

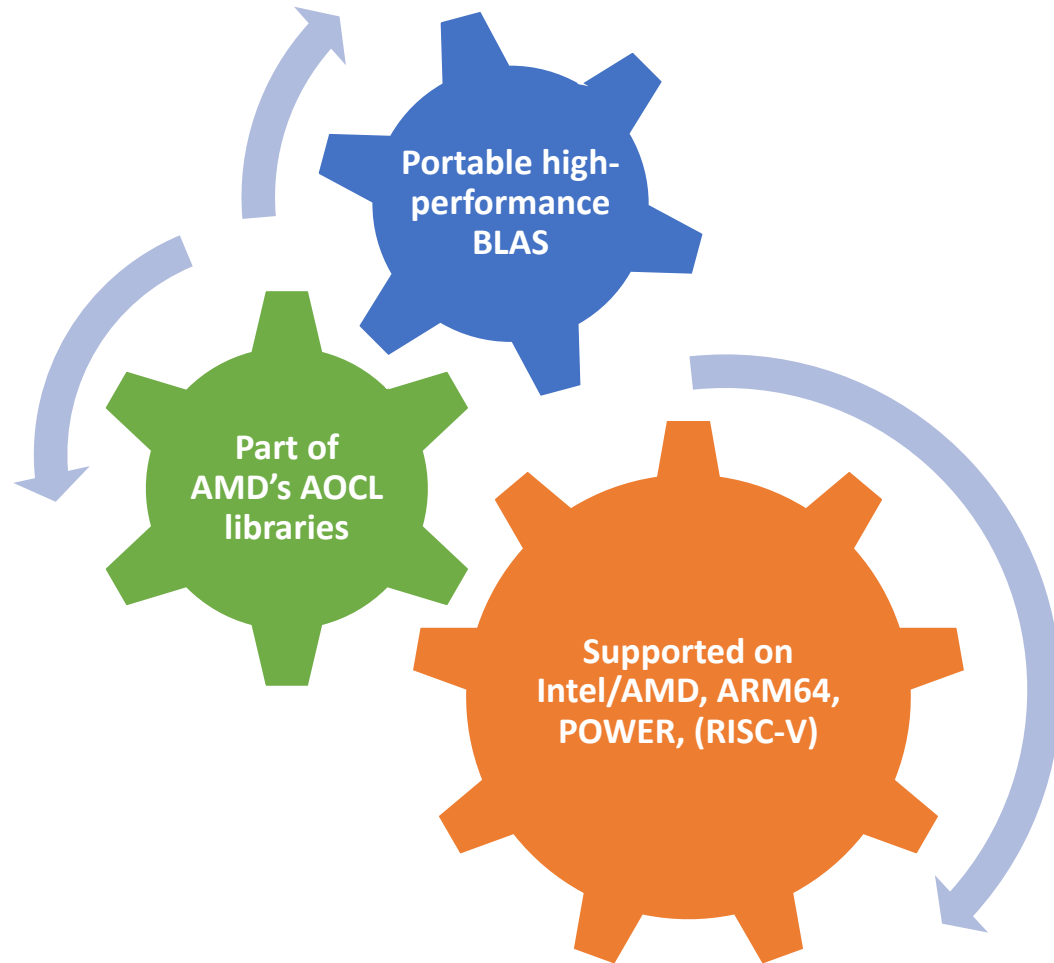


Some History...

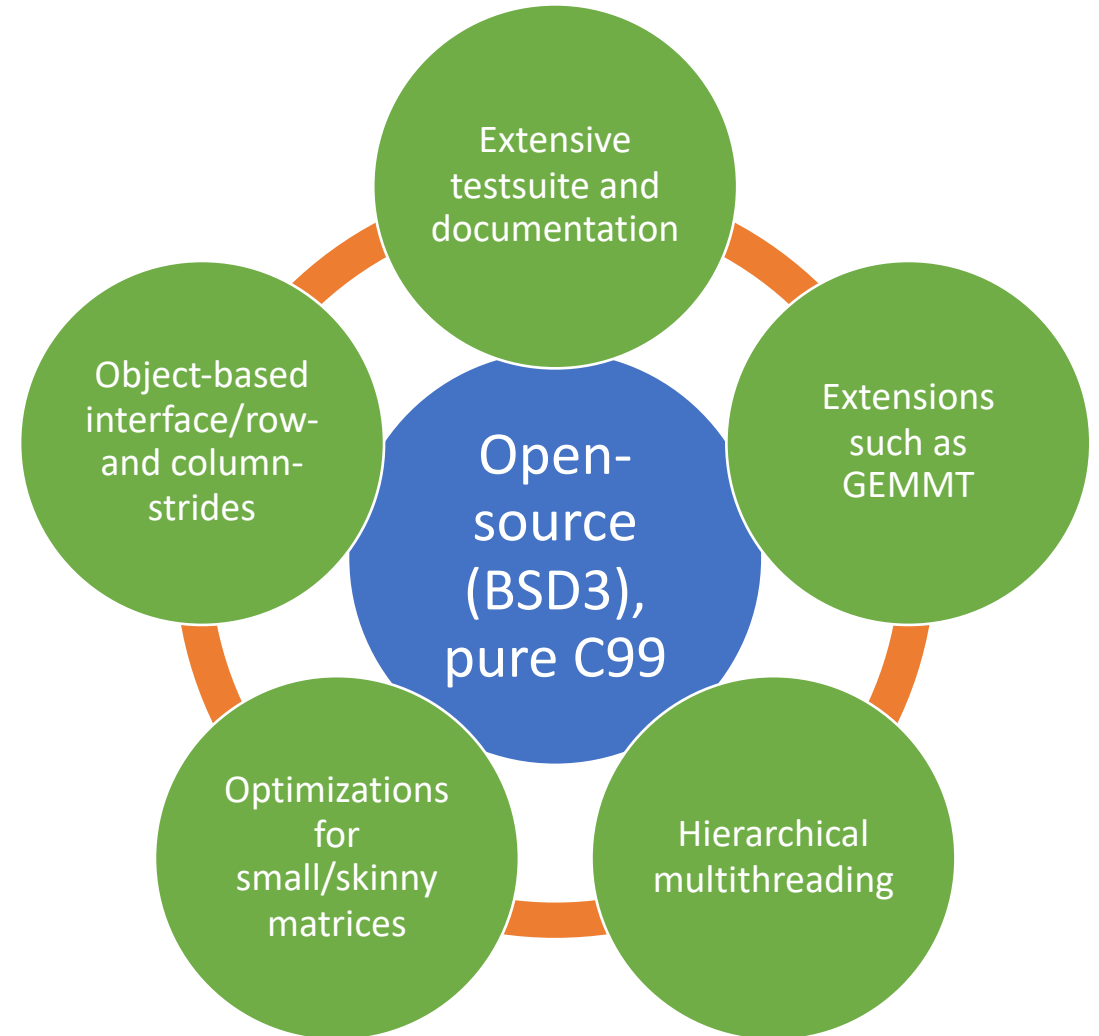
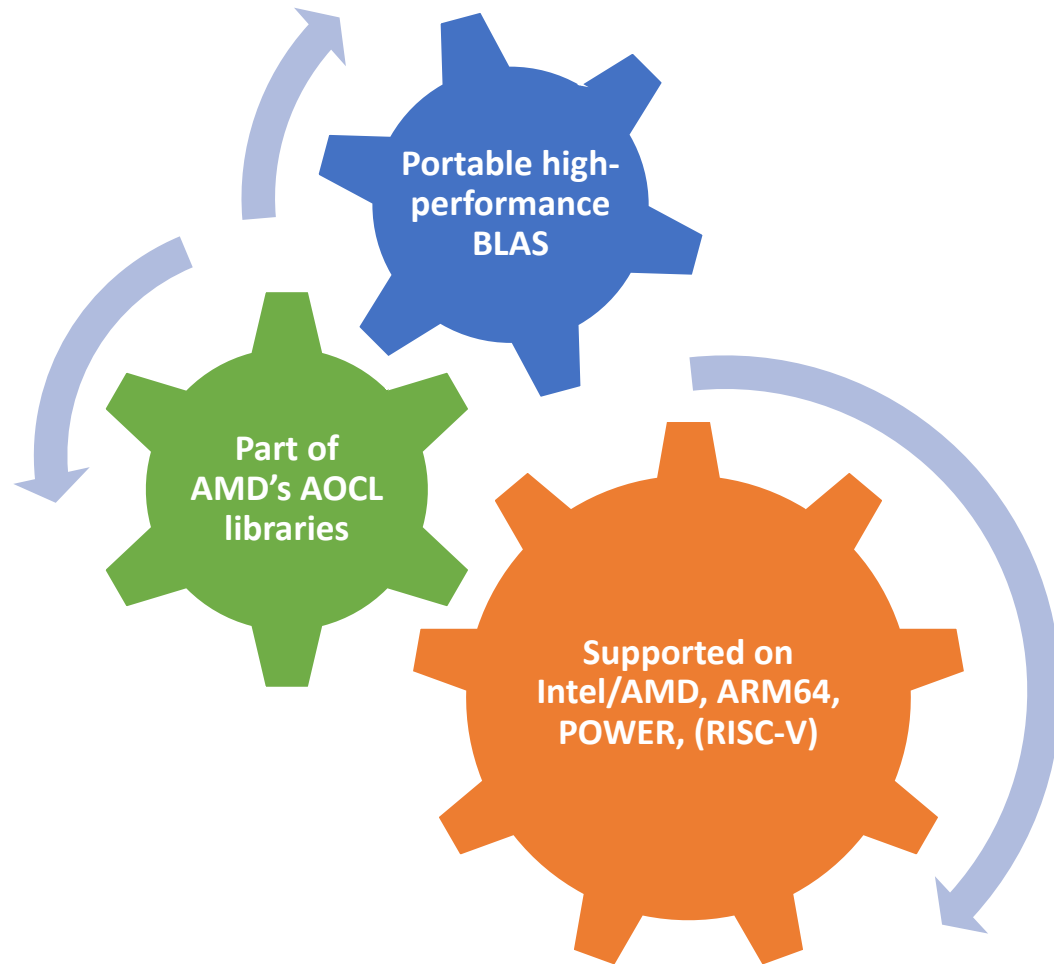
- 2012 • First NSF SI² award. Subsequent SI²/CSSI awards in 2016 and 2020.
- 2012 • First BLIS commit in Github. BLIS existed before this within libflame.
- 2013 • First BLIS retreat.
- 2014 • Multi-threading support.
- 2016 • Runtime multi-architecture support.
- 2019 • “Skinny/unpacked” GEMM
- 2020 • SIAM Activity Group Best Paper Prize
- 2023 • James H. Wilkinson Prize for Numerical Software

What *IS* BLIS?

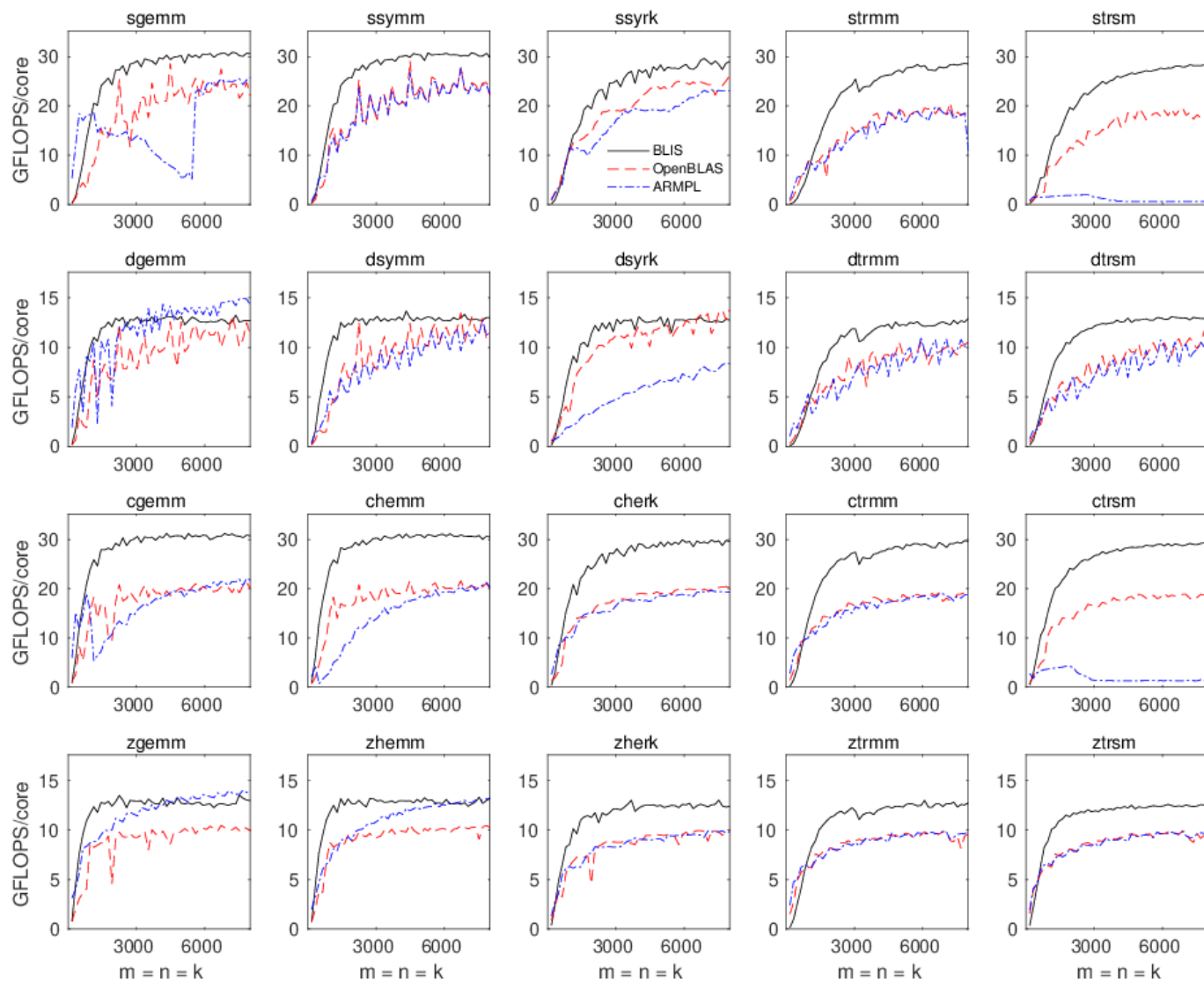
What *IS* BLIS?



What *IS* BLIS?



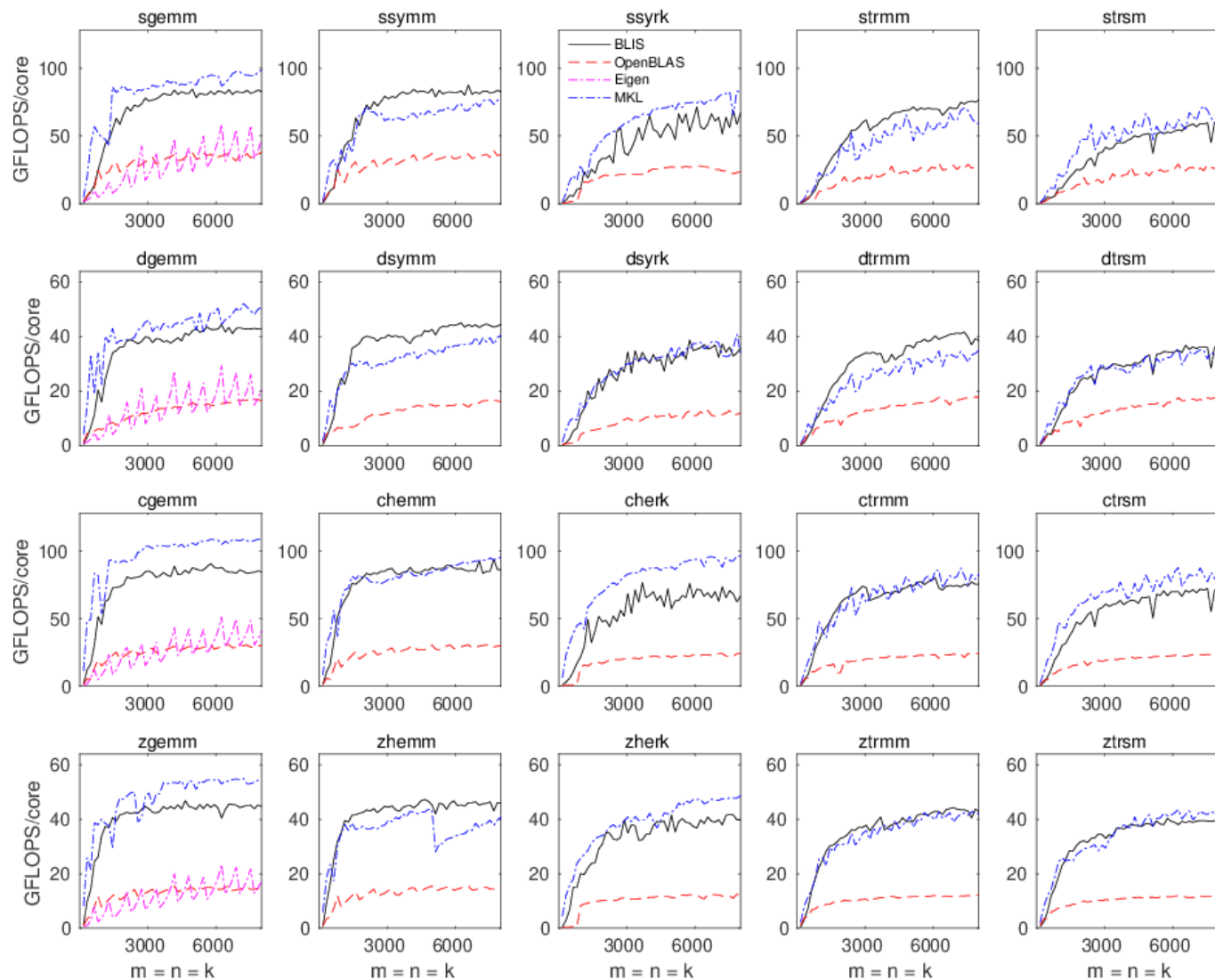
What *IS* BLIS?



ThunderX2

—— BLIS ——
- - - - The - - - -
- - - - Other - - - -
- - - - Guys - - - -

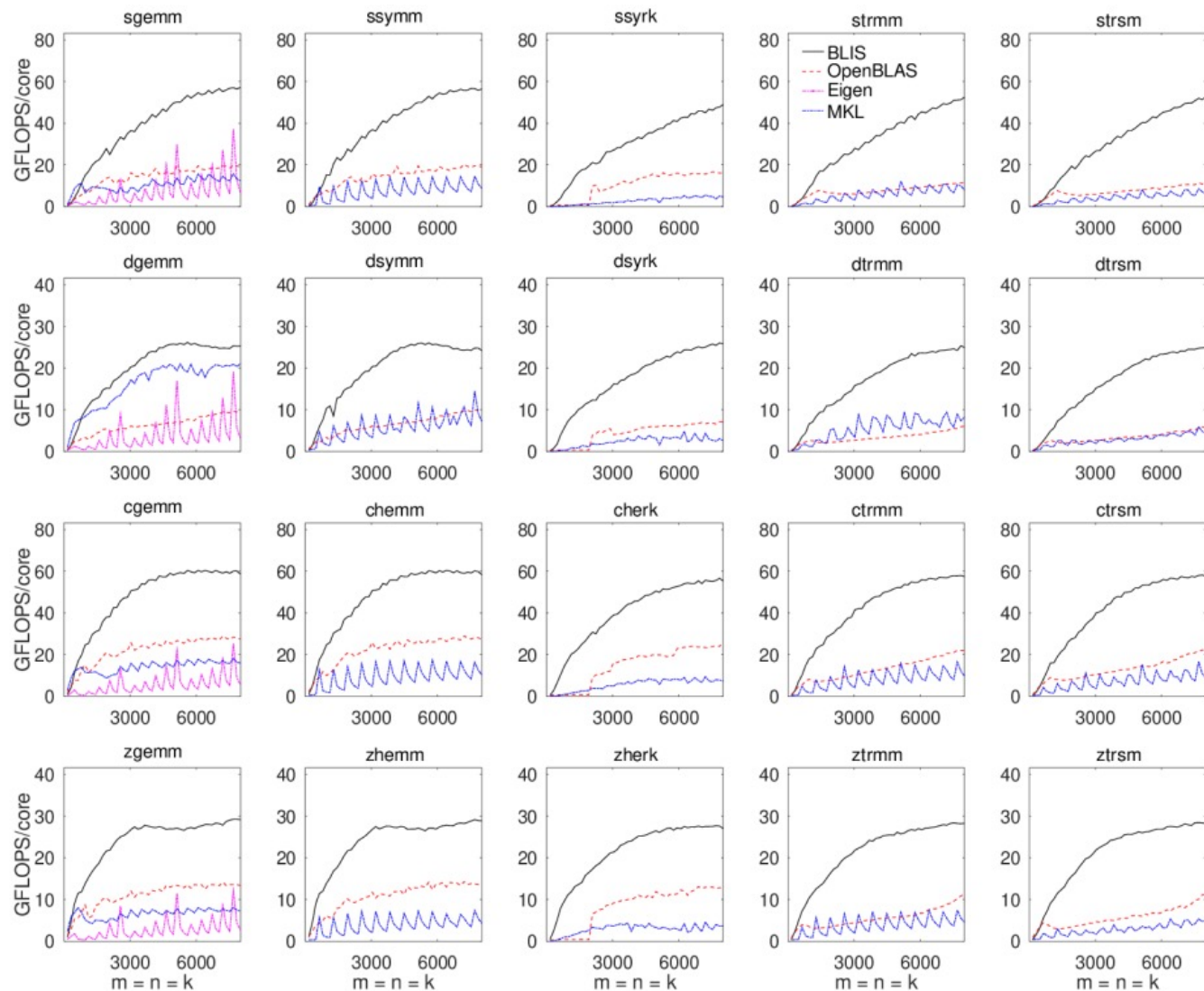
What *IS* BLIS?



Skylake X

—— BLIS ——
- - - - The - - - -
- - - - Other - - - -
- - - - Guys - - - -

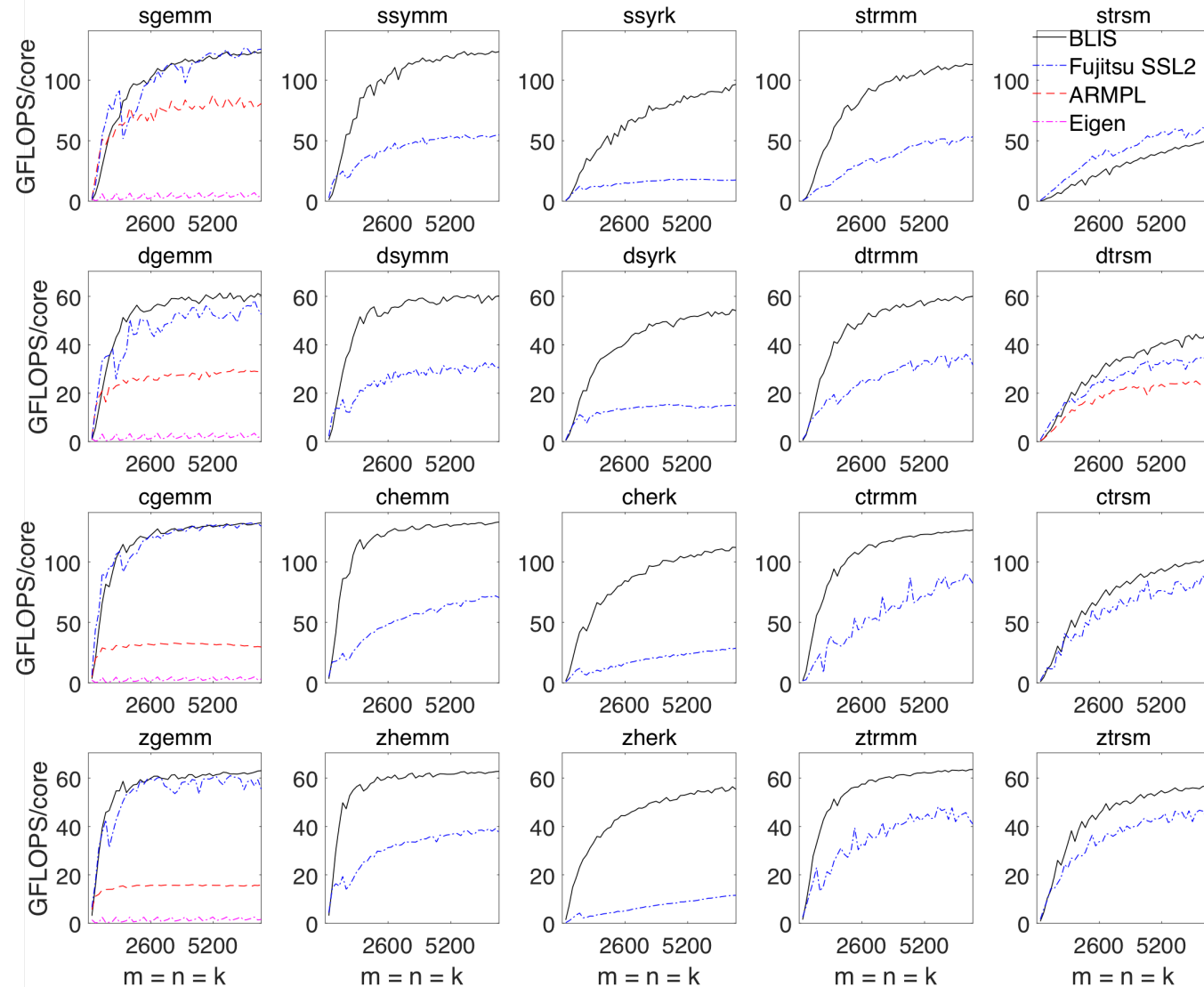
What *IS* BLIS?



Zen 2 (EPYC)

— BLIS —
- - - The - - -
- - - Other - - -
- - - Guys - - -

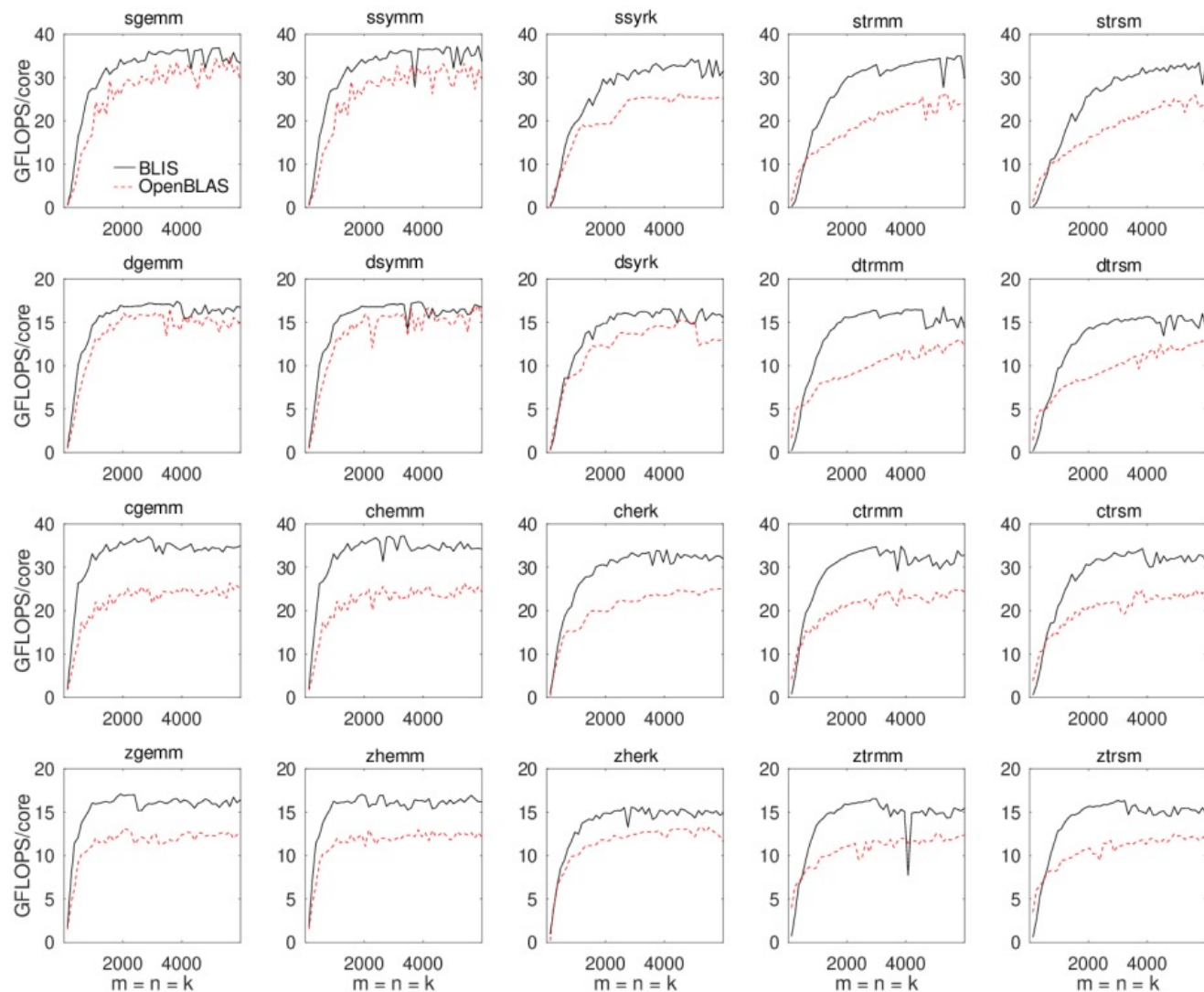
What *IS* BLIS?



A64fx

—— BLIS ——
- - - - The - - - -
- - - - Other - - - -
- - - - Guys - - - -

What *IS* BLIS?



Neoverse N1

—— BLIS ——
- - - - The - - - -
- - - - Other - - - -
- - - - Guys - - - -

What *IS* BLIS?

1M and other “induced methods”

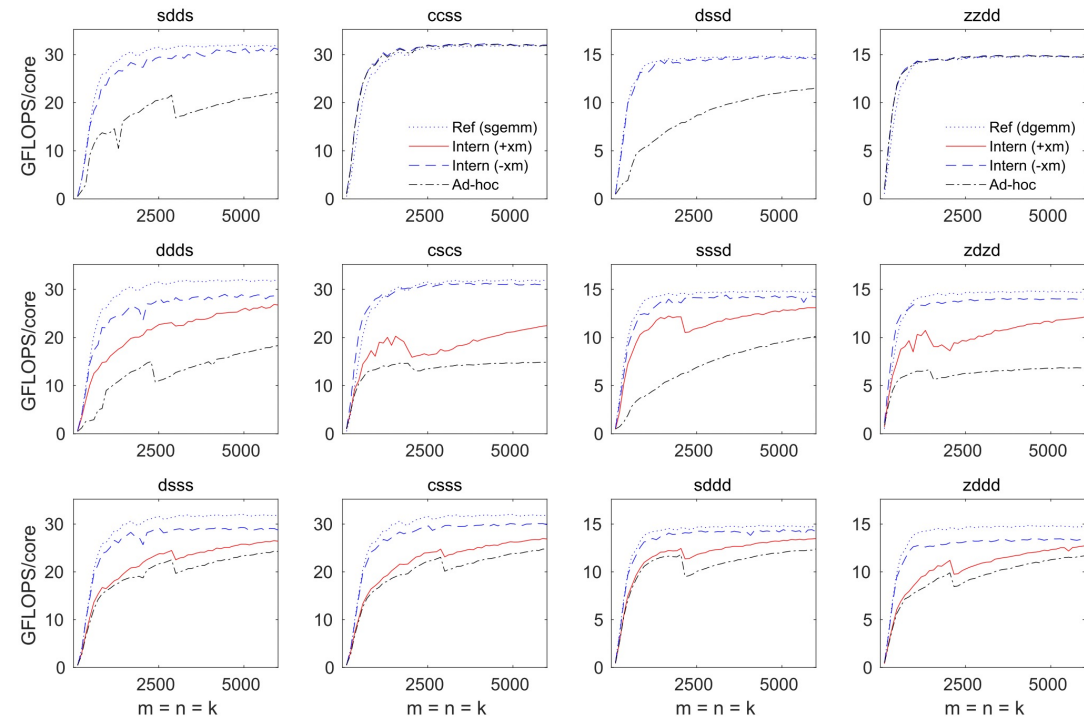
$$C^r := C^r + A^r B^r - A^i B^i,$$
$$C^i := C^i + A^i B^r + A^r B^i.$$



$$\begin{pmatrix} \gamma^r \\ \gamma^i \end{pmatrix} += \begin{pmatrix} \alpha^r & -\alpha^i \\ \alpha^i & \alpha^r \end{pmatrix} \begin{pmatrix} \beta^r \\ \beta^i \end{pmatrix}$$



Mixed-precision/mixed-domain **GEMM**

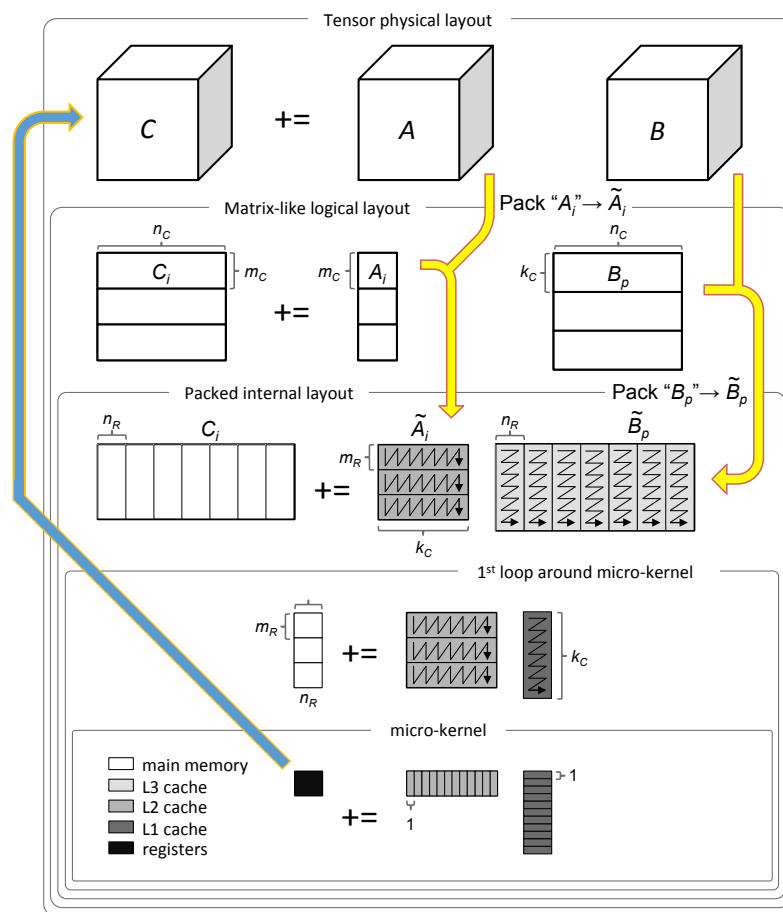
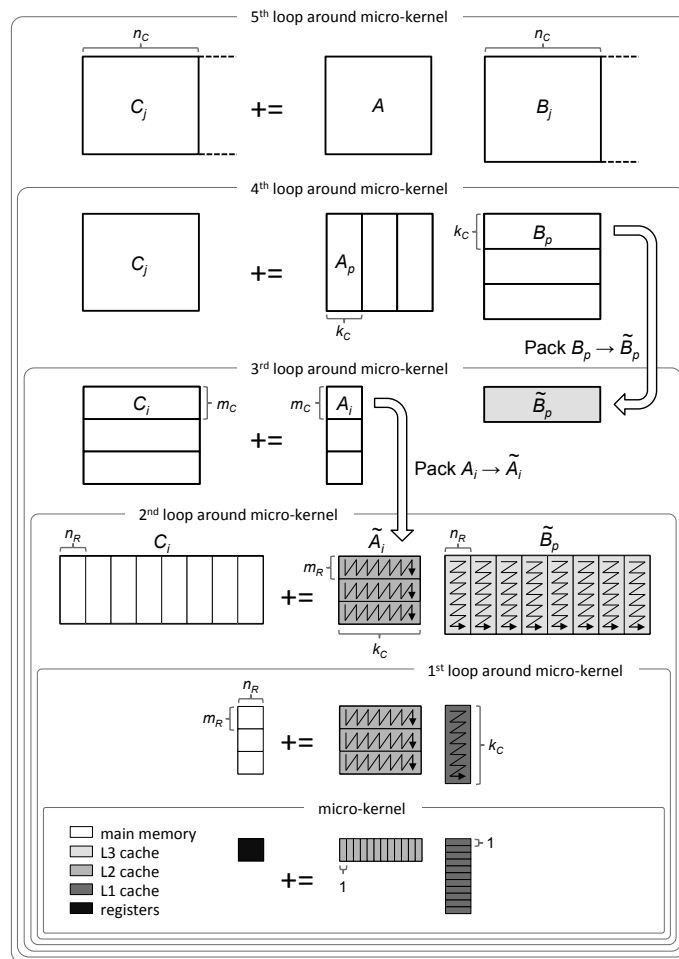
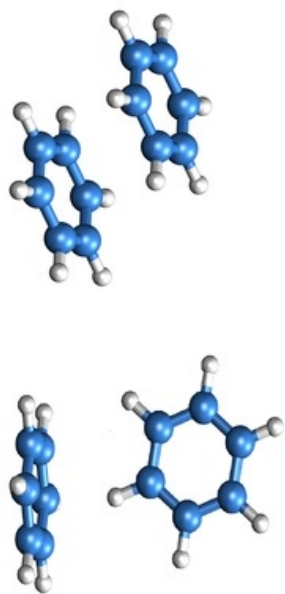


Some History...

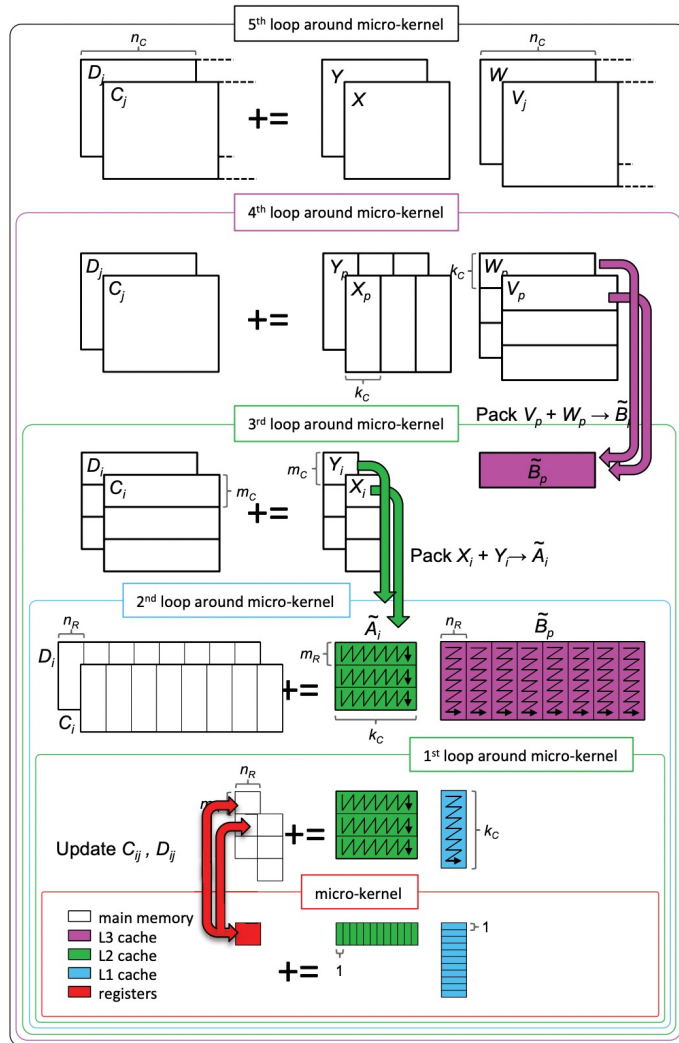
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What *SHOULD* BLIS Be?

$$W_{ic}^{ak} = t_{im}^{ae} \cdot V_{ec}^{mk}$$



What *SHOULD* BLIS Be?

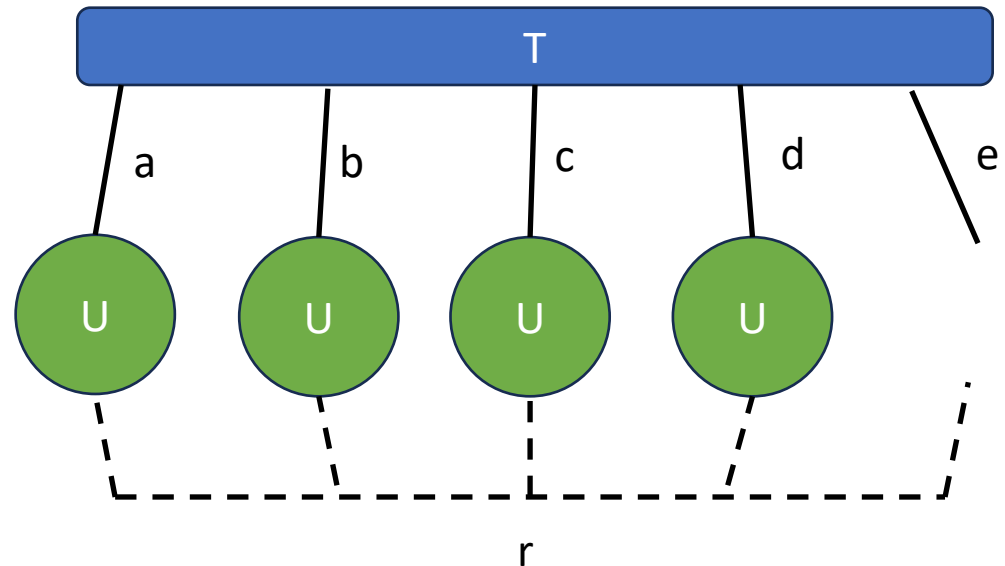


$$\begin{aligned}
 M_0 &= \alpha(A_{00} + A_{11})(B_{00} + B_{11}); & C_{00} &+= M_0; C_{11} &+= M_0; \\
 M_1 &= \alpha(A_{10} + A_{11})B_{00}; & C_{10} &+= M_1; C_{11} &- = M_1; \\
 M_2 &= \alpha A_{00}(B_{01} - B_{11}); & C_{01} &+= M_2; C_{11} &+= M_2; \\
 M_3 &= \alpha A_{11}(B_{10} - B_{00}); & C_{00} &+= M_3; C_{10} &+= M_3; \\
 M_4 &= \alpha(A_{00} + A_{01})B_{11}; & C_{01} &+= M_4; C_{00} &- = M_4; \\
 M_5 &= \alpha(A_{10} - A_{00})(B_{00} + B_{01}); & C_{11} &+= M_5; \\
 M_6 &= \alpha(A_{01} - A_{11})(B_{10} + B_{11}); & C_{00} &+= M_6;
 \end{aligned}$$

What *SHOULD* BLIS Be?

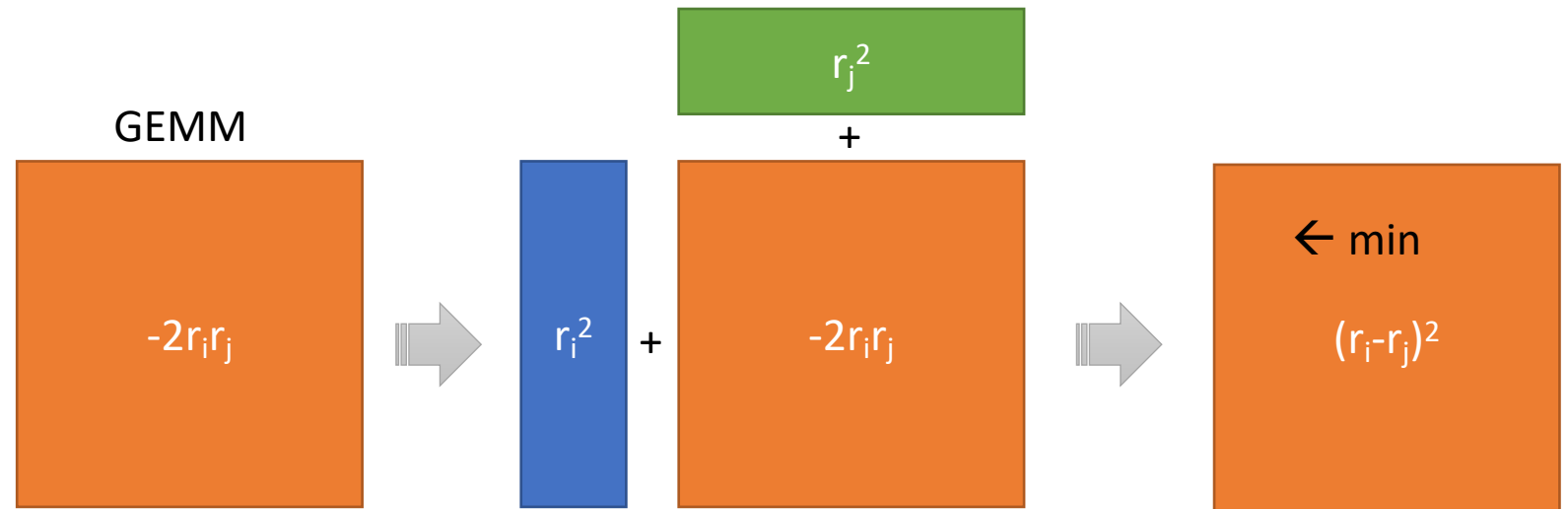
$$G_{re} = \sum_{bcd} \left[\sum_a (U_{r;b} U_{r;c} U_{r;d}) U_{ra} T_{ae;bcd} \right]$$

C += DAB

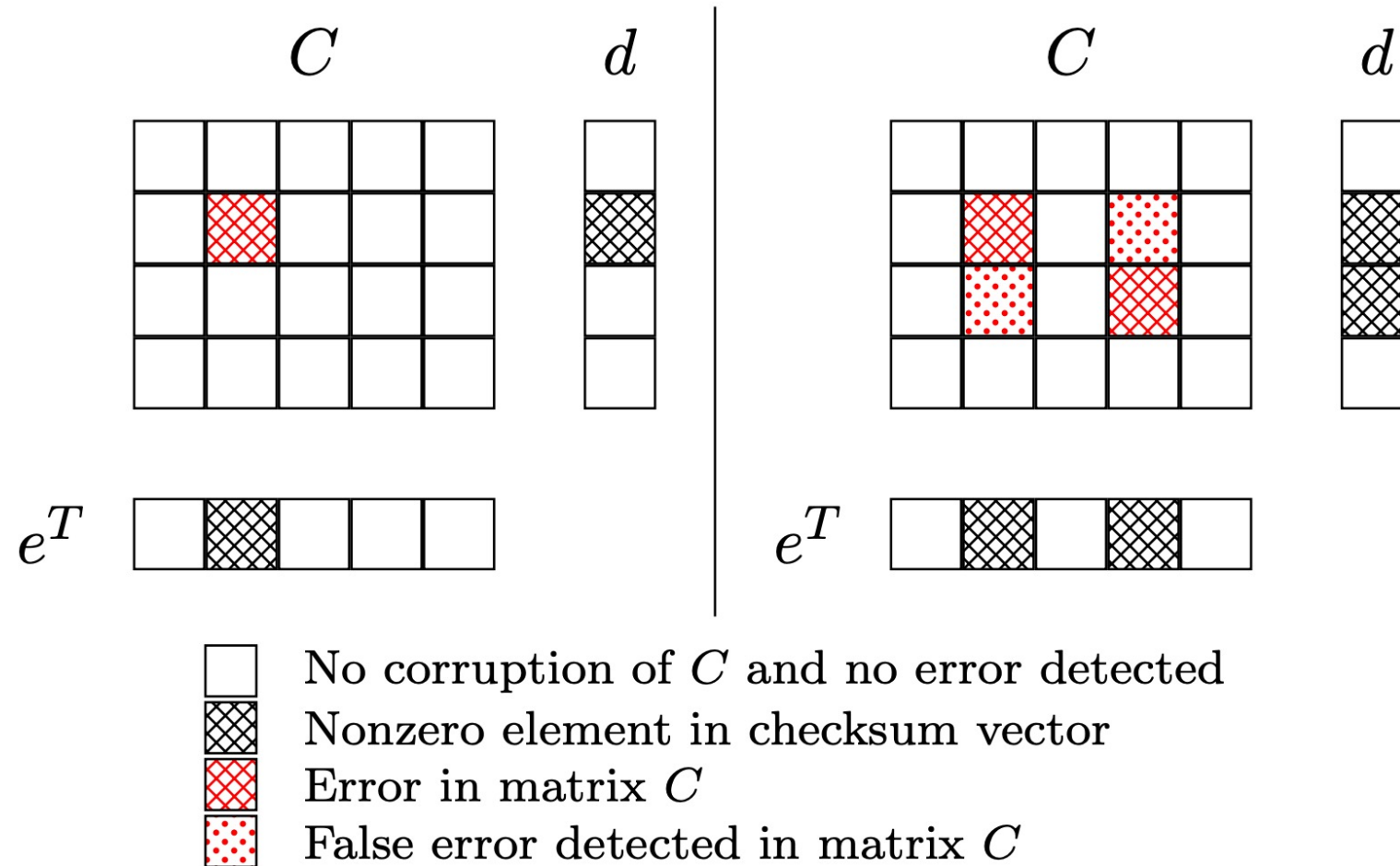


What *SHOULD* BLIS Be?

$$KNN_i = \left\{ \underset{j}{\text{k-min}} |r_i - r_j|^2 \right\}_{k=1}^K$$

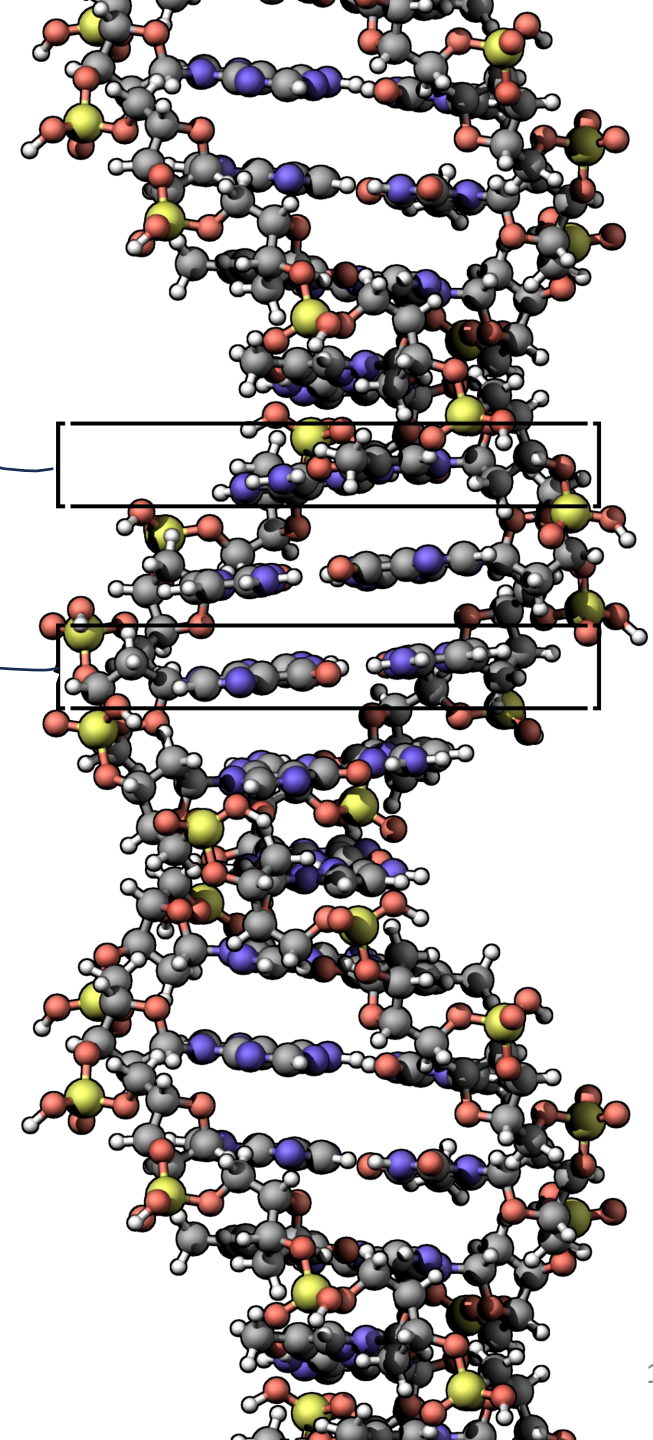


What *SHOULD* BLIS Be?



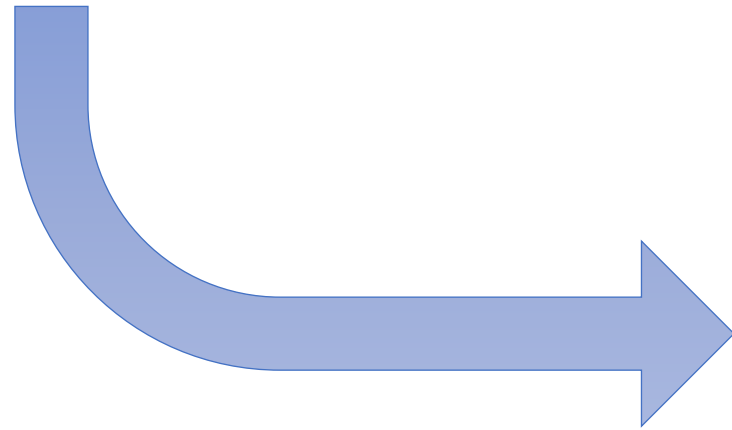
What *SHOULD* BLIS Be?

0	1	0	1	0	0	0	0	...	0	1	0	1	Sample
⋮												⋮	
0	0	0	1	1	1	1	0	...	1	1	0	0	
0	1	0	0	1	0	1	0	...	1	1	0	1	
⋮												⋮	
1	0	1	0	0	1	1	0	...	1	0	1	0	padding
1	0	0	1	0	1	1	1	...	1	0	1	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	
				SNP									

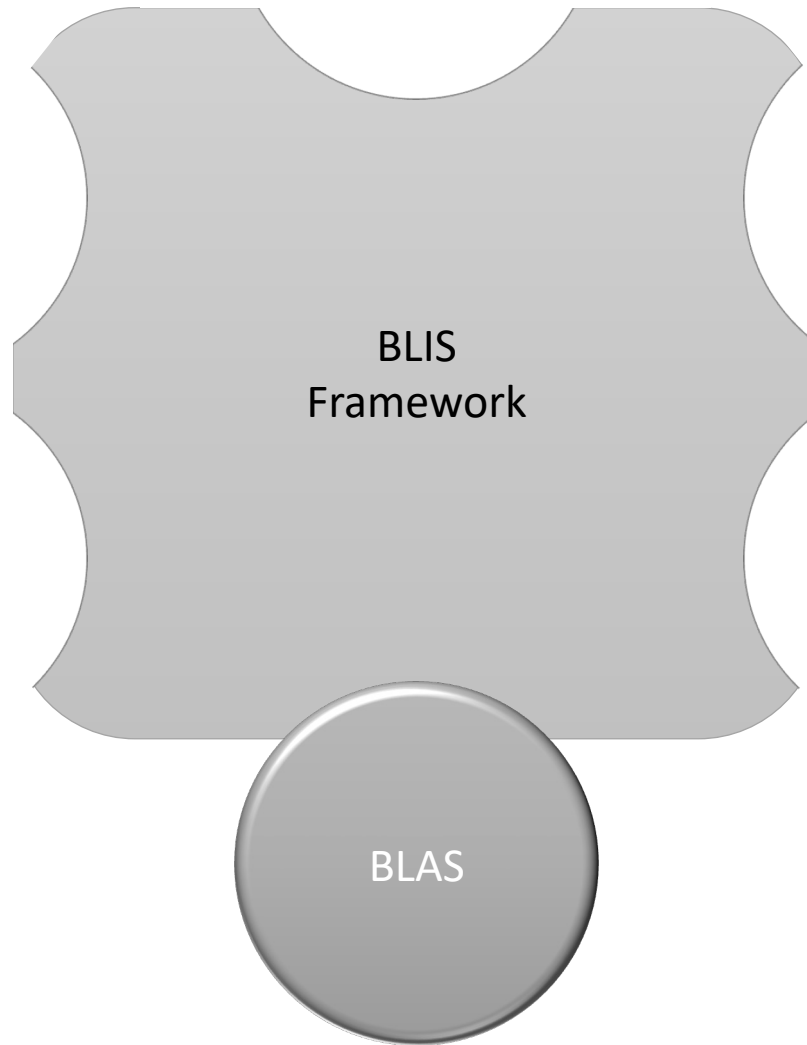


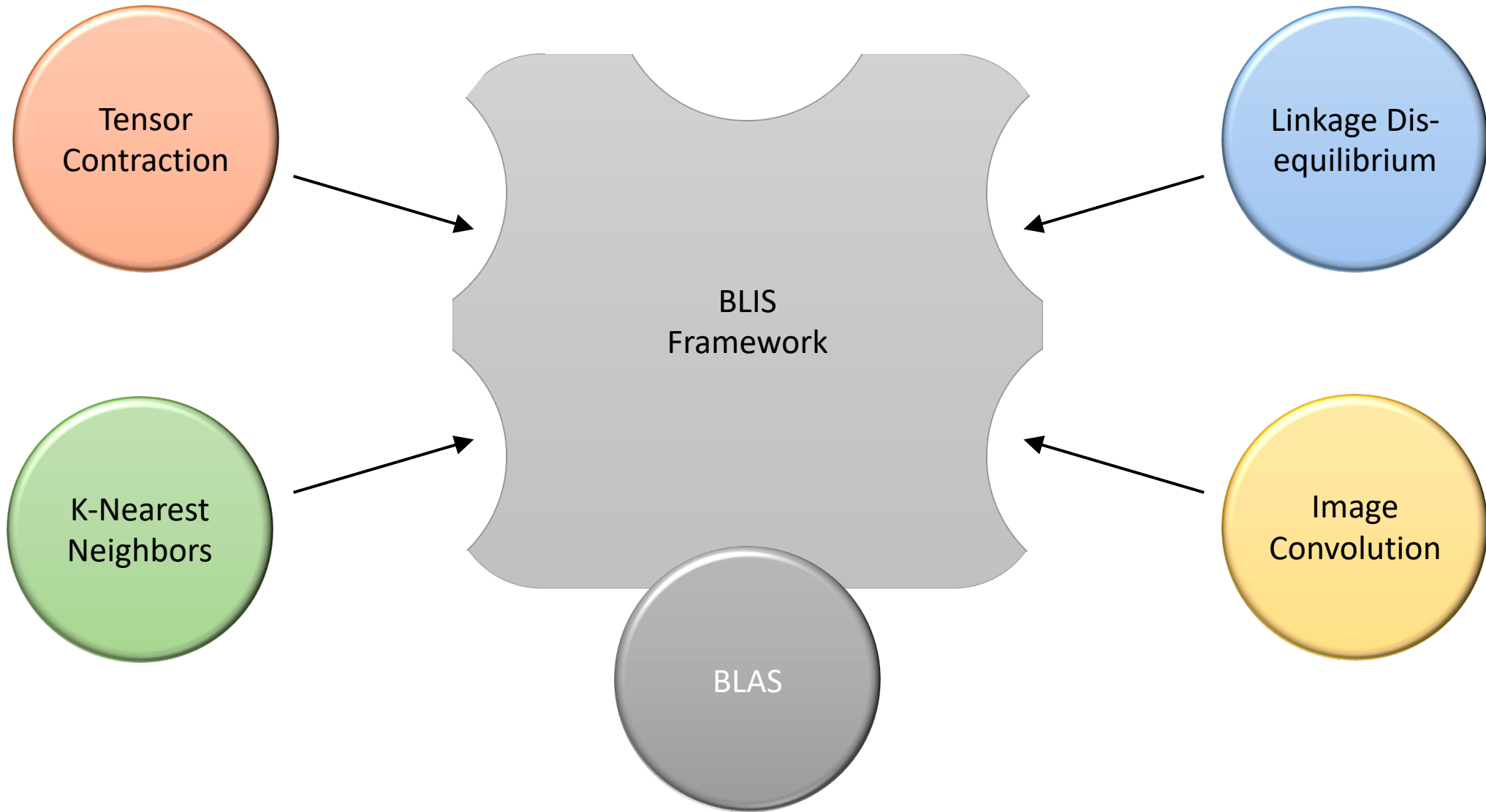
What *SHOULD* BLIS Be?

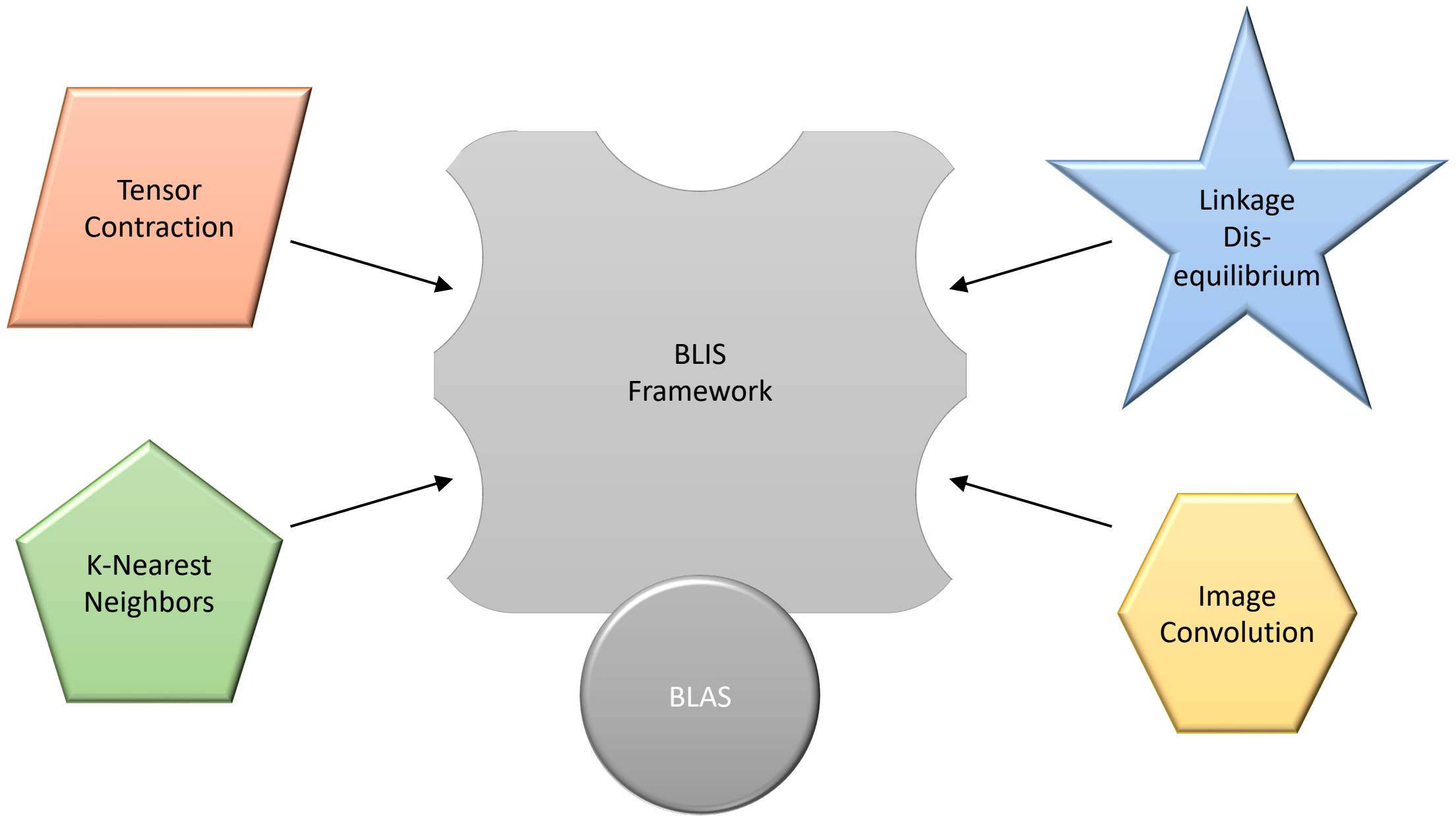
Is it “like GEMM but different”?

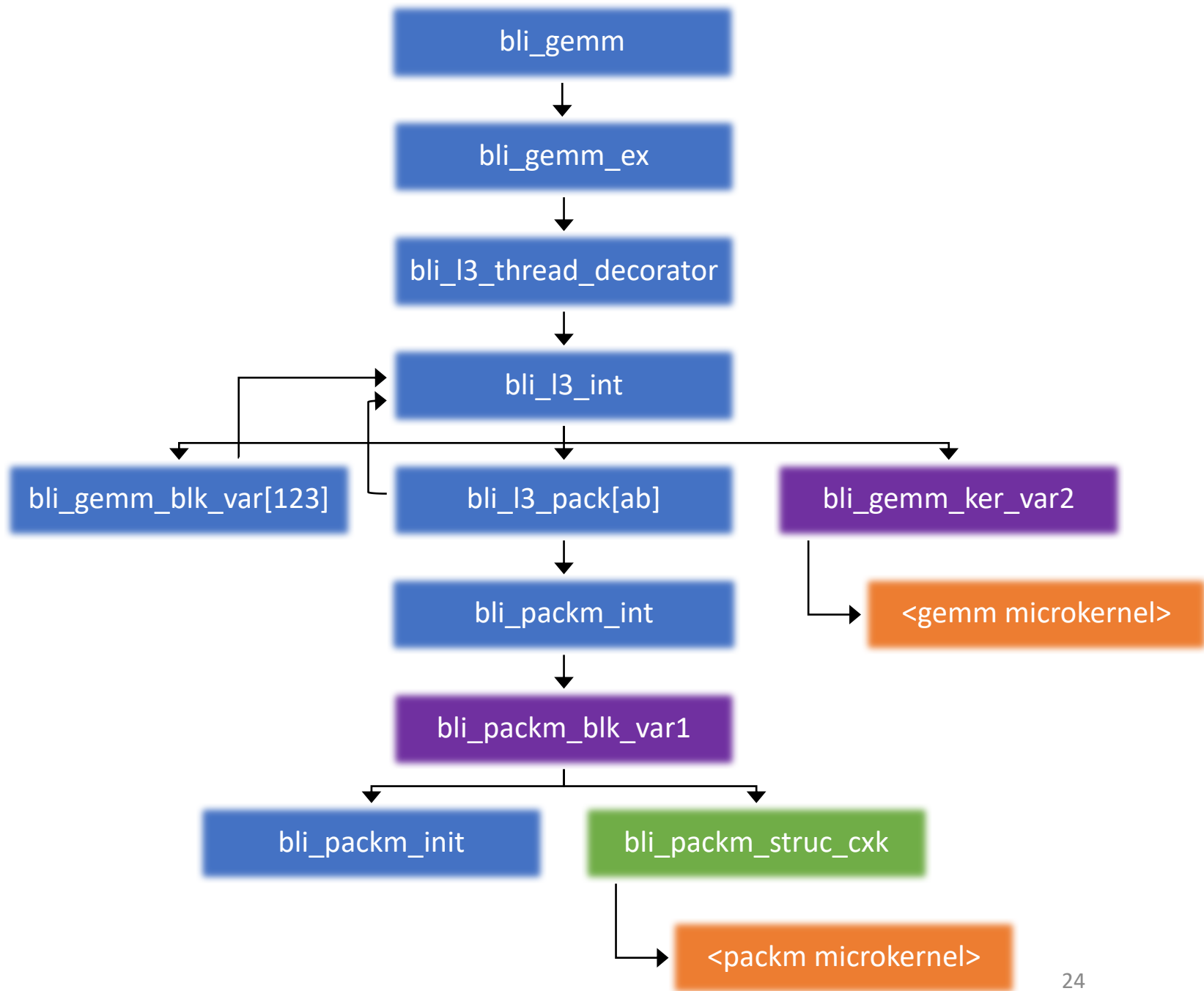
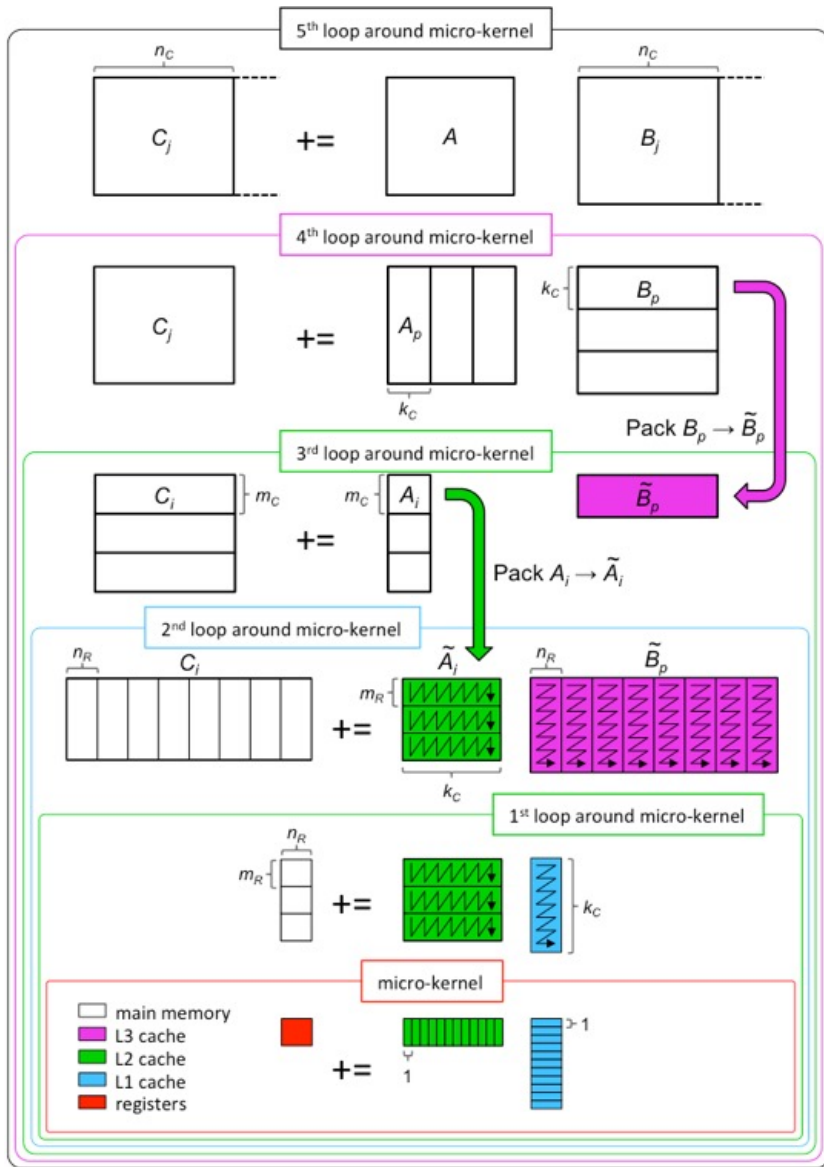


YES!









cntl_t* **control_tree**

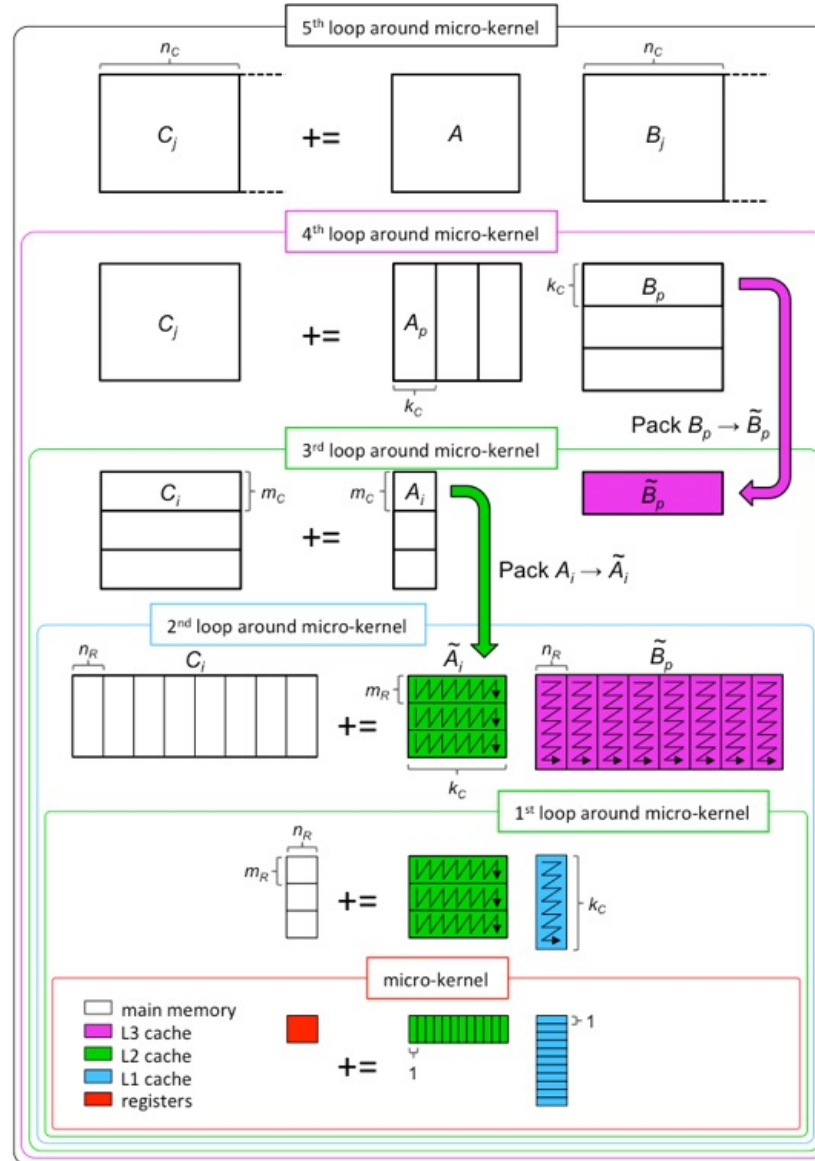
```

struct cntl_s
{
    // Basic fields
    // (usually required).
    opid_t      family;
    bszid_t     bszid;
    void_fp     var_func;
    struct cntl_s* sub_prenode;
    struct cntl_s* sub_node;

    // Optional fields (needed
    // only by some operations
    // such as packm).
    void*       params;

    // Internal fields that
    // track "cached" data.
    mem_t       pack_mem;
};
typedef struct cntl_s cntl_t;

```

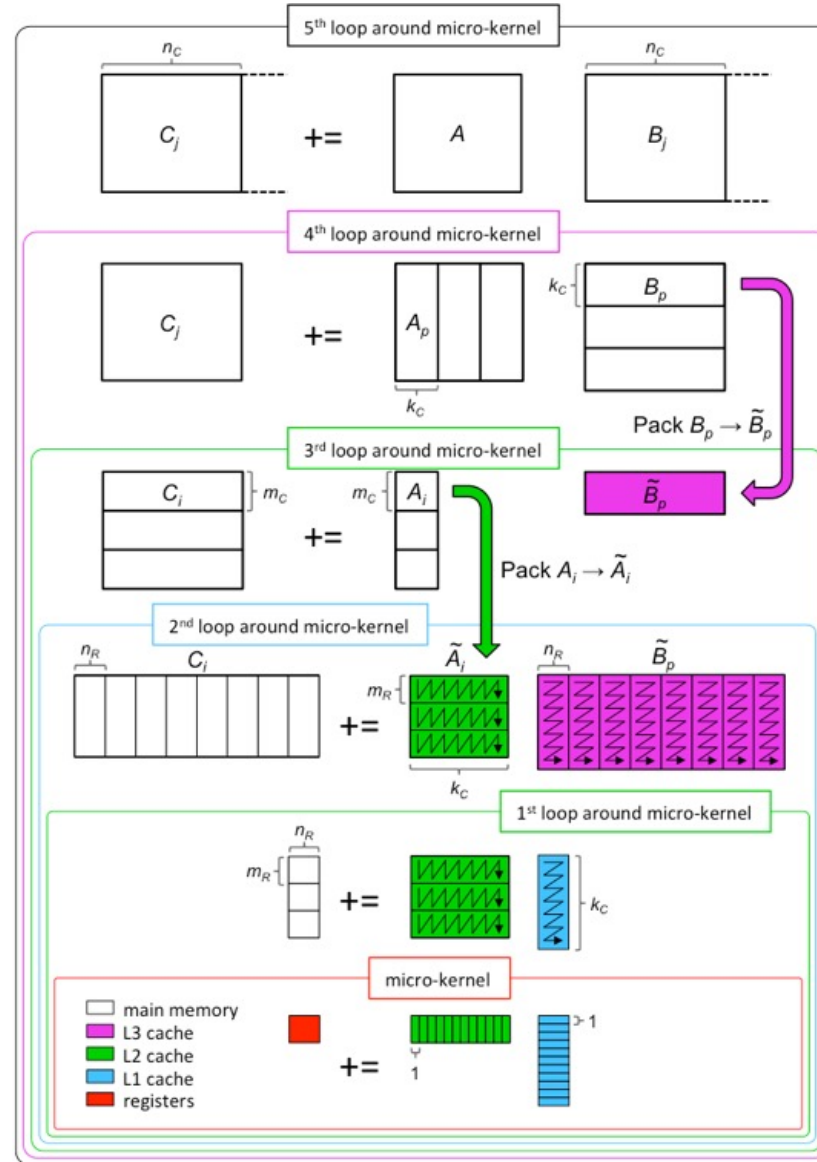


cntl_t* **control_tree**

```
struct cntl_s
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    bsid_t      bsid;
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    struct cntl_s* sub_node;

    // Optional fields (needed
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    void*       params;

    // Internal fields that
    // track "cached" data.
    mem_t       pack_mem;
};
typedef struct cntl_s cntl_t;
```



thrinfo_t*
thread_control_tree

```
struct thrinfo_s
{
    // The thread communicator for the
    // other threads sharing the same
    // work at this level.
    thrcomm_t*      ocomm;

    // Our thread id within the
    // ocomm thread communicator.
    dim_t           ocomm_id;

    // The number of distinct threads
    // used to parallelize the loop.
    dim_t           n_way;

    // What we're working on.
    dim_t           work_id;

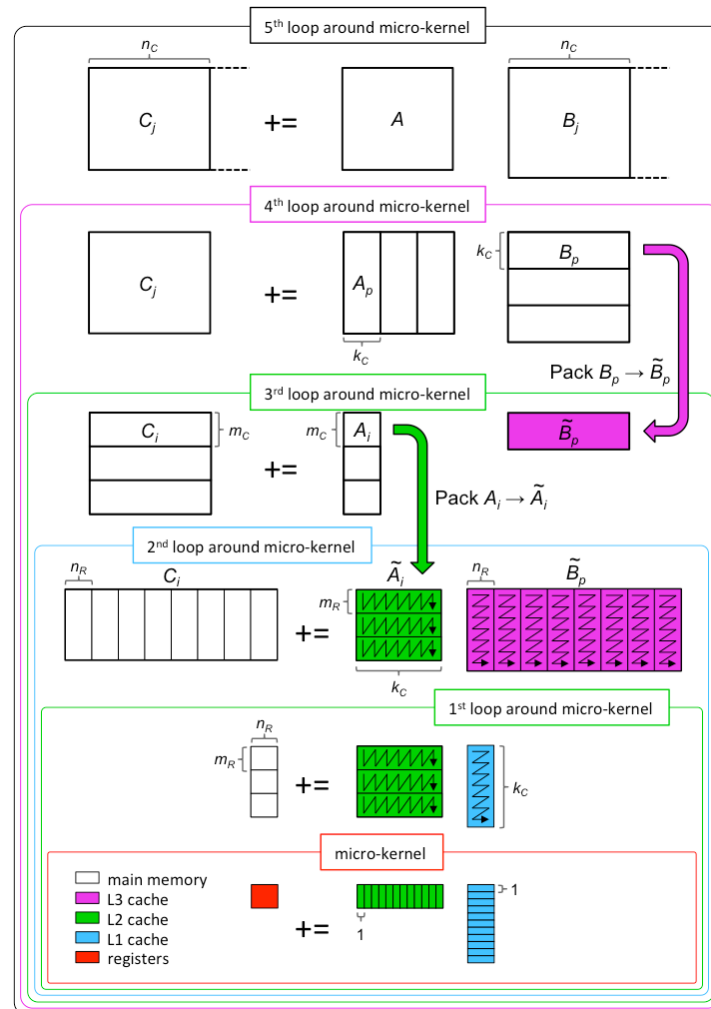
    bool            free_comm;
    bsid_t          bsid;

    struct thrinfo_s* sub_prenode;
    struct thrinfo_s* sub_node;
};
typedef struct thrinfo_s thrinfo_t;
```



A Vision Five(+) Years in the Making

A Vision Five Years in the Making: 2018

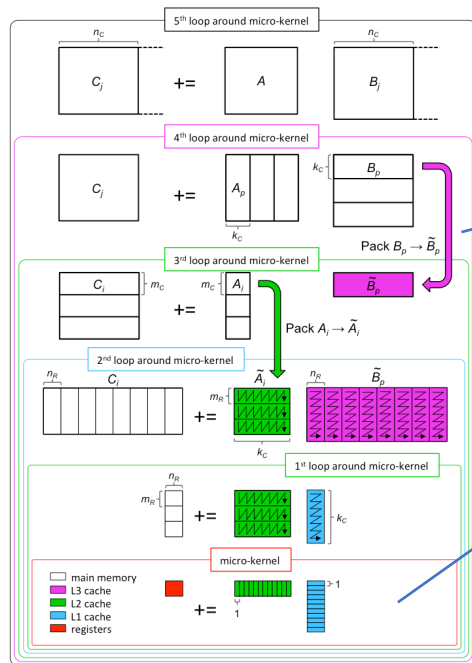


X.partition(...)

X.pack(...)

X.accum_ukr(...)

A Vision Five Years in the Making: 2019



X.pack (...)

- Scale row and/or columns?
- Cwise product?
- Generate on-the-fly: Toeplitz, Hankel, etc. (convolution)?
- Special ukr structure?
- Why not!

X.gemm_ker (...) or X.gemm_ukr (...)

- Accumulate cwise product (matrix dot)?
- Apply filter (e.g. Gaussian kernel)?
- $\alpha = (AB)_{ij}(B^T A^T)_{ij} = C_{ij}C_{ji}$?
(no I'm not kidding, I need this one)

High-level algorithm (FLAMEy part):
Composable pieces, loops, whatever

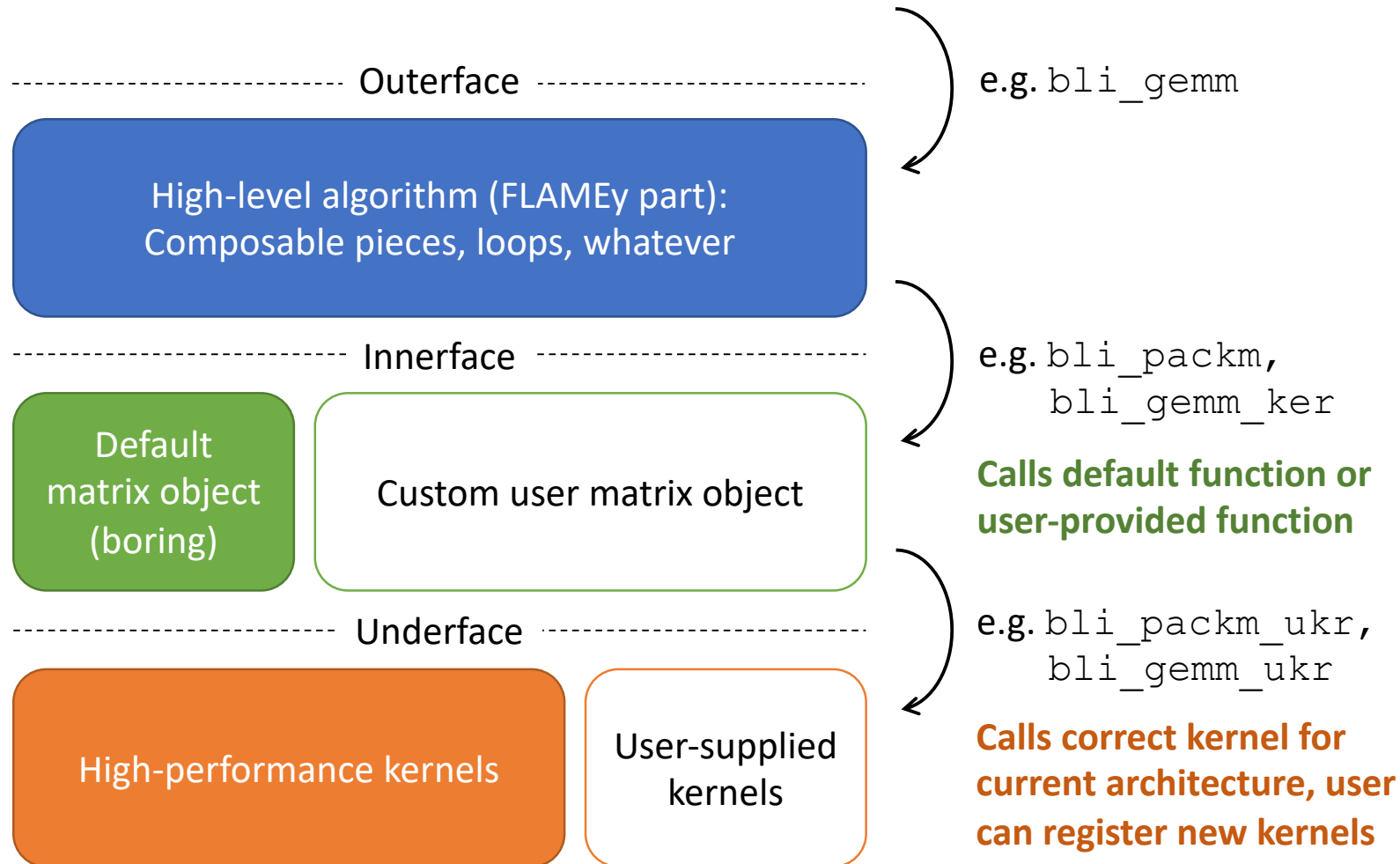
Default
matrix object
(boring)

Custom user matrix object

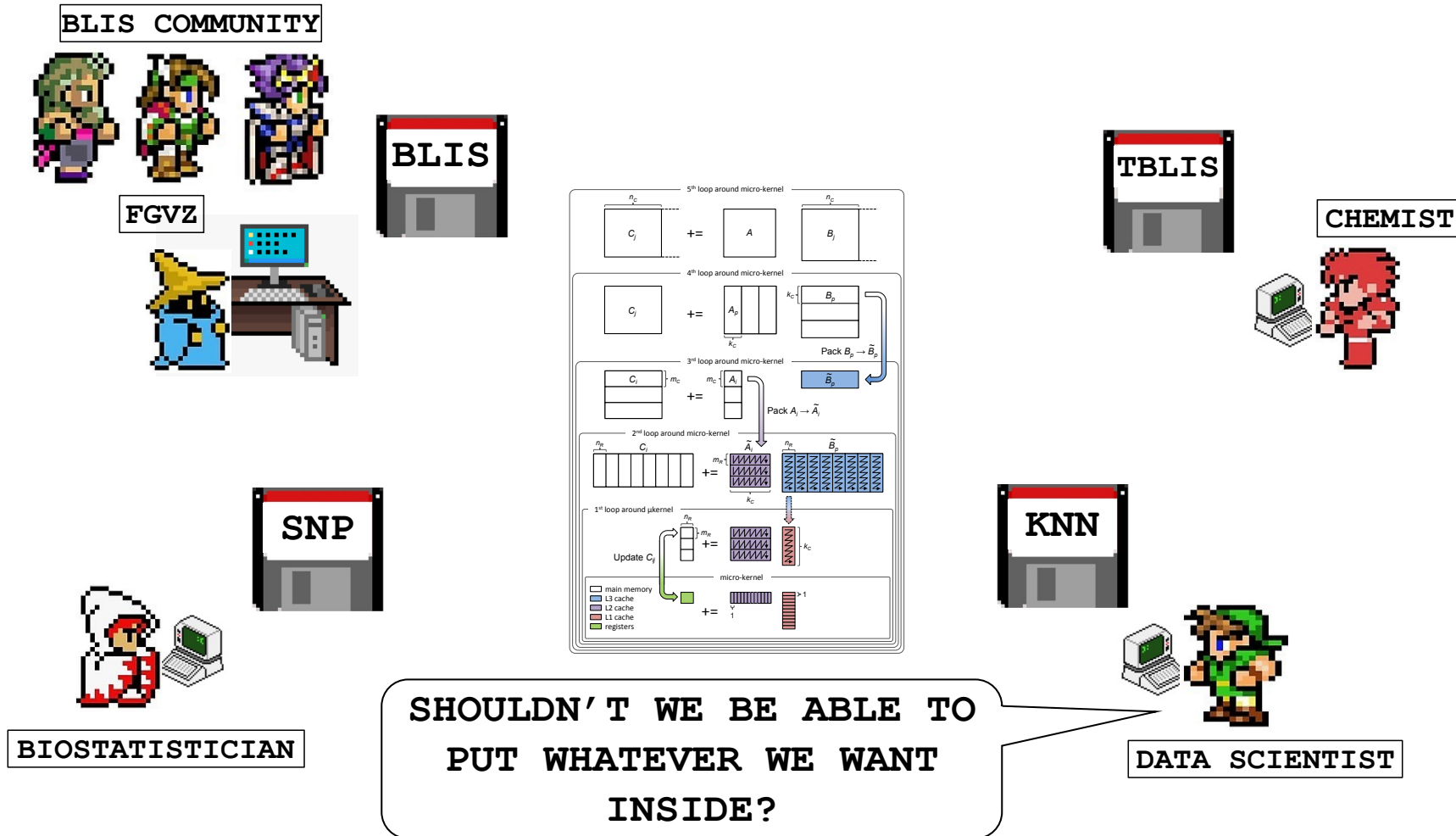
High-performance kernels

User-supplied
kernels

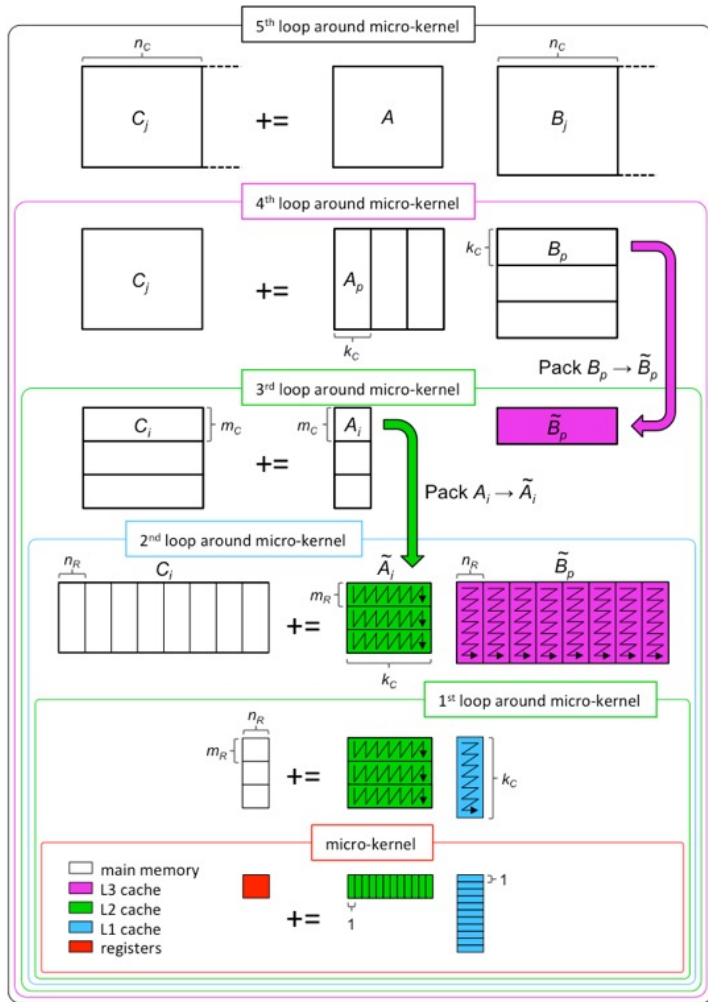
A Vision Five Years in the Making: 2020



A Vision Five Years in the Making: 2021



A Vision Five Years in the Making: 2022



```

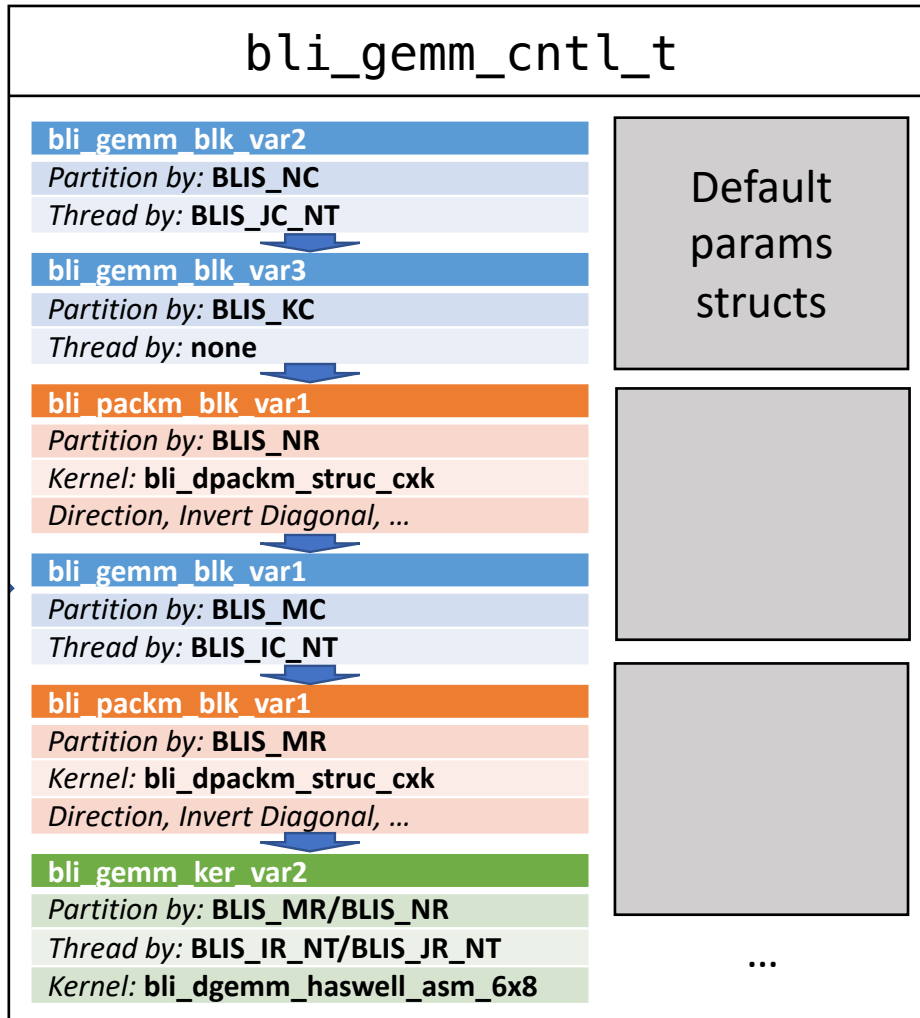
bli_gemm_blk_var2
Partition by: BLIS_NC
Thread by: BLIS_JC_NT
↓
bli_gemm_blk_var3
Partition by: BLIS_KC
Thread by: none
↓
bli_packm_blk_var1
Partition by: BLIS_NR
Kernel: bli_dpackm_struc_cxk
Direction, Invert Diagonal, ...
↓
bli_gemm_blk_var1
Partition by: BLIS_MC
Thread by: BLIS_IC_NT
↓
bli_packm_blk_var1
Partition by: BLIS_MR
Kernel: bli_dpackm_struc_cxk
Direction, Invert Diagonal, ...
↓
bli_gemm_ker_var2
Partition by: BLIS_MR/BLIS_NR
Thread by: BLIS_IR_NT/BLIS_JR_NT
Kernel: bli_dgemm_haswell_asm_6x8
    
```

Phase I: A Control Tree API

Complete revamp of the control tree structure:

- **ALL** parameters used by variants (partitioning, packing, macrokernel) encoded using the parameters struct. No more blocksize or microkernel IDs.
- Type information is stored and used to update the control tree with new information (user-defined kernels, blocksizes, etc.).
- 1m is built-in: most user-defined modifications continue to work with 1m.
- Users can supply blocksizes/kernels from the context, or arbitrary integers/pointers.
- For gemm-like operations, the control tree is stored entirely on the stack and shared by all threads.
- Users can supply additional operation-specific data via opaque structs.

Phase I: A Control Tree API



```
bli_gemm_cntl_set_pack[ab]_var(...)
bli_gemm_cntl_set_pack[ab]_ukr(...)
bli_gemm_cntl_set_pack[ab]_params(...)
bli_gemm_cntl_set_pack[ab]_schema(...)
```

```
bli_gemm_cntl_set_{mc,nc,kc}(...)
bli_gemm_cntl_set_{mr,nr}(...)
```

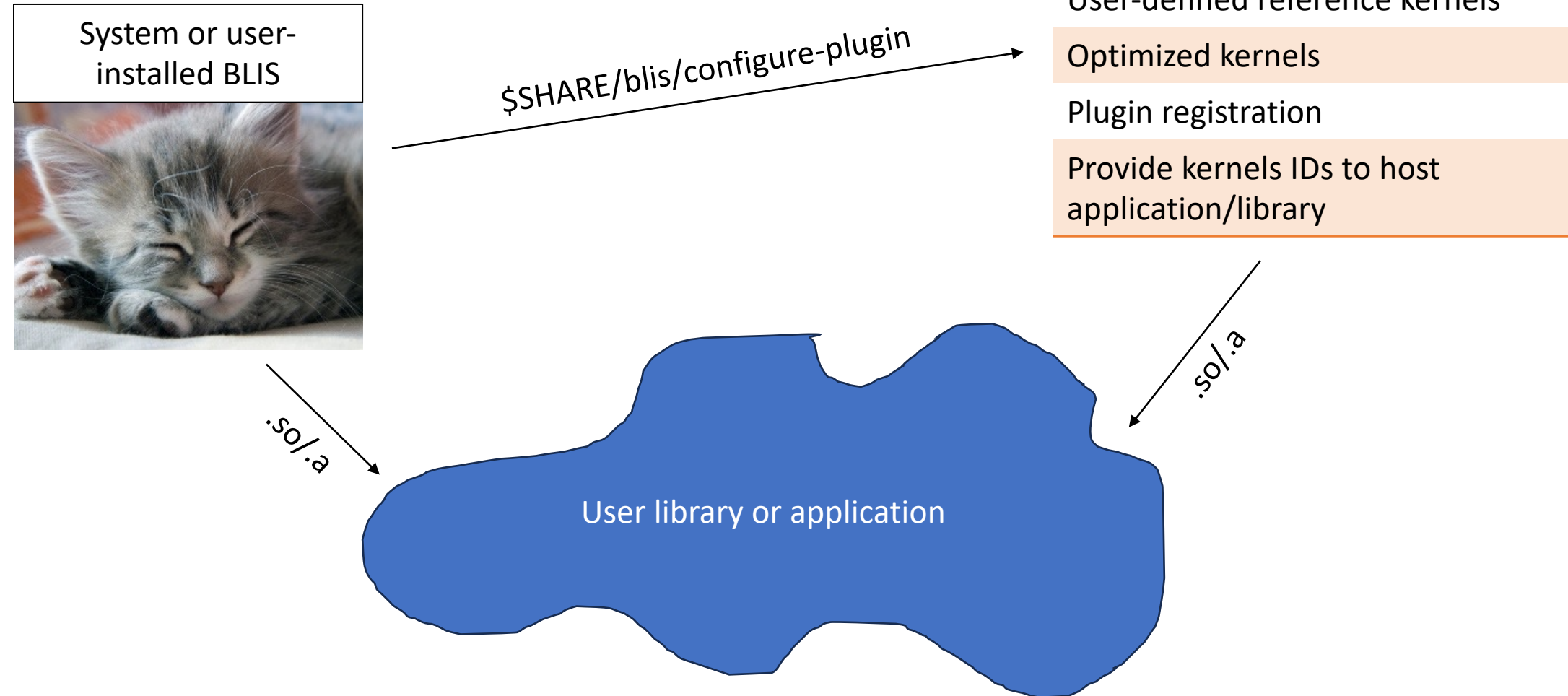
```
bli_gemm_cntl_set_ukr(...)
bli_gemm_cntl_set_params(...)
```

Phase II: External Plugin API

Users will often want to use custom kernels. With plugins...

- Users can code kernels outside of BLIS and register them to the context.
- Have reference kernels compiled for all configured architectures using the original BLIS compiler and flags.
- Provide optimized kernels for architectures of choice. These will override reference kernels when running on the matching architecture.
- Also provide blocksizes and kernel preferences.
- Register a (near-)unlimited number of new kernels, blocksizes, and preferences.
- Kernels are compiled into a static and/or shared object for inclusion in the user's codebase.

Phase II: External Plugin API



Phase III: Mixed-Precision/Mixed-Domain

Most BLIS kernels are now explicitly mixed-precision

- Currently supported: all combinations of sdcz
- Packing (reference kernels are templated C99 code)
- Microkernel (reference mixed-precision microkernel is a type-casting wrapper kernel)
- Real $\leftrightarrow$ complex usually not needed (handled by induced methods)
- Soon to be extended to Level 1 and 2 BLAS

Phase III: Mixed-Precision/Mixed-Domain

Storage precision/domain (C matrix)

**Example: optimization for
GEMM microkernel**

Hand-optimized

Reference (wrapper ukr)

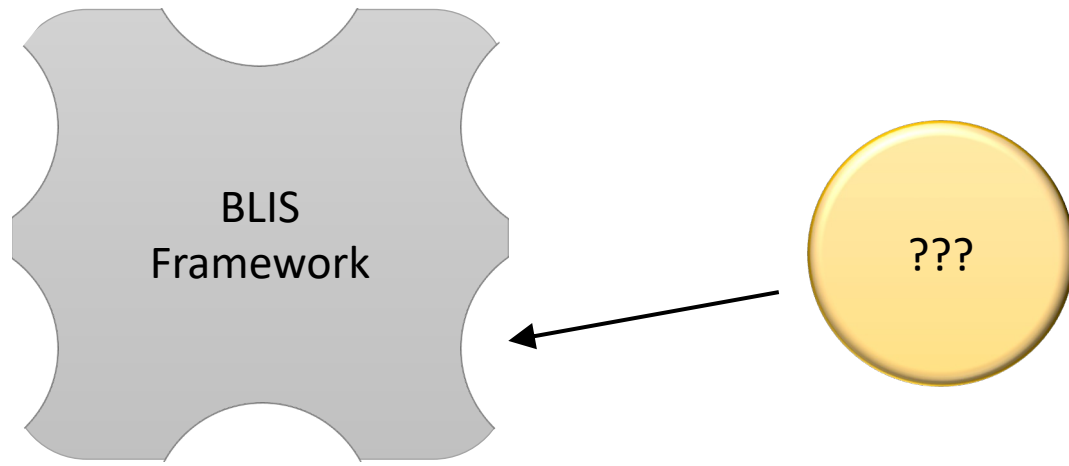
Not used

Computational precision		s	d	c	z
	s	Hand-optimized	Hand-optimized	Reference (wrapper ukr)	Reference (wrapper ukr)
	d	Hand-optimized	Hand-optimized	Reference (wrapper ukr)	Reference (wrapper ukr)
	c	Reference (wrapper ukr)	Reference (wrapper ukr)	Hand-optimized	Hand-optimized
	z	Reference (wrapper ukr)	Reference (wrapper ukr)	Hand-optimized	Hand-optimized

Phase IV: Expanded Precision (In Progress)

Framework support for reduced (e.g. float16, bfloat16, int8/16) and extended (e.g. double-double, float128) support.

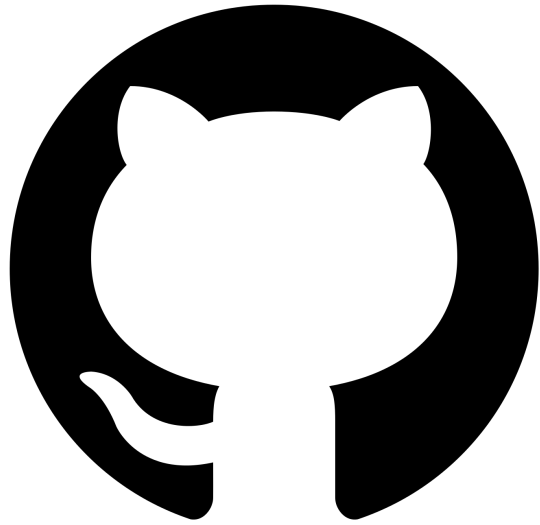
- For “core” types: full reference kernel support, either via HW support or software implementation/upcasting when not available.
- Mixed-precision with standard and other expanded types.
- Rich macro library for Level 0 (scalar) operations.
- Potentially: run-time registration of new types!



Demo of plugins and control tree
API tomorrow after lunch!



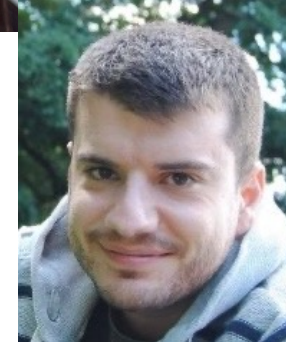
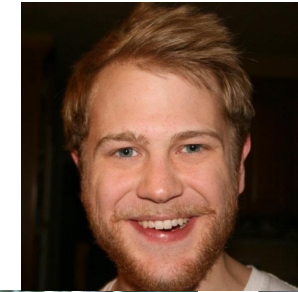
BLIS is a Community Project



115+ contributors



Bryan
Marker



AMD

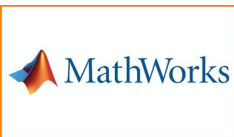
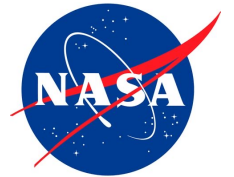
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ORACLE®

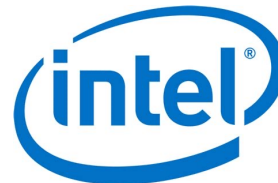


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