

BLIS Day the 13th: **BLIS out of BLAS**

Devin Matthews, Southern Methodist University





Outline

- **Part I:** BLIS Yesterday and Today
- **Part II:** TBLIS
- **Part III:** Revisiting libflame



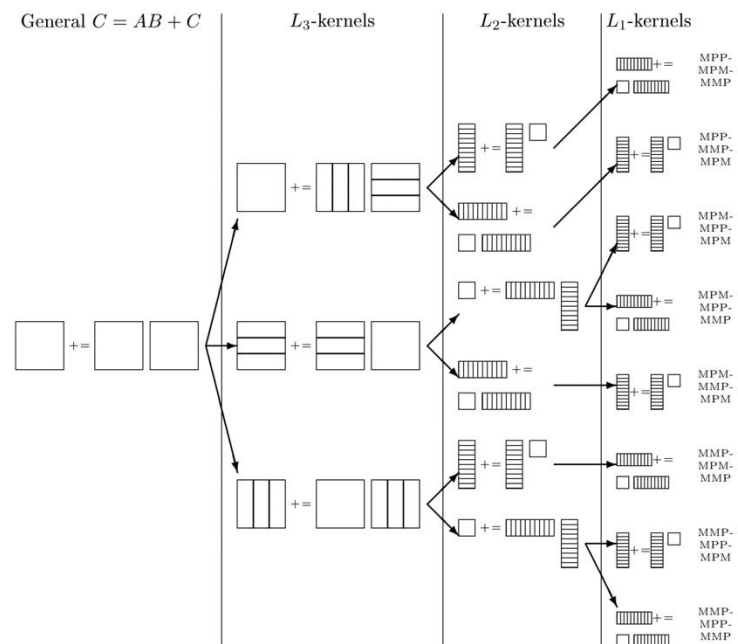


Figure 3: Possible algorithms for matrices in memory level L_3 given all L_2 -kernels.

ITX-GEMM



John Gunnels (UT, now NVIDIA)



Greg Henry (Intel, now NVIDIA)

High-performance implementation of the level-3 BLAS (GotoBLAS)

Authors:  Kazushige Goto,  Robert Van De Geijn [Authors Info & Claims](#)

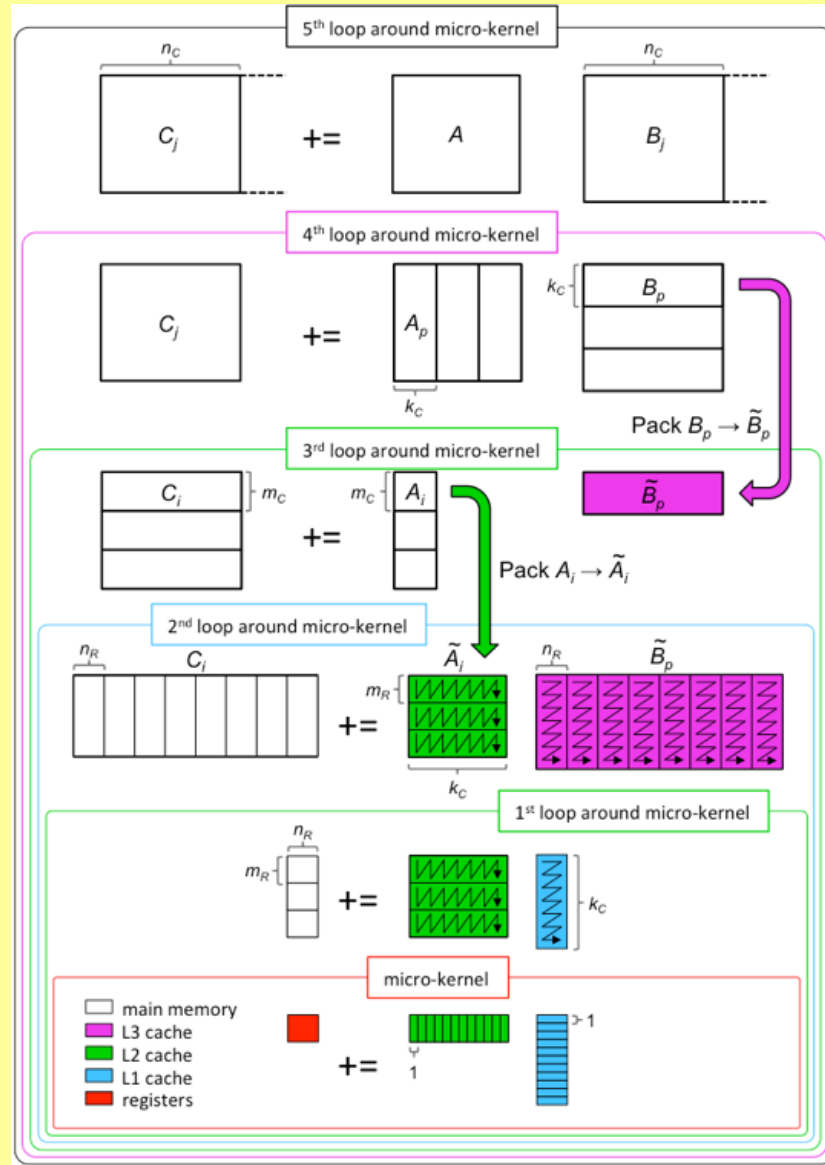
ACM Transactions on Mathematical Software, Volume 35, Issue 1 • Article No.: 4, pp 1–14 • <https://doi.org/10.1145/1377603.1377607>

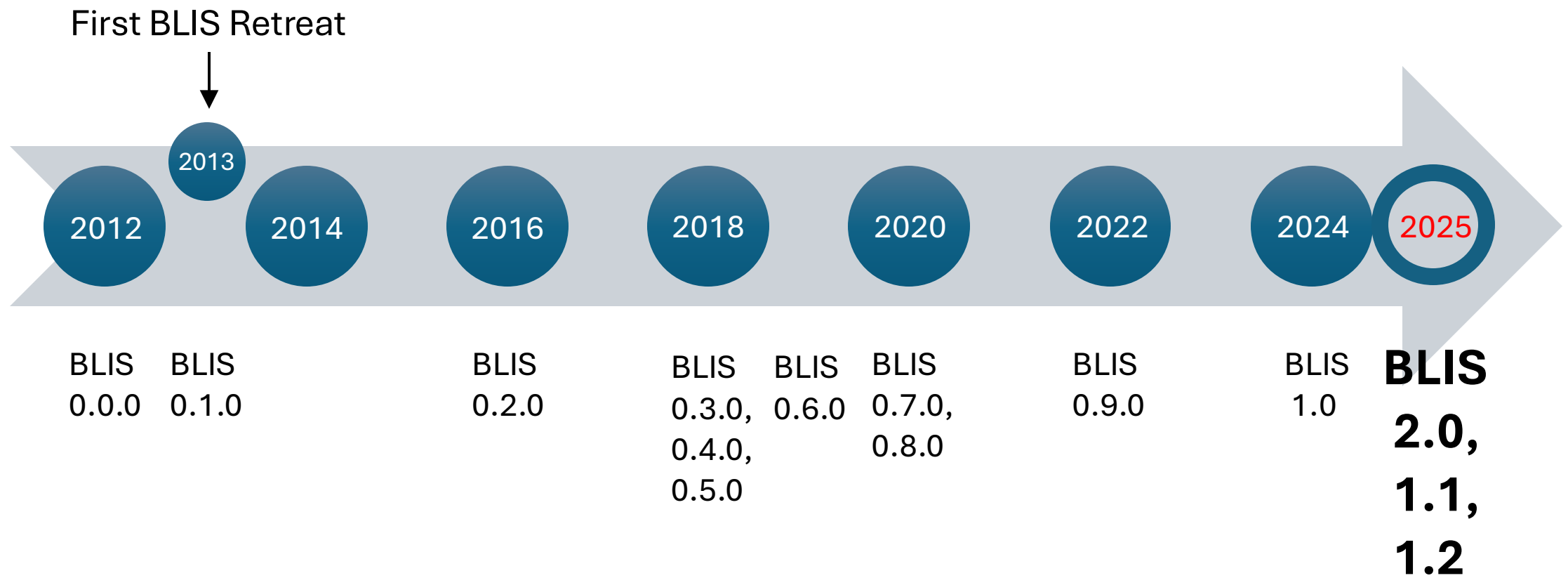
Published: 25 July 2008 [Publication History](#)

 Check for updates

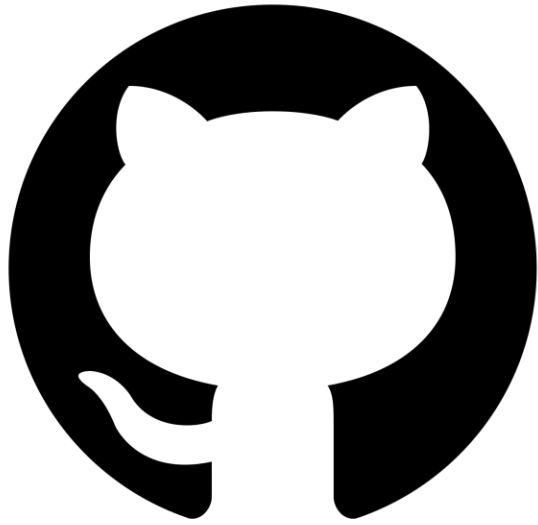
 205  1,534

   [eReader](#) [PDF](#)





BLIS is a Community Project



141+ contributors



Bryan
Marker



AMD

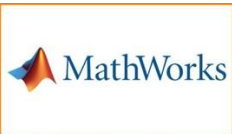
ARM®

ORACLE®



Microsoft

NEC



IBM



HUAWEI



Qualcomm



NVIDIA®

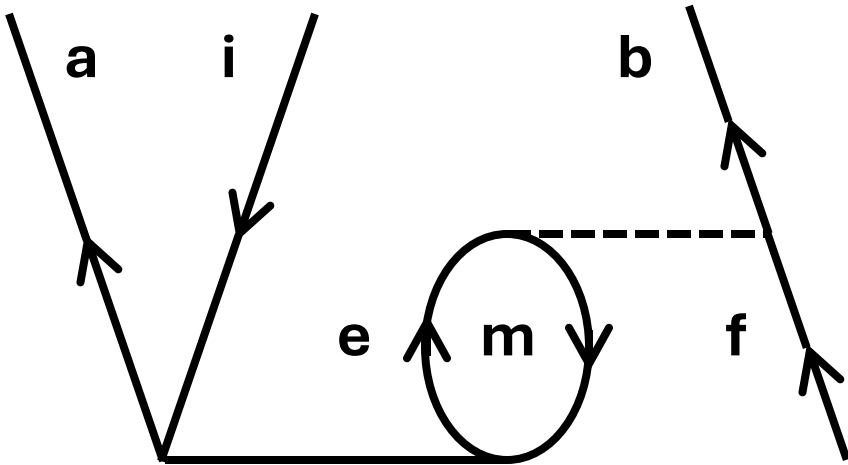


ODEN INSTITUTE
FOR COMPUTATIONAL ENGINEERING & SCIENCES



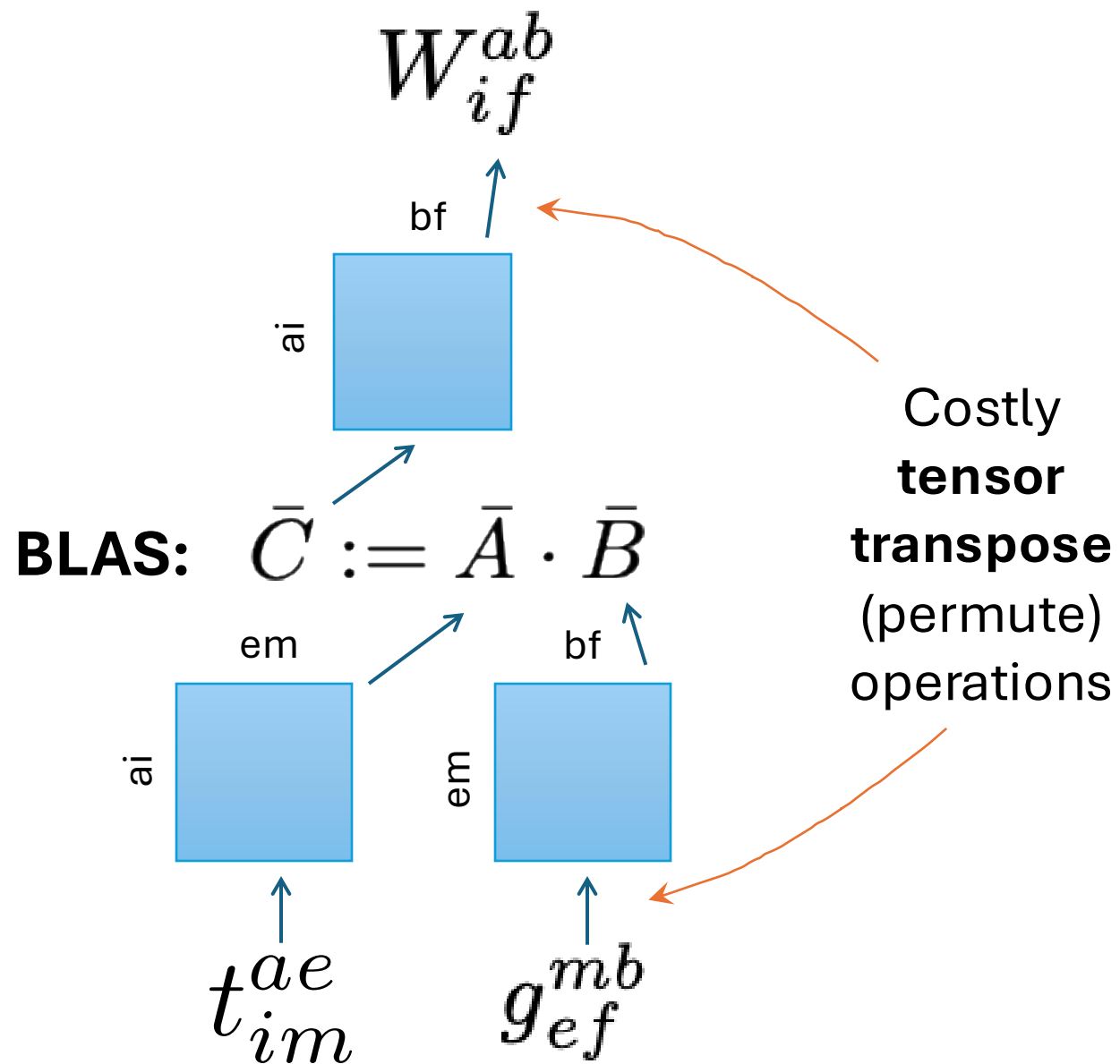
The Tensor-Based Library Instantiation Software (TBLIS)

a.k.a. “Tensor BLIS”



$$W_{if}^{ab} \leftarrow 2t_{im}^{ae} g_{ef}^{mb}$$

```
tblis::mult(2.0, t2, "aeim",
            g, "mbef",
            1.0, W, "abif");
```



“Transpose-Transpose-GEMM-Transpose” (TTGT)

$$\mathcal{C}_{abif} := \mathcal{A}_{aeim} \cdot \mathcal{B}_{mbej}$$



$$\tilde{C}_{IJ} := \tilde{A}_{IP} \cdot \tilde{B}_{PJ}$$

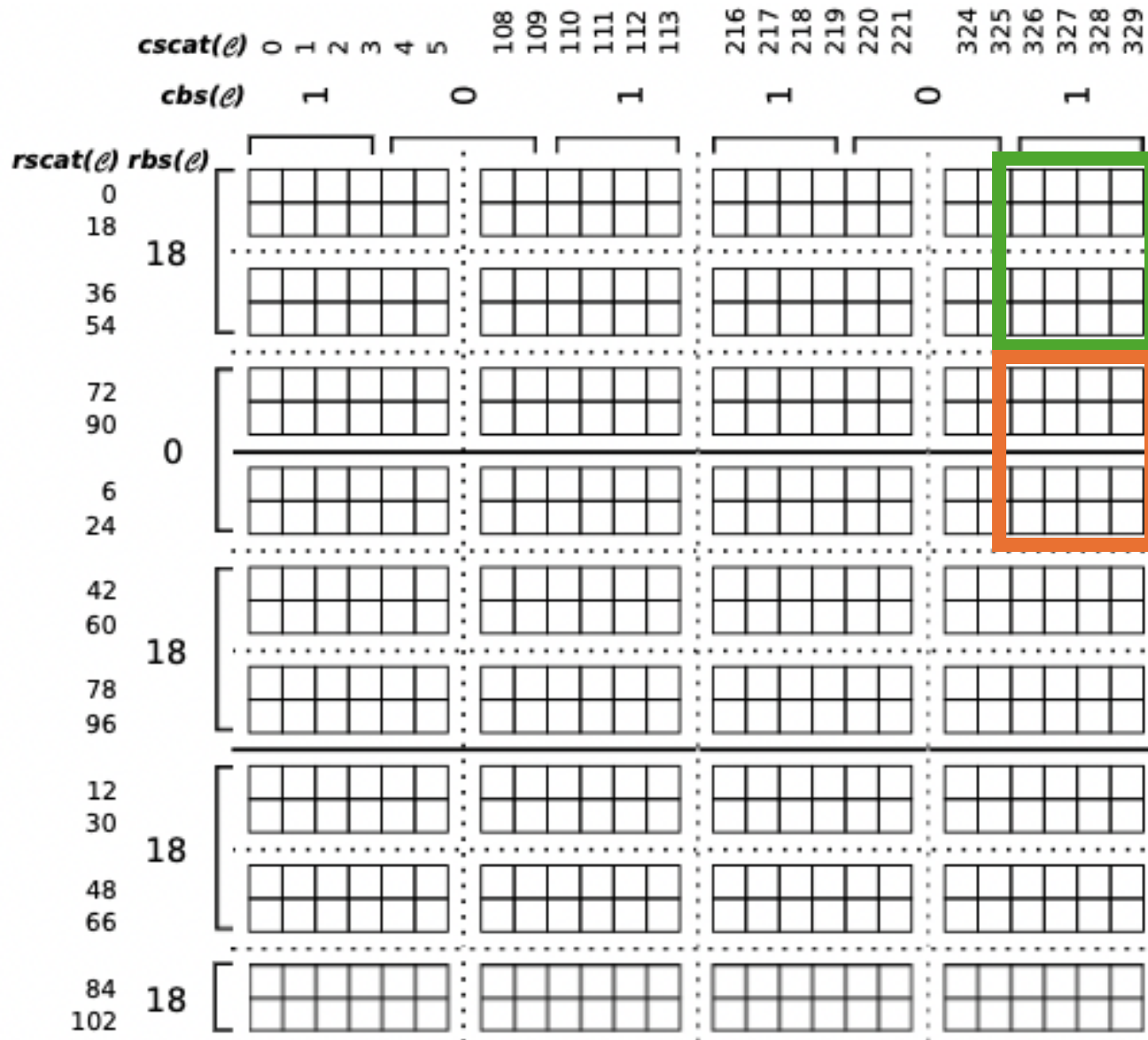
		cscat(ℓ)																													
		0	1	2	3	4	5	108	109	110	111	112	113	216	217	218	219	220	221	324	325	326	327	328	329						
		cbs(ℓ)																													
		1					0					1					1					0					1				
rscat(ℓ)	rbs(ℓ)																														
0	18																														
18																															
36																															
54																															
72	0																														
90																															
6																															
24																															
42	18																														
60																															
78																															
96																															
12	18																														
30																															
48																															
66																															
84	18																														
102																															

“Block-Scatter Matrix Tensor Contraction” (BSMTC)

$$\mathcal{C}_{abif} := \mathcal{A}_{aeim} \cdot \mathcal{B}_{mbej}$$



$$\tilde{C}_{IJ} := \tilde{A}_{IP} \cdot \tilde{B}_{PJ}$$



Well-defined row and column strides:
Use **standard matrix-multiply microkernel**

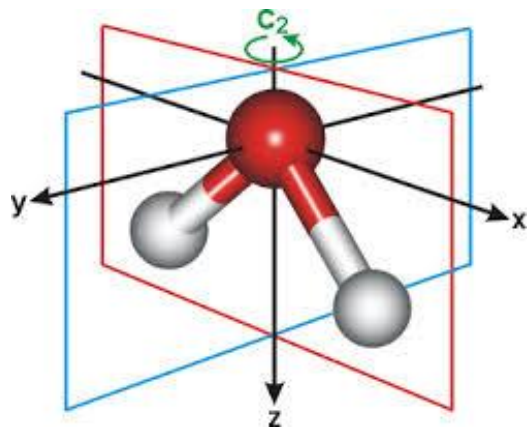
Irregular "jump" along rows and/or columns:
Write from matrix-multiply microkernel into temporary buffer, then write back using **scatter vectors**.

TBLIS 2.0

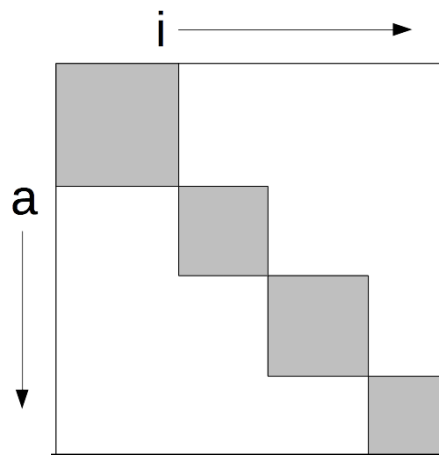
- Uses a **BLIS plugin** to leverage optimized kernels across x86_64 (AMD, Intel), ARM, IBM, etc.
- Significantly improved complex number support (1m).
- New C++ API which supports various front-ends/tensor manipulation libraries (e.g. MArray, Eigen::Tensor, xtensor, Armadillo, std::mdspan). Compatibility mode for TBLIS 1.x is supported.
- New CMake build system makes integration and packaging easier.
- C++20 (☺/☹)



The Direct Product Decomposition



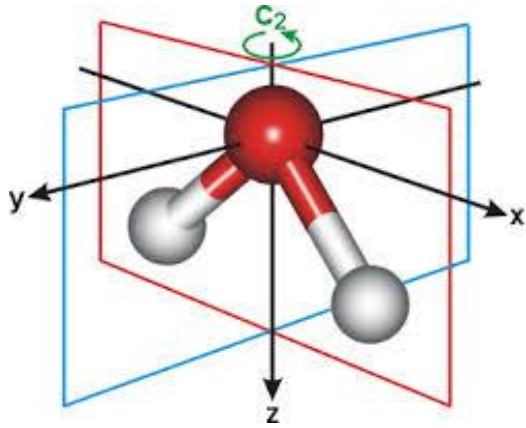
$$T_i^a \longrightarrow$$
$$\Gamma_a \otimes \Gamma_i = \Gamma_T$$



Point Group Symmetry

Cost savings proportional to g^2
(g = number of irreducible representations/blocks).

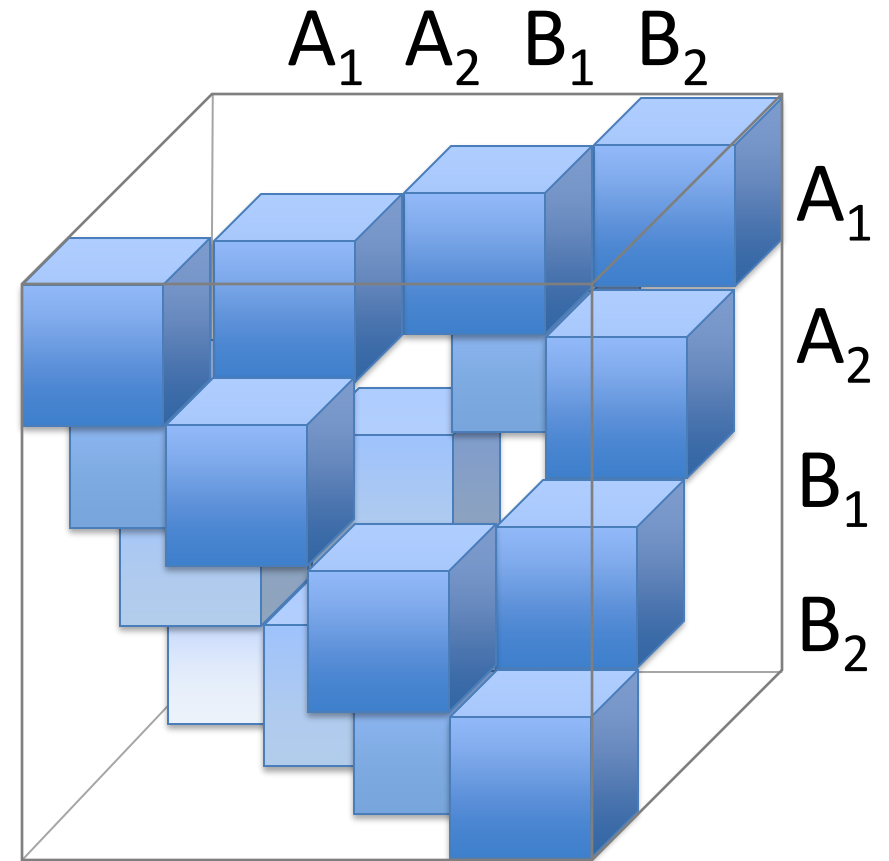
The Direct Product Decomposition



Z_{abc}

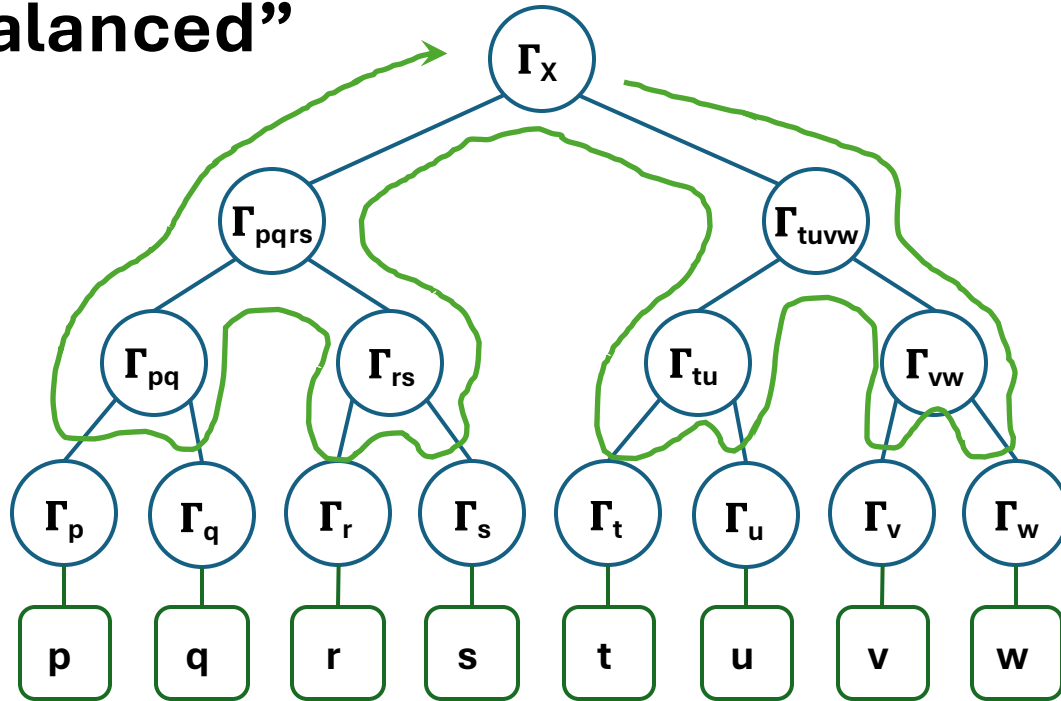


$$\Gamma_a \otimes \Gamma_b \otimes \Gamma_c = \Gamma_Z$$



The Direct Product Decomposition

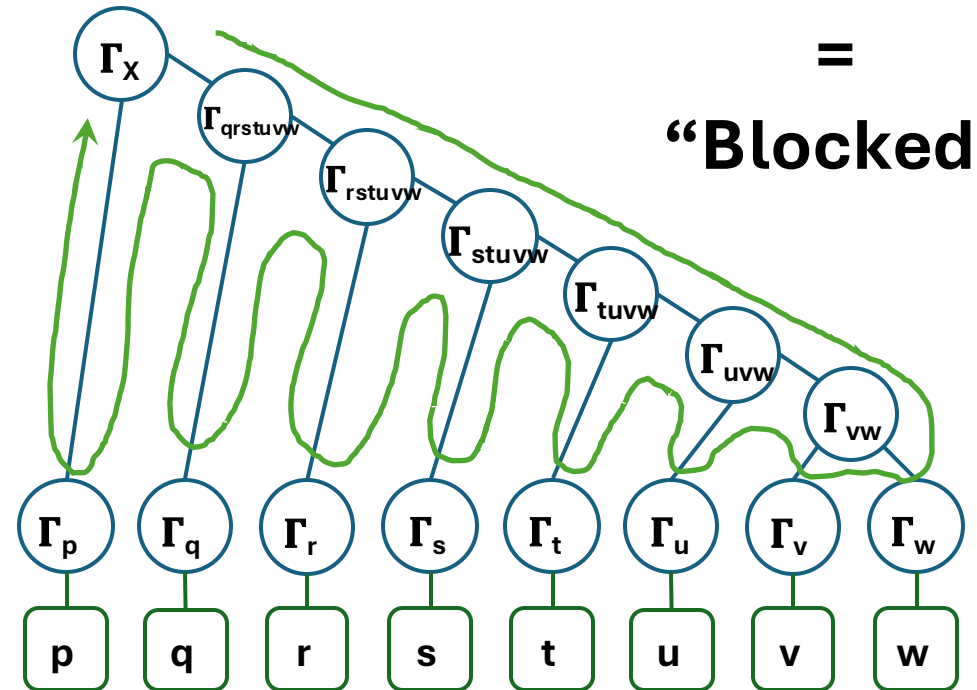
“Balanced”



“Postfix”

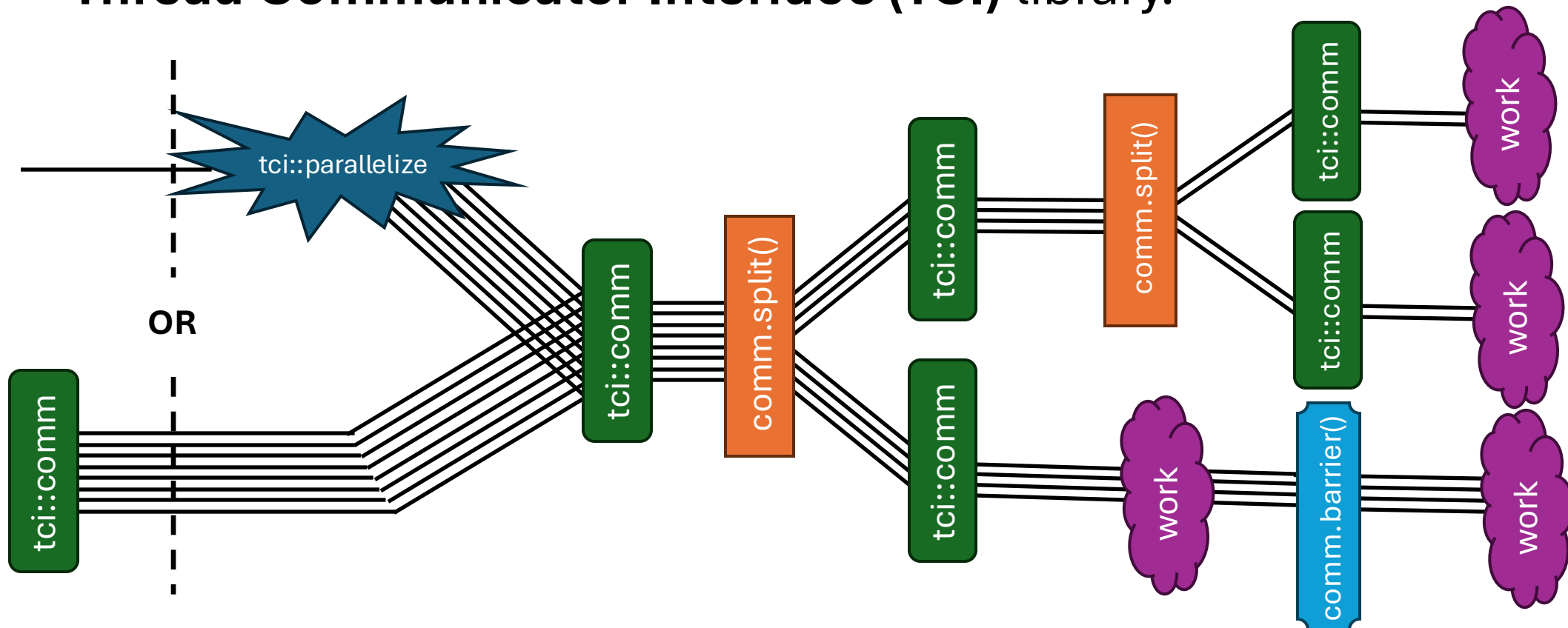
=

“Blocked”



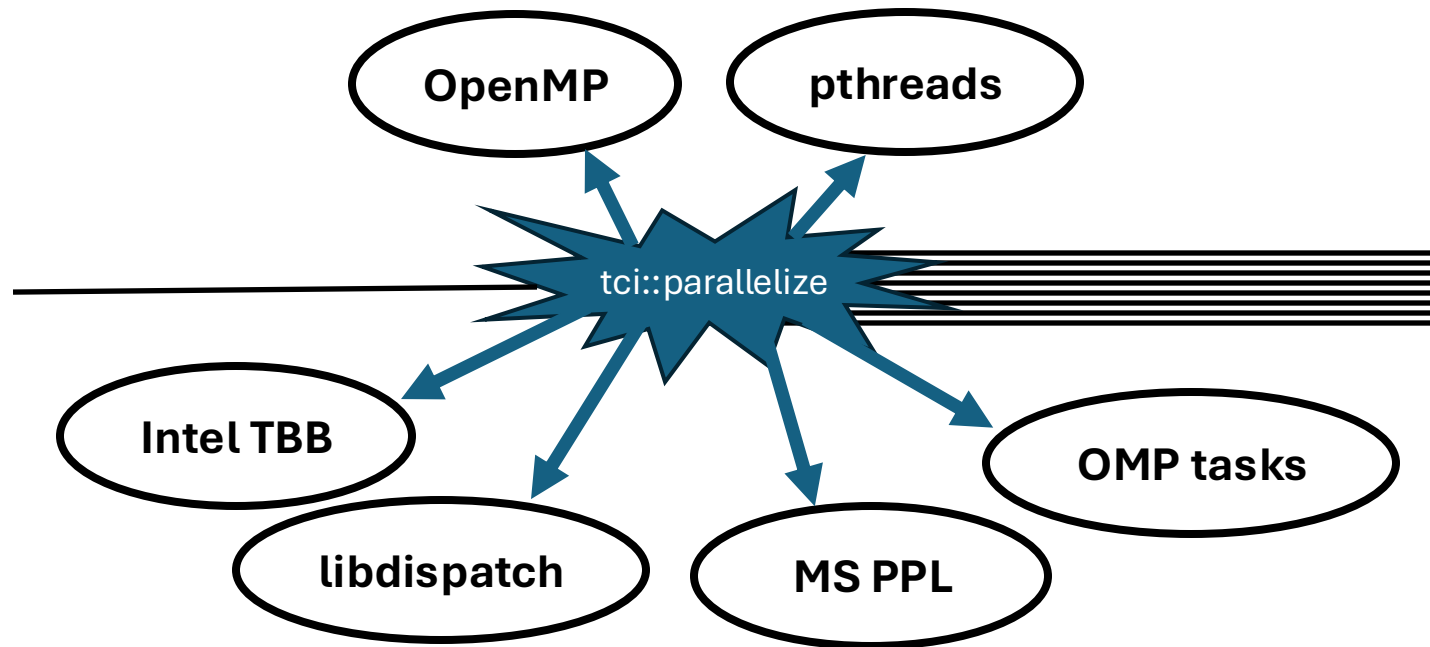
Multi-threading

- Threading over multiple cores (up to 100s) uses the external **Thread Communicator Interface (TCI)** library.



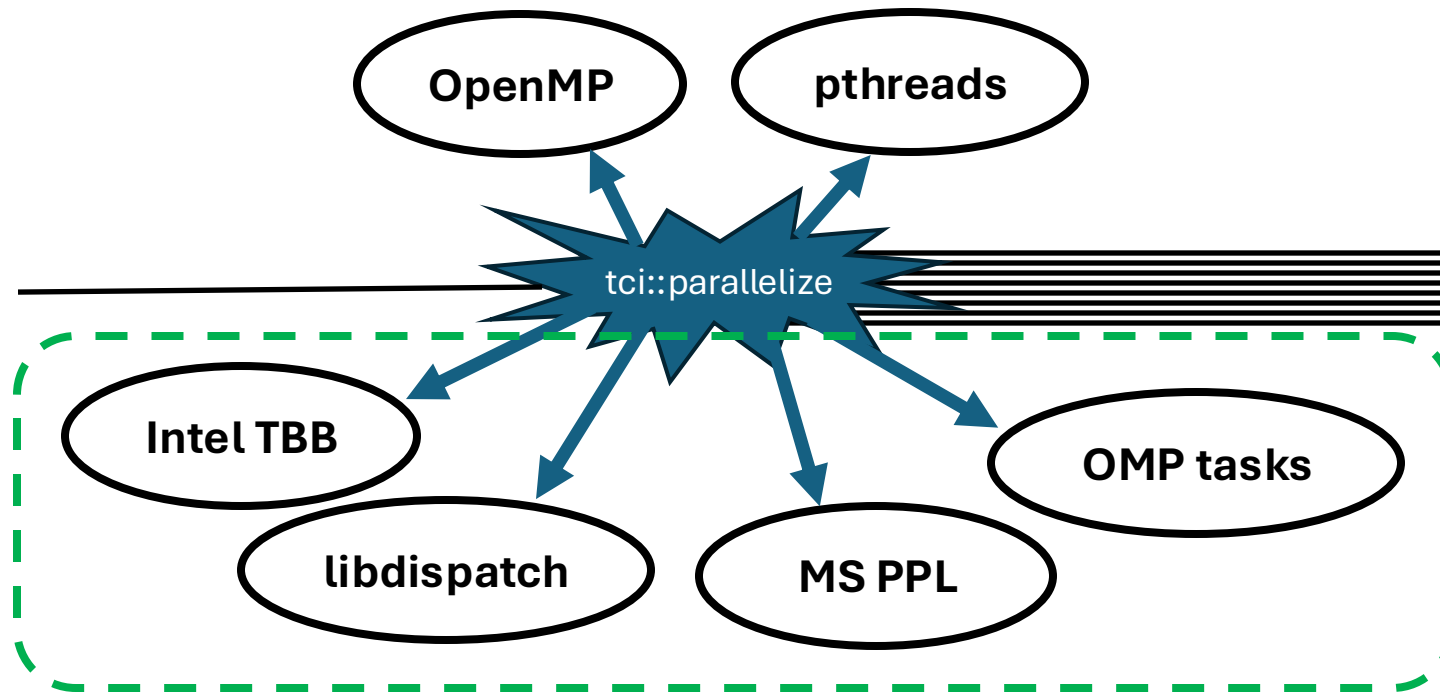
Multi-threading

- Threading over multiple cores (up to 100s) uses the external **Thread Communicator Interface (TCI)** library.



Multi-threading

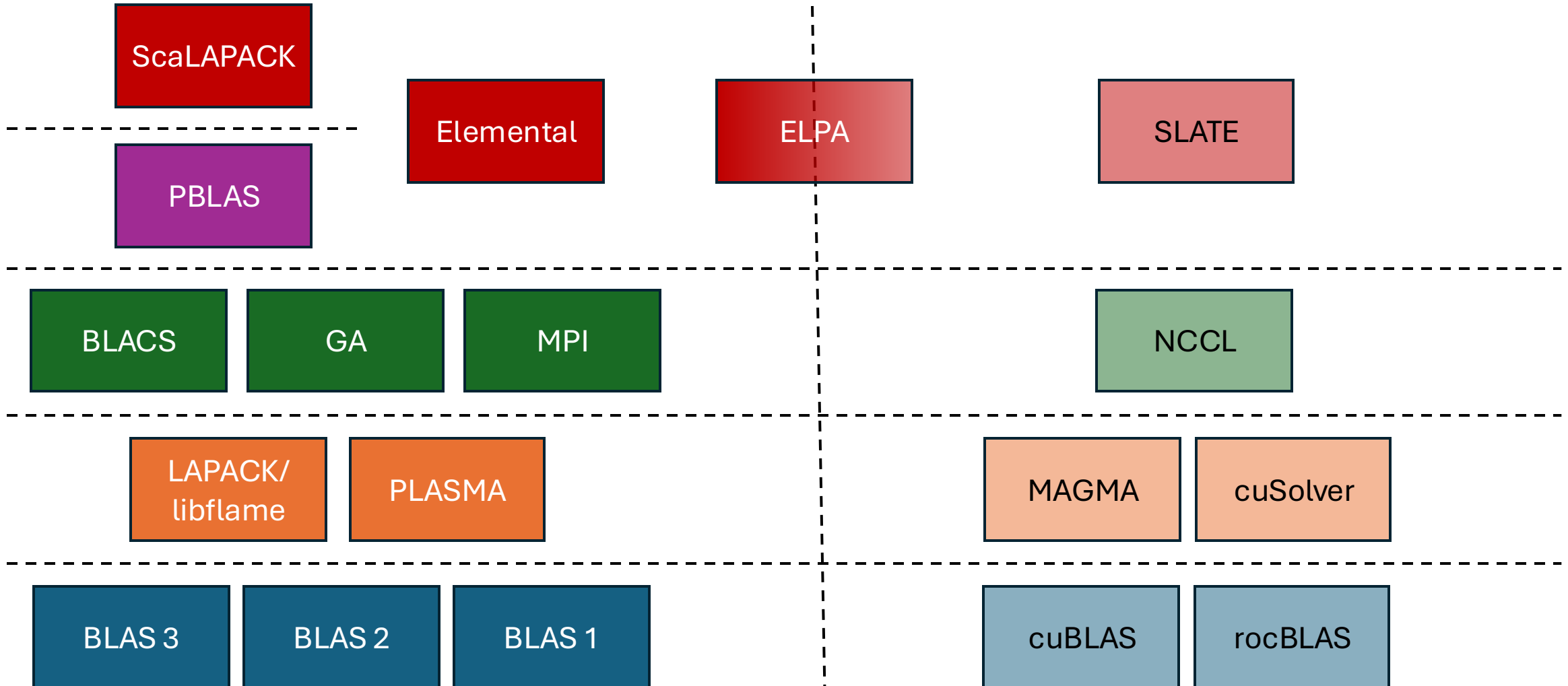
- Threading over multiple cores (up to 100s) uses the external **Thread Communicator Interface (TCI)** library.



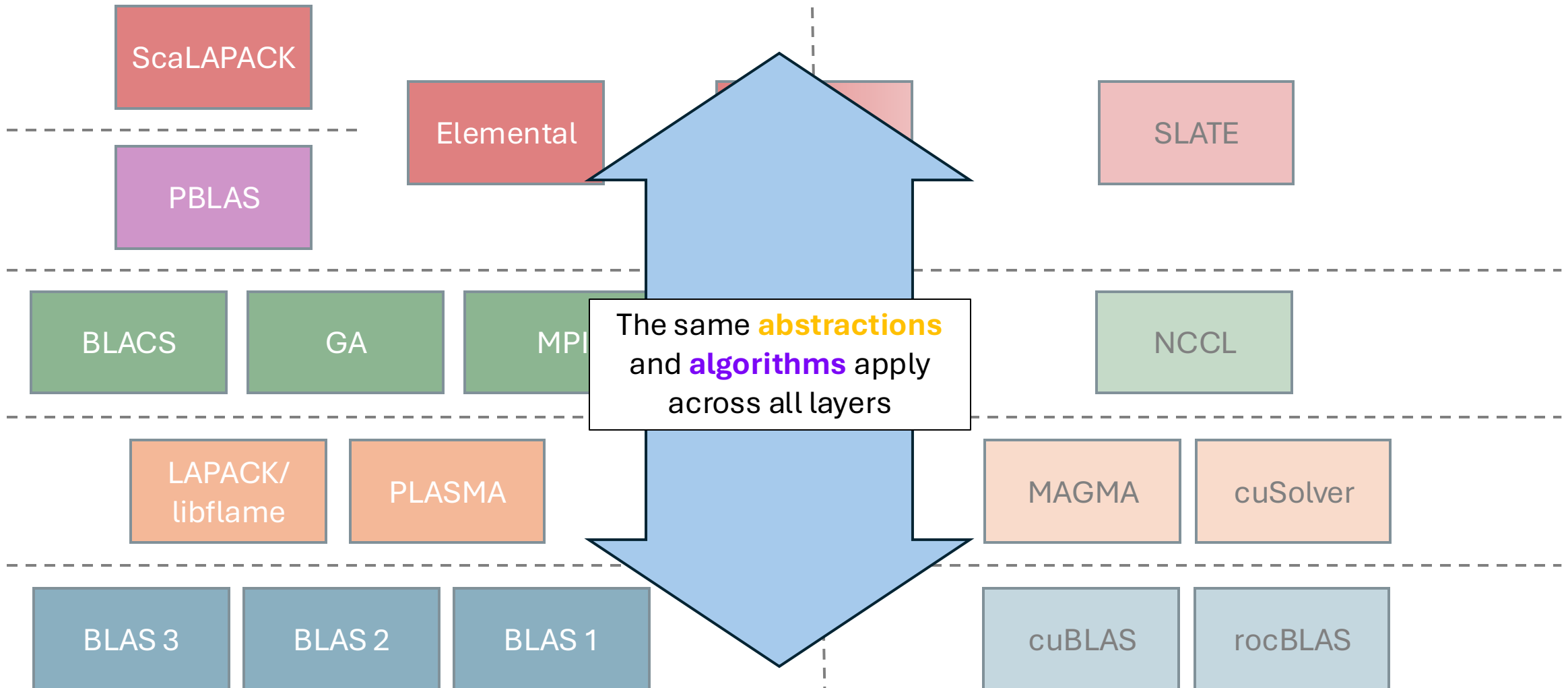
Revisiting libflame et al.:

All in the FAMILIES

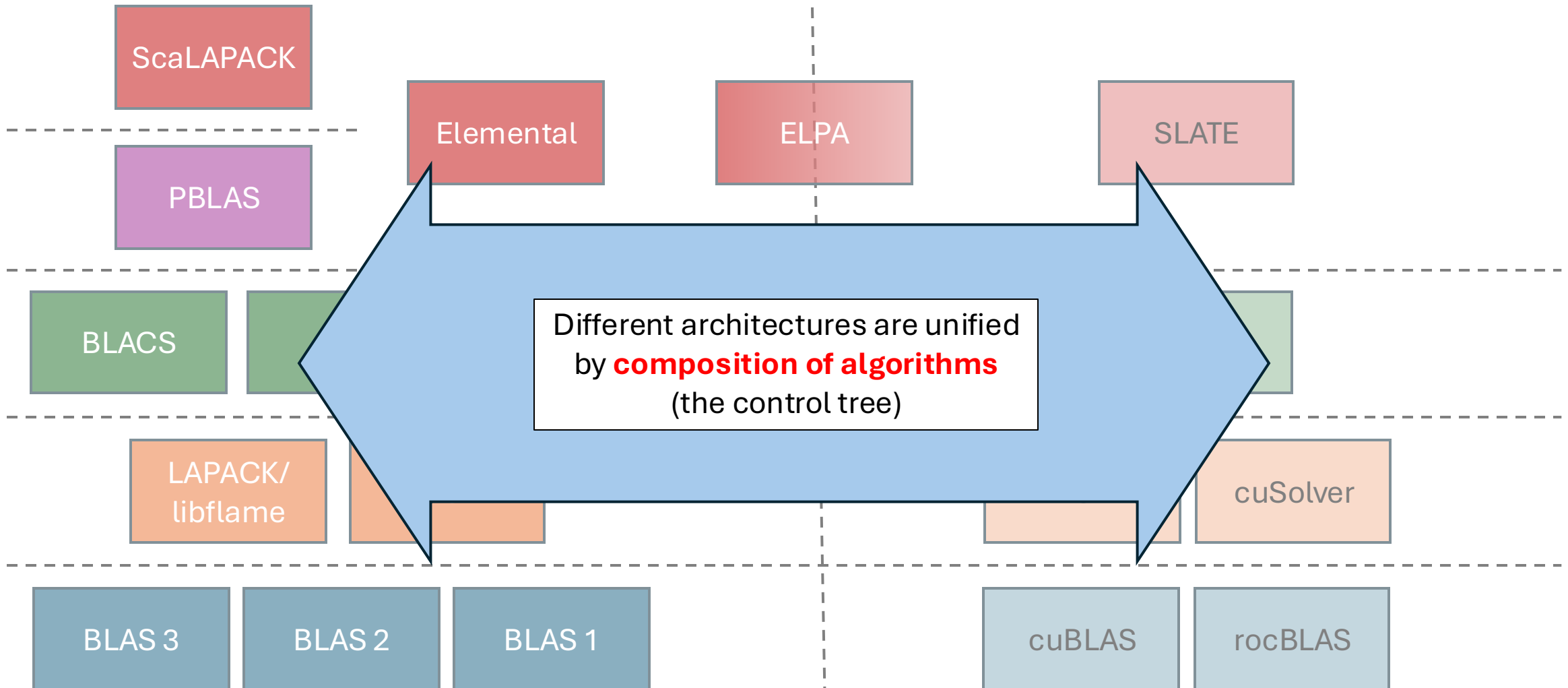
The Traditional DLA Stack



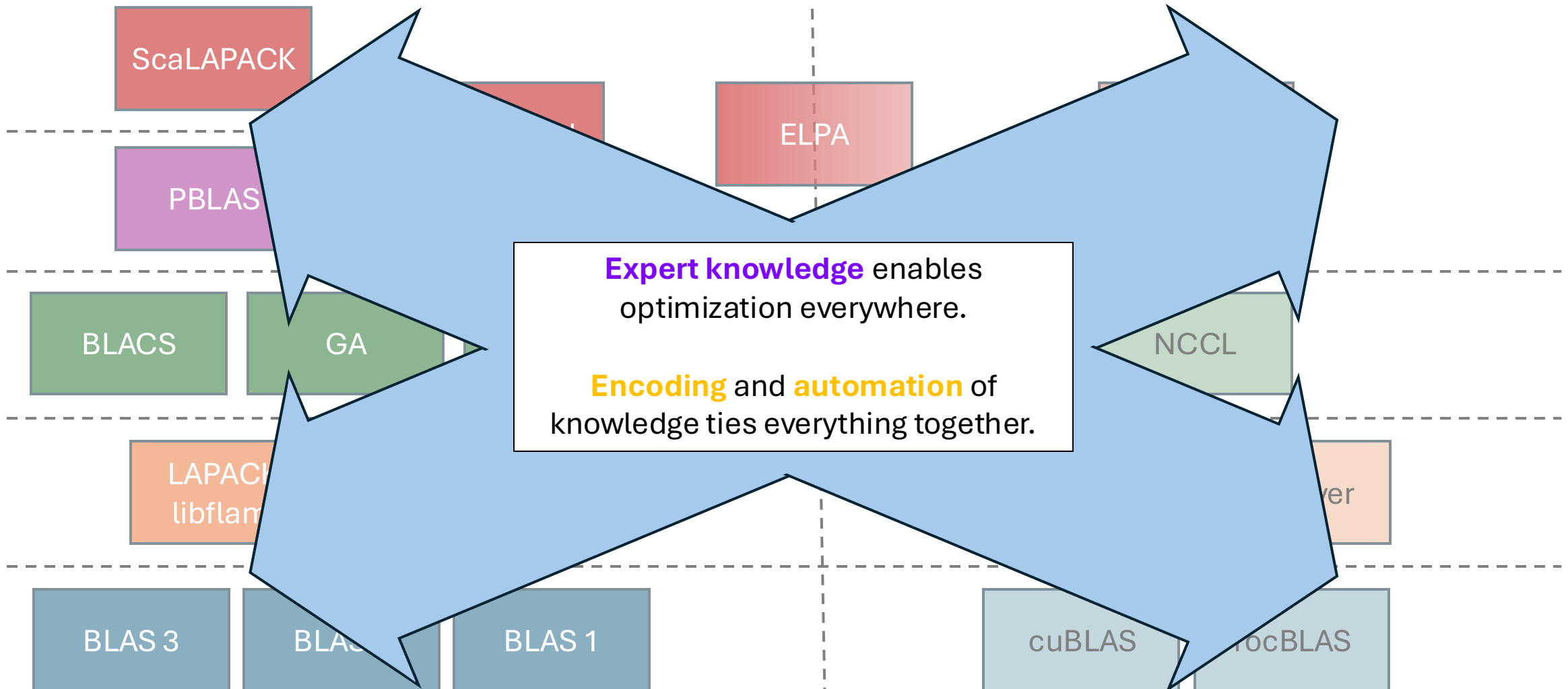
The Traditional DLA Stack



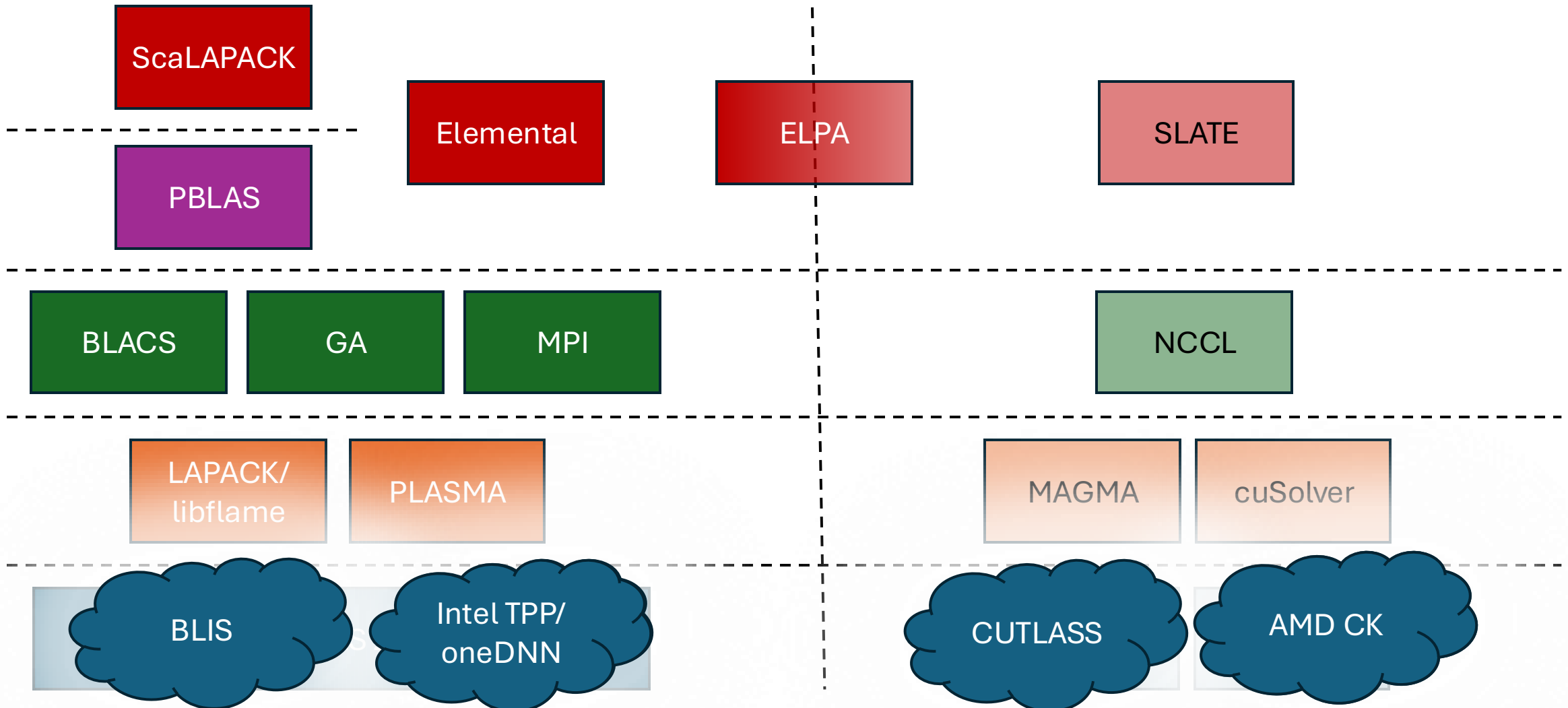
The Traditional DLA Stack

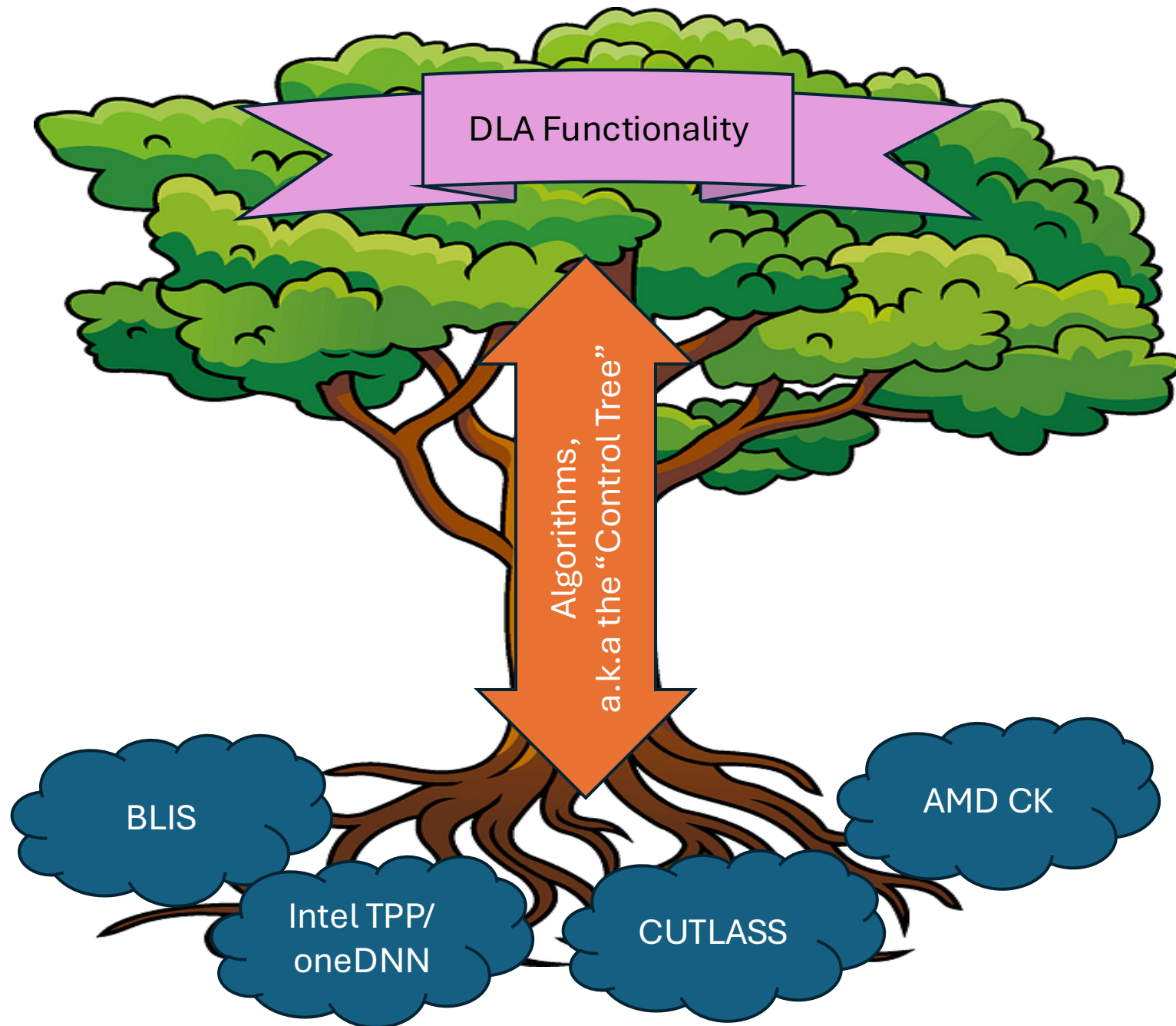


The Traditional DLA Stack



Recent Developments





FAMLIES



$$A = LL^H \quad (1)$$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c|c} L_{TL} & * \\ \hline L_{BL} & L_{BR} \end{array} \right) \wedge \left(\begin{array}{c|c} \hat{A}_{TL} = L_{TL} L_{TL}^H & * \\ \hline \hat{A}_{BL} = L_{BL} L_{BL}^H & A_{BR} = L_{BL} L_{BL}^H + L_{BR} L_{BR}^H \end{array} \right) \quad (2)$$

Invariant	
1	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L_{TL} & * \\ \hline L_{BL} & L_{BR} \end{array} \right) \wedge \hat{A}_{TL} = L_{TL} L_{TL}^H$
2	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L_{TL} & * \\ \hline L_{BL} & L_{BR} \end{array} \right) \wedge \hat{A}_{TL} = L_{TL} L_{TL}^H$
3	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L_{TL} & * \\ \hline L_{BL} & L_{BR} \end{array} \right) \wedge \hat{A}_{TL} = L_{TL} L_{TL}^H$

Step	Algorithm: $A := \text{CHOL}(A)$
1	$k \leftarrow A$
2	$A \leftarrow \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} - \begin{pmatrix} L_{TL} & * \\ L_{BL} & L_{BR} \end{pmatrix} \begin{pmatrix} L_{TL}^H & 0 \\ 0 & 0 \end{pmatrix}$
3	$\text{while } m(A_{TL}) < m(A) \text{ do}$
2.3	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c} L_{TL} & * \\ \hline L_{BL} & L_{BR} \end{array} \right) \begin{pmatrix} L_{TL}^H & 0 \\ 0 & 0 \end{pmatrix} \wedge m(A_{TL}) < m(A)$
5a	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c} L_{TL} & * \\ \hline L_{BL} & L_{BR} \end{array} \right) \begin{pmatrix} L_{TL}^H & 0 \\ 0 & 0 \end{pmatrix}$
6	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c} L_{TL} & * \\ \hline L_{BL} & L_{BR} \end{array} \right) \begin{pmatrix} L_{TL}^H & 0 \\ 0 & 0 \end{pmatrix}$
7	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c} L_{TL} & * \\ \hline L_{BL} & L_{BR} \end{array} \right) \begin{pmatrix} L_{TL}^H & 0 \\ 0 & 0 \end{pmatrix}$
5b	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c} L_{TL} & * \\ \hline L_{BL} & L_{BR} \end{array} \right) \begin{pmatrix} L_{TL}^H & 0 \\ 0 & 0 \end{pmatrix}$
2	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c} L_{TL} & * \\ \hline L_{BL} & L_{BR} \end{array} \right) \begin{pmatrix} L_{TL}^H & 0 \\ 0 & 0 \end{pmatrix}$
2.3	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c} L_{TL} & * \\ \hline L_{BL} & L_{BR} \end{array} \right) \begin{pmatrix} L_{TL}^H & 0 \\ 0 & 0 \end{pmatrix}$
5b	$k \leftarrow k - 1$

Algorithm: $A := \text{CHOL_BLK}(A)$
$A \rightarrow \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}$
where A_{TL} is 0×0
while $m(A_{TL}) < m(A)$ do
Determine block size b
$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c c} A_{00} & A_{01} \ A_{02} \\ \hline A_{10} & A_{11} \ A_{12} \\ A_{20} & A_{21} \ A_{22} \end{array} \right)$
Variant 1:
$A_{10} := A_{10} L_{00}^{-H}$
$A_{11} := A_{11} - A_{10} A_{00}^{-1} A_{10}^H$
$A_{11} := \text{Chol.unb}(A_{11})$
Variant 2:
$A_{11} := A_{11} - A_{10} A_{00}^{-1} A_{10}^H$
$A_{11} := \text{Chol.unb}(A_{11})$
$A_{21} := A_{21} - A_{20} A_{00}^{-1} A_{10}^H$
$A_{21} := A_{21} L_{11}^{-H}$
Variant 3:
$A_{11} := \text{Chol.unb}(A_{11})$
$A_{21} := A_{21} L_{11}^{-H}$
$A_{22} := A_{22} - A_{21} A_{11}^{-1} A_{21}^H$
$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} \ A_{02} \\ \hline A_{10} & A_{11} \ A_{12} \\ A_{20} & A_{21} \ A_{22} \end{array} \right)$
endwhile

Step	Algorithm: $A := \text{CHOL_BLK}(A)$
1	$k \leftarrow A$
2	$A \leftarrow \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} - \begin{pmatrix} L_{TL} & * \\ L_{BL} & L_{BR} \end{pmatrix} \begin{pmatrix} L_{TL}^H & 0 \\ 0 & 0 \end{pmatrix}$
3	$\text{while } m(A_{TL}) < m(A) \text{ do}$
2.3	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c} L_{TL} & * \\ \hline L_{BL} & L_{BR} \end{array} \right) \begin{pmatrix} L_{TL}^H & 0 \\ 0 & 0 \end{pmatrix} \wedge m(A_{TL}) < m(A)$
5a	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c} L_{TL} & * \\ \hline L_{BL} & L_{BR} \end{array} \right) \begin{pmatrix} L_{TL}^H & 0 \\ 0 & 0 \end{pmatrix}$
6	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c} L_{TL} & * \\ \hline L_{BL} & L_{BR} \end{array} \right) \begin{pmatrix} L_{TL}^H & 0 \\ 0 & 0 \end{pmatrix}$
7	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c} L_{TL} & * \\ \hline L_{BL} & L_{BR} \end{array} \right) \begin{pmatrix} L_{TL}^H & 0 \\ 0 & 0 \end{pmatrix}$
5b	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c} L_{TL} & * \\ \hline L_{BL} & L_{BR} \end{array} \right) \begin{pmatrix} L_{TL}^H & 0 \\ 0 & 0 \end{pmatrix}$
2	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c} L_{TL} & * \\ \hline L_{BL} & L_{BR} \end{array} \right) \begin{pmatrix} L_{TL}^H & 0 \\ 0 & 0 \end{pmatrix}$
2.3	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c} L_{TL} & * \\ \hline L_{BL} & L_{BR} \end{array} \right) \begin{pmatrix} L_{TL}^H & 0 \\ 0 & 0 \end{pmatrix}$
5b	$k \leftarrow k - 1$

```

FLA_Part_2x2( A,      &ATL, &ATR,
               &ABL, &ABR,      0, 0, FLA_TL );

while ( FLA_Obj_length( ATL ) < FLA_Obj_length( A ) ){

    b = FLA_Determine_blocksize( ABR, FLA_BR, FLA_Cntl_blocksize( cntl ) );

    FLA_Repart_2x2_to_3x3( ATL, /**/ ATR,      &A00, /**/ &A01, &A02,
                           /* ***** */ /* ***** */
                           &A10, /**/ &A11, &A12,
                           ABL, /**/ ABR,      &A20, /**/ &A21, &A22,
                           b, b, FLA_BR );

    /*-----*/

    // A11 = chol( A11 )
    r_val = FLA_Chol_internal( FLA_LOWER_TRIANGULAR, A11,
                              FLA_Cntl_sub_chol( cntl ) );

    if ( r_val != FLA_SUCCESS )
        return ( FLA_Obj_length( A00 ) + r_val );

    // A21 = A21 * inv( tril( A11 ) )
    FLA_Trsm_internal( FLA_RIGHT, FLA_LOWER_TRIANGULAR,
                      FLA_CONJ_TRANSPOSE, FLA_NONUNIT_DIAG,
                      FLA_ONE, A11, A21,
                      FLA_Cntl_sub_trsm( cntl ) );

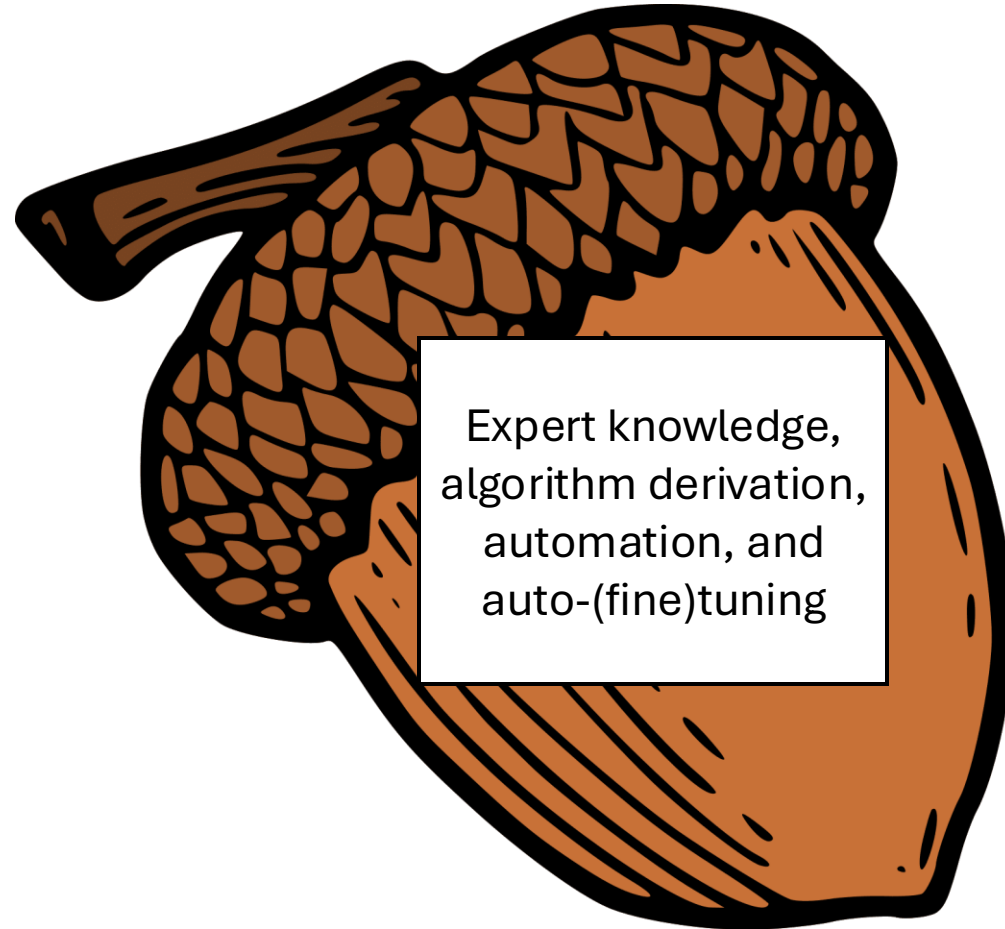
    /*-----*/

    // A22 = A22 - A21 * A21'
    FLA_Herk_internal( FLA_LOWER_TRIANGULAR, FLA_NO_TRANSPOSE,
                      FLA_MINUS_ONE, A21, FLA_ONE, A22,
                      FLA_Cntl_sub_herk( cntl ) );

    /*-----*/

    FLA_Cont_with_3x3_to_2x2( &ATL, /**/ &ATR,      A00, A01, /**/ A02,
                              /* ***** */ /* ***** */
                              &ABL, /**/ &ABR,      A10, A11, /**/ A12,
                              FLA_TL );
}
    
```

The Future DLA Stack?



Hopefully we'll see you at the
FAMILIES Retreat 2026!