CS 378: Computer Game Technology

Physics for Games

Spring 2012

Game Physics – Basic Areas

- Point Masses
  - Particle simulation
  - Collision response
- Rigid-Bodies
  - Extensions to non-points
- Soft Body Dynamic Systems
- Articulated Systems and Constraints
- Collision Detection

Physics Engines

- API for collision detection
- API for kinematics (motion but no forces)
- API for dynamics

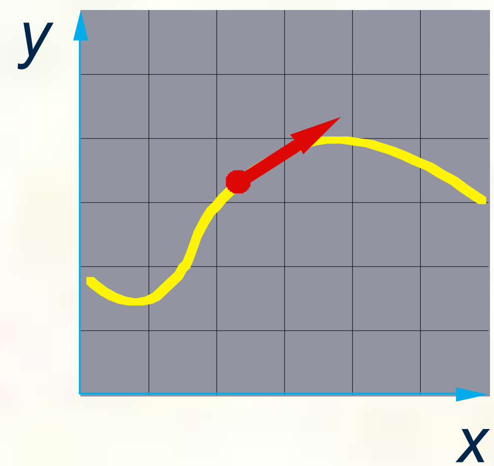
- Examples
  - Box2d
  - Bullet
  - ODE (Open Dynamics Engine)
  - PhysX
  - Havok
  - Etc.

Particle dynamics and particle systems

- A **particle system** is a collection of point masses that obeys some physical laws (e.g, gravity, heat convection, spring behaviors, …).
- Particle systems can be used to simulate all sorts of physical phenomena:

Particle in a flow field

- We begin with a single particle with:
  - Position, $\vec{\mathbf{x}} = \begin{bmatrix} x \\ y \end{bmatrix}$
  - Velocity, $\vec{\mathbf{v}} = \dot{\mathbf{x}} = \dfrac{d\vec{\mathbf{x}}}{dt} = \begin{bmatrix} dx/dt \\ dy/dt \end{bmatrix}$

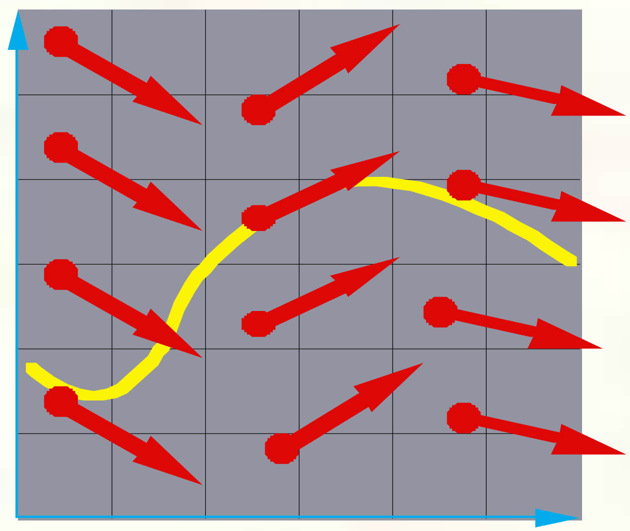


- Suppose the velocity is actually dictated by some driving function **g**:

$$\dot{\mathbf{x}} = \mathrm{g}(\vec{\mathbf{x}}, t)$$

Vector fields

- At any moment in time, the function **g** defines a vector field over **x**:

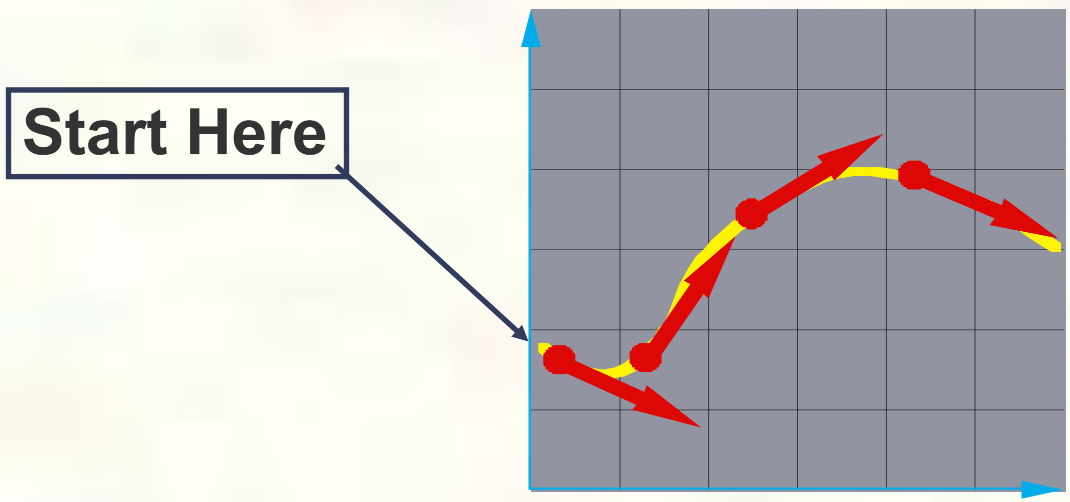


- How does our particle move through the vector field?

Diff eqs and integral curves

- The equation $$\dot{\mathbf{x}} = g(\vec{\mathbf{x}},t)$$

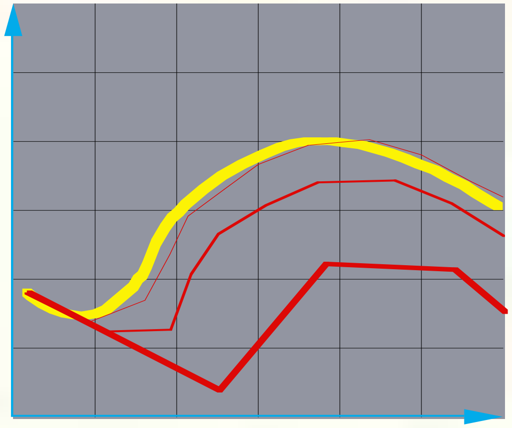
  is actually a **first order differential equation**.
- We can solve for **x** through time by starting at an initial point and stepping along the vector field:



- This is called an **initial value problem** and the solution is called an **integral curve**.

Eulers method

- One simple approach is to choose a time step, Δt, and take linear steps along the flow: $$\vec{\mathbf{x}}(t+\Delta t) = \vec{\mathbf{x}}(t) + \Delta t \cdot \dot{\vec{\mathbf{x}}}(t) = \vec{\mathbf{x}}(t) + \Delta t \cdot g(\vec{\mathbf{x}}, t)$$
- Writing as a time iteration: $$\vec{\mathbf{x}}^{i+1} = \vec{x}^{i} + \Delta t \cdot \vec{\mathbf{v}}^{i}$$
- This approach is called **Euler's method** and looks like:



- Properties:
  - Simplest numerical method
  - Bigger steps, bigger errors. Error ~ $O(\Delta t^2)$.
- Need to take pretty small steps, so not very efficient. Better (more complicated) methods exist, e.g., "Runge-Kutta" and "implicit integration."

Particle in a force field

- Now consider a particle in a force field **f**.
- In this case, the particle has:
  - Mass, m
  - Acceleration, $\vec{\mathbf{a}} \equiv \ddot{\mathbf{x}} = \frac{d\vec{\mathbf{v}}}{dt} = \frac{d^2\vec{\mathbf{x}}}{dt^2}$
- The particle obeys Newton's law: $\vec{\mathbf{f}} = m\vec{\mathbf{a}} = m\ddot{\mathbf{x}}$
- The force field **f** can in general depend on the position and velocity of the particle as well as time.
- Thus, with some rearrangement, we end up with:

$$\ddot{\mathbf{x}} = \frac{\vec{\mathbf{f}}(\vec{\mathbf{x}},\dot{\mathbf{x}},t)}{m}$$

Second order equations

This equation:

$$\ddot{\mathbf{x}} = \frac{\vec{\mathbf{f}}(\vec{\mathbf{x}},\dot{\mathbf{x}},t)}{m}$$

is a **second order differential equation**.

Our solution method, though, worked on first order differential equations.

We can rewrite this as:

$$\begin{bmatrix} \dot{\mathbf{x}} = \vec{\mathbf{v}} \\ \dot{\mathbf{v}} = \dfrac{\vec{\mathbf{f}}(\vec{\mathbf{x}},\vec{\mathbf{v}},t)}{m} \end{bmatrix}$$

where we have added a new variable **v** to get a pair of coupled first order equations.

Phase space

$$\begin{bmatrix} \vec{\mathbf{x}} \\ \vec{\mathbf{v}} \end{bmatrix}$$

- Concatenate **x** and **v** to make a 6-vector: position in **phase space**.

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix}$$

- Taking the time derivative: another 6-vector.

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \vec{\mathbf{v}} \\ \vec{\mathbf{f}}/m \end{bmatrix}$$

- A vanilla 1st-order differential equation.

Differential equation solver

Starting with:

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \vec{\mathbf{v}} \\ \vec{\mathbf{f}}/m \end{bmatrix}$$

Applying Euler's method:

$$\vec{\mathbf{x}}(t+\Delta t) = \vec{\mathbf{x}}(t) + \Delta t \cdot \dot{\mathbf{x}}(t)$$
$$\dot{\mathbf{x}}(t+\Delta t) = \dot{\mathbf{x}}(t) + \Delta t \cdot \ddot{\mathbf{x}}(t)$$

And making substitutions:

$$\vec{\mathbf{x}}(t+\Delta t) = \vec{\mathbf{x}}(t) + \Delta t \cdot \vec{\mathbf{v}}(t)$$
$$\dot{\mathbf{x}}(t+\Delta t) = \dot{\mathbf{x}}(t) + \Delta t \cdot \vec{\mathbf{f}}(\vec{\mathbf{x}},\dot{\mathbf{x}},t)/m$$

Writing this as an iteration, we have:

$$\vec{\mathbf{x}}^{i+1} = \vec{x}^{i} + \Delta t \cdot \vec{\mathbf{v}}^{i}$$
$$\vec{\mathbf{v}}^{i+1} = \vec{\mathbf{v}}^{i} + \Delta t \cdot \frac{\vec{\mathbf{f}}^{i}}{m}$$

Again, performs poorly for large Δt.

Particle structure

How do we represent a particle?

Position in phase space

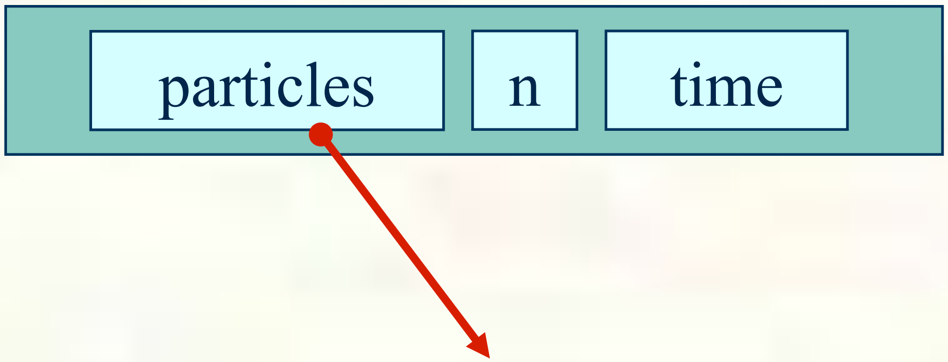
$$\begin{bmatrix} \mathbf{x} \\ \mathbf{v} \\ \mathbf{f} \\ m \end{bmatrix}$$

- $\mathbf{x}$ ← position
- $\mathbf{v}$ ← velocity
- $\mathbf{f}$ ← force accumulator
- m ← mass

Single particle solver interface

Particle systems

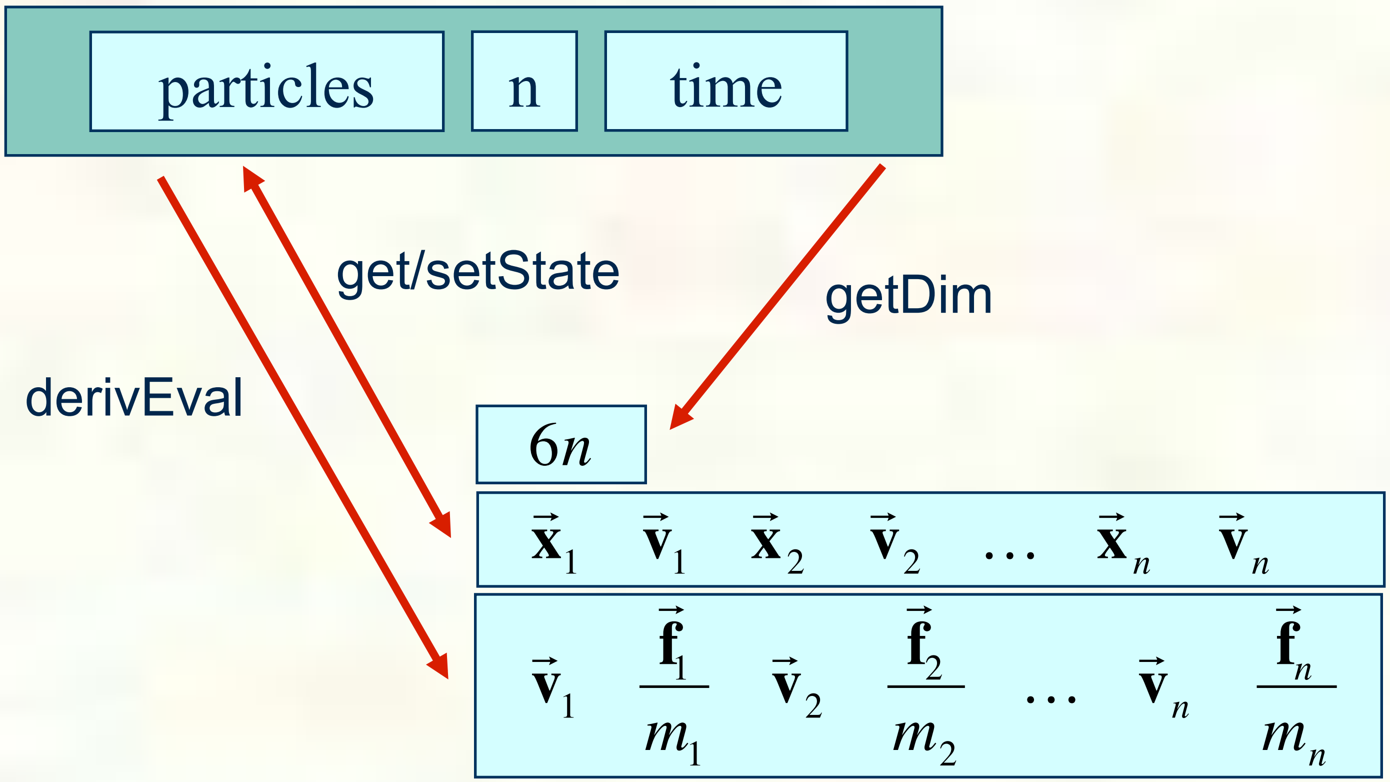
In general, we have a particle system consisting of *n* particles to be managed over time:



$$\begin{bmatrix} \vec{\mathbf{x}}_1 \\ \vec{\mathbf{v}}_1 \\ \vec{\mathbf{f}}_1 \\ m_1 \end{bmatrix} \begin{bmatrix} \vec{\mathbf{x}}_2 \\ \vec{\mathbf{v}}_2 \\ \vec{\mathbf{f}}_2 \\ m_2 \end{bmatrix} \cdots \begin{bmatrix} \vec{\mathbf{x}}_n \\ \vec{\mathbf{v}}_n \\ \vec{\mathbf{f}}_n \\ m_n \end{bmatrix}$$

Particle system solver interface

For *n* particles, the solver interface now looks like:



Particle system diff. eq. solver

We can solve the evolution of a particle system again using the Euler method:

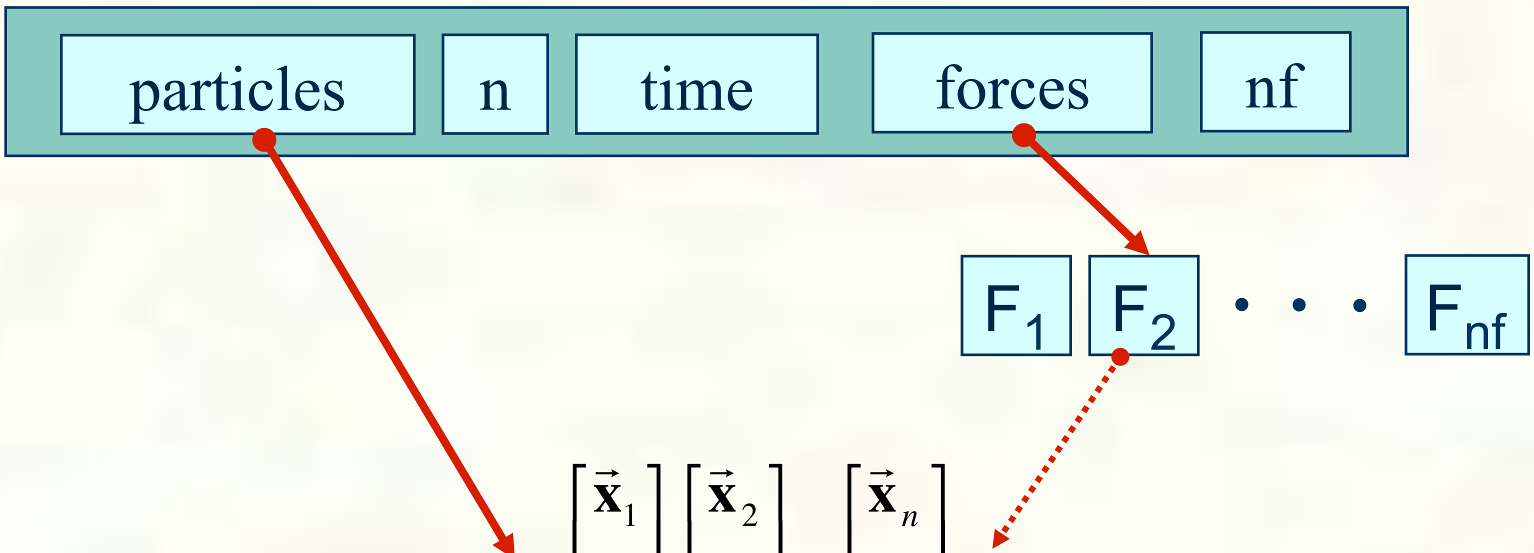
$$\begin{bmatrix} \vec{\mathbf{x}}_1^{i+1} \\ \vec{\mathbf{v}}_1^{i+1} \\ \vdots \\ \vec{\mathbf{x}}_n^{i+1} \\ \vec{\mathbf{v}}_n^{i+1} \end{bmatrix} = \begin{bmatrix} \vec{\mathbf{x}}_1^{i} \\ \vec{\mathbf{v}}_1^{i} \\ \vdots \\ \vec{\mathbf{x}}_n^{i} \\ \vec{\mathbf{v}}_n^{i} \end{bmatrix} + \Delta t \begin{bmatrix} \vec{\mathbf{v}}_1^{i} \\ \vec{\mathbf{f}}_1^{i} / m_1 \\ \vdots \\ \vec{\mathbf{v}}_n^{i} \\ \vec{\mathbf{f}}_n^{i} / m_n \end{bmatrix}$$

Forces

- Each particle can experience a force which sends it on its merry way.
- Where do these forces come from? Some examples:
  - Constant (gravity)
  - Position/time dependent (force fields)
  - Velocity-dependent (drag)
  - Combinations (Damped springs)
- How do we compute the net force on a particle?

Particle systems with forces

- Force objects are black boxes that point to the particles they influence and add in their contributions.
- We can now visualize the particle system with force objects:



$$\begin{bmatrix} \vec{\mathbf{x}}_1 \\ \vec{\mathbf{v}}_1 \\ \vec{\mathbf{f}}_1 \\ m_1 \end{bmatrix} \begin{bmatrix} \vec{\mathbf{x}}_2 \\ \vec{\mathbf{v}}_2 \\ \vec{\mathbf{f}}_2 \\ m_2 \end{bmatrix} \cdots \begin{bmatrix} \vec{\mathbf{x}}_n \\ \vec{\mathbf{v}}_n \\ \vec{\mathbf{f}}_n \\ m_n \end{bmatrix}$$

Gravity and viscous drag

The force due to **gravity** is simply:

$$\vec{\mathbf{f}}_{grav} = m\vec{\mathbf{G}}$$

```
p->f += p->m * F->G
```

Often, we want to slow things down with **viscous drag**:

$$\vec{\mathbf{f}}_{drag} = -k\vec{\mathbf{v}}$$

```
p->f -= F->k * p->v
```

Damped spring

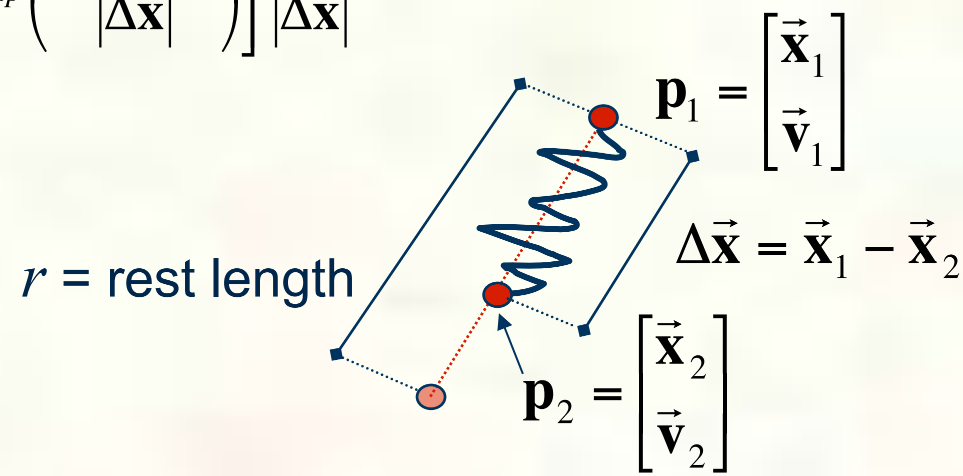
Recall the equation for the force due to a spring: $f = -k_{spring}(|\Delta\vec{\mathbf{x}}| - r)$

We can augment this with damping: $f = -\left[k_{spring}(|\Delta\vec{\mathbf{x}}| - r) + k_{damp}|\vec{\mathbf{v}}|\right]$

The resulting force equations for a spring between two particles become:

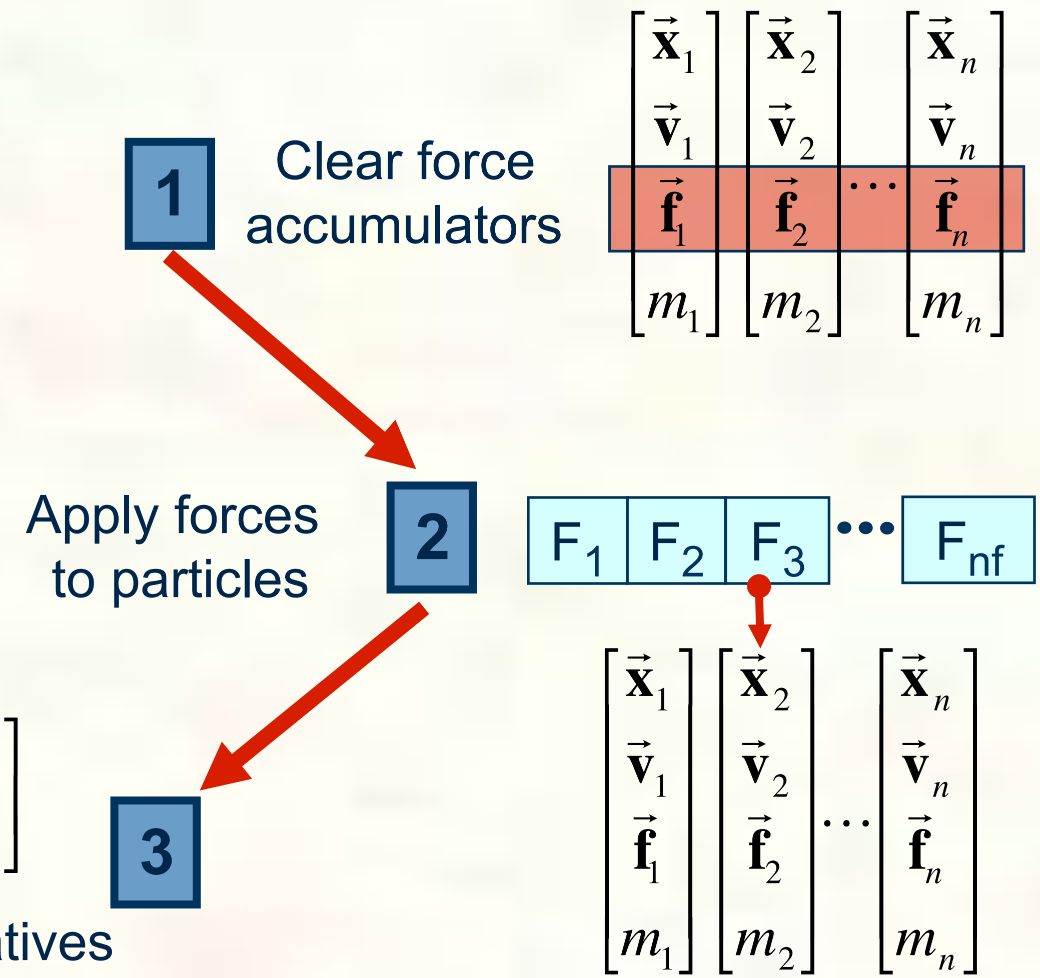
$$\vec{\mathbf{f}}_1 = -\left[k_{spring}\left(|\Delta\vec{\mathbf{x}}| - r\right) + k_{damp}\left(\frac{\Delta\vec{\mathbf{v}} \bullet \Delta\vec{\mathbf{x}}}{|\Delta\vec{\mathbf{x}}|}\right)\right]\frac{\Delta\vec{\mathbf{x}}}{|\Delta\vec{\mathbf{x}}|}$$

$$\vec{\mathbf{f}}_2 = -\vec{\mathbf{f}}_1$$



derivEval

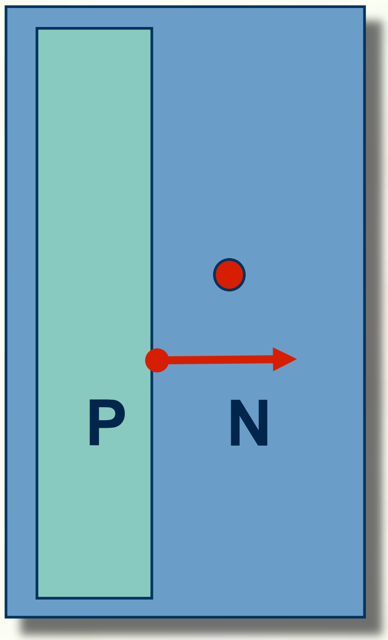
- Clear forces
  - Loop over particles, zero force accumulators
- Calculate forces
  - Sum all forces into accumulators
- Return derivatives
  - Loop over particles, return **v** and **f**/*m*



$$\begin{bmatrix} \vec{\mathbf{v}}_1 \\ \vec{\mathbf{f}}_1 / m_1 \end{bmatrix} \begin{bmatrix} \vec{\mathbf{v}}_2 \\ \vec{\mathbf{f}}_2 / m_2 \end{bmatrix} \cdots \begin{bmatrix} \vec{\mathbf{v}}_n \\ \vec{\mathbf{f}}_n / m_n \end{bmatrix}$$

Return derivatives to solver

Bouncing off the walls

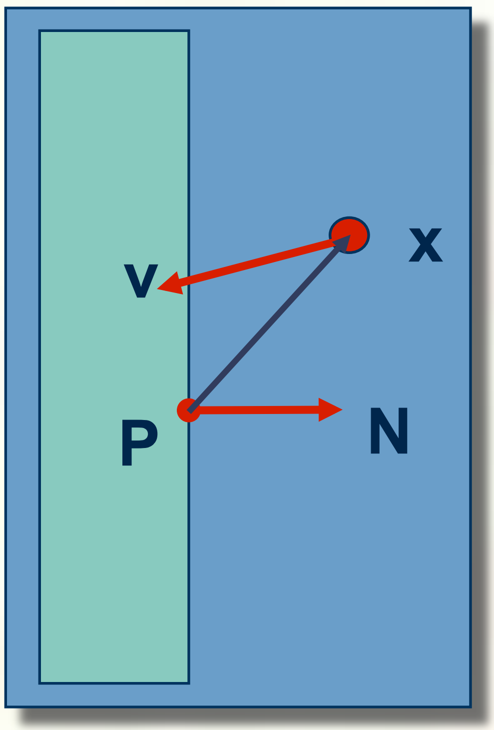


- Add-on for a particle simulator
- For now, just simple point-plane collisions

A plane is fully specified by any point **P** on the plane and its normal **N**.

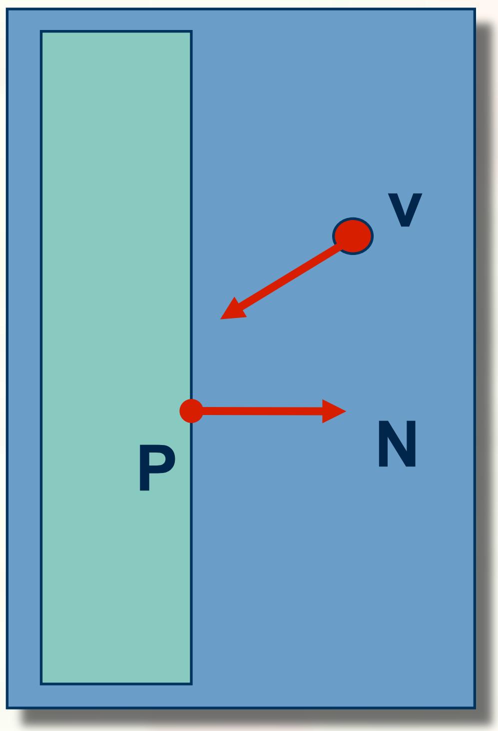
Collision Detection

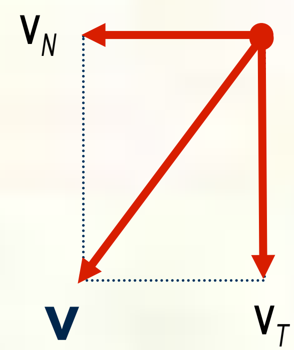
How do you decide when you've crossed a plane?



Normal and tangential velocity

To compute the collision response, we need to consider the normal and tangential components of a particle's velocity.

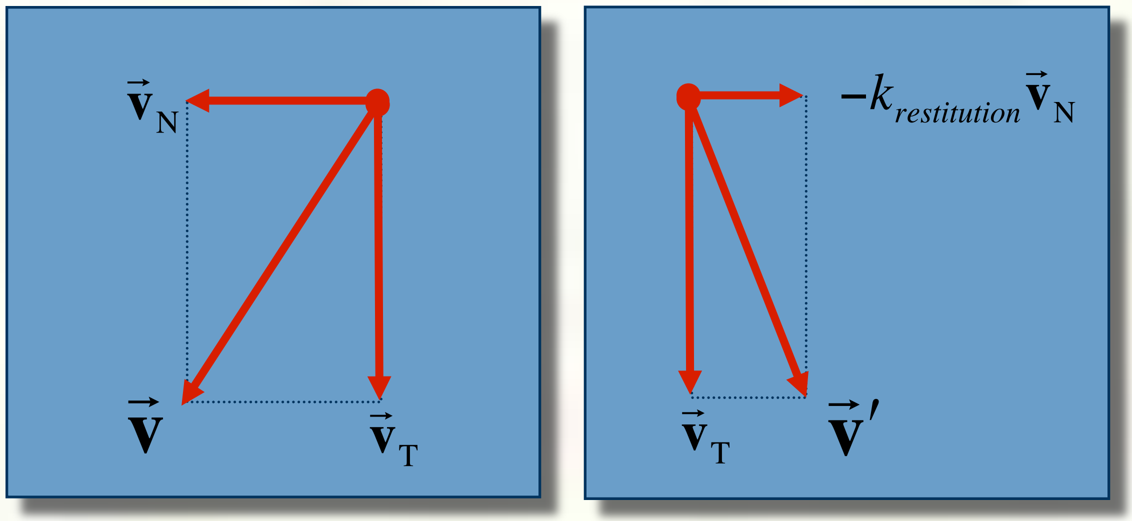




$$\vec{\mathbf{v}}_N = \left(\vec{\mathbf{N}} \bullet \vec{\mathbf{v}}\right) \vec{\mathbf{N}}$$

$$\vec{\mathbf{v}}_T = \vec{\mathbf{v}} - \vec{\mathbf{v}}_N$$

Collision Response



before after

$$\vec{\mathbf{v}}' = \vec{\mathbf{v}}_T - k_{restitution}\vec{\mathbf{v}}_N$$

Without backtracking, the response may not be enough to bring a particle to the other side of a wall.
In that case, detection should include a velocity check: