Point and Line Clipping

Clipping: Remove points outside a region of interest.

- Want to discard everything that's outside of our window...

Point clipping: Remove points outside window.

- A point is either entirely inside the region or not.

Line clipping: Remove portion of line segment outside window.

- Line segments can straddle the region boundary.
- The Liang-Barsky algorithm efficiently clips line segments against a halfspace.
- Halfspaces can be combined to bound a convex region.
- Use *outcodes* to organize combination of halfspaces.
- Can use some of the ideas in Liang-Barsky to clip points.

Polygon clipping: Remove portion of polygon outside window.

- Polygons can straddle the region boundary.
- Concave polygons can be broken into multiple parts.
- Sutherland-Hodgemen algorithm deals with all cases efficiently.
- Built upon efficient line segment clipping.

Parametric representation of line:

$$P(t) = (1 - t)P_0 + P_1$$

or equivalently:

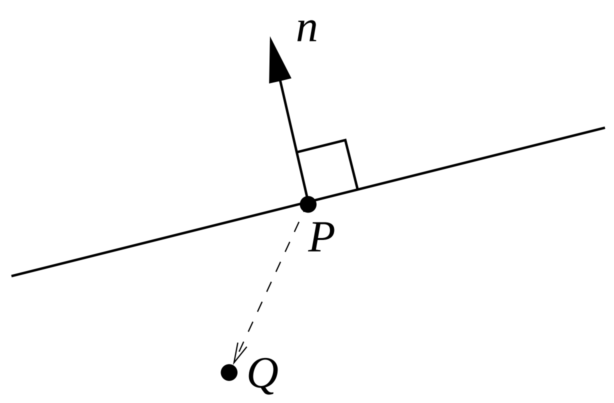
$$P(t) = P_0 + t(P_1 - P_0)$$

- P_0 and P_1 are non-coincident points.
- For $t \in \mathbf{R}$, $P(t)$ defines an infinite line.
- For $t \in [0, 1]$, $P(t)$ defines a line segment from P_0 to P_1.
- Good for generating points on a line.
- Not so good for testing if a given point is on a line.

Implicit representation of line:

$$\ell(Q) = (Q - P) \cdot \vec{n}$$

- P is a point on the line.
- $\vec{n}$ is a vector perpendicular to the line.
- $\ell(Q)$ gives us the signed distance from any point Q to the line.
- The sign of $\ell(Q)$ tells us if Q is on the left or right of the line, relative to the direction of $\vec{n}$.
- If $\ell(Q)$ is zero, then Q is on the line.
- Use same form for the implicit representation of a halfspace.

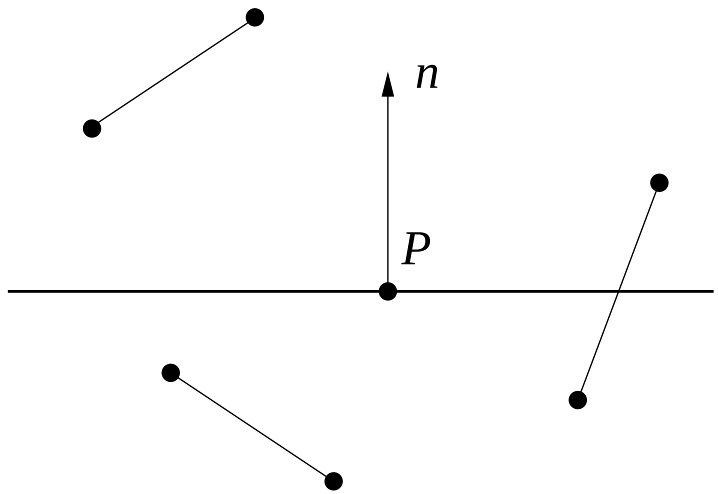


Clipping a point to a halfspace:

- Represent a window edge implicitly...
- Use the implicit form of a line to classify a point Q.
- Must choose a convention for the normal: point to the *inside*.
- Check the sign of $\ell(Q)$:
  - If $\ell(Q) > 0$, the Q is inside.
  - Otherwise clip (discard) Q; it is on, or outside.

Clipping a line segment to a halfspace: There are three cases:

- The line segment is entirely inside.
- The line segment is entirely outside.
- The line segment is partially inside and partially outside.



Do the easy stuff first: We can devise easy (and fast!) tests for the first two cases:

- $(P_0 - P) \cdot \vec{n} < 0$ AND $(P_1 - P) \cdot \vec{n} < 0 \Longrightarrow$ Outside
- $(P_0 - P) \cdot \vec{n} > 0$ AND $(P_1 - P) \cdot \vec{n} > 0 \Longrightarrow$ Inside

We will also need to decide whether "on the boundary" is inside or outside.

Trivial tests are important in computer graphics:

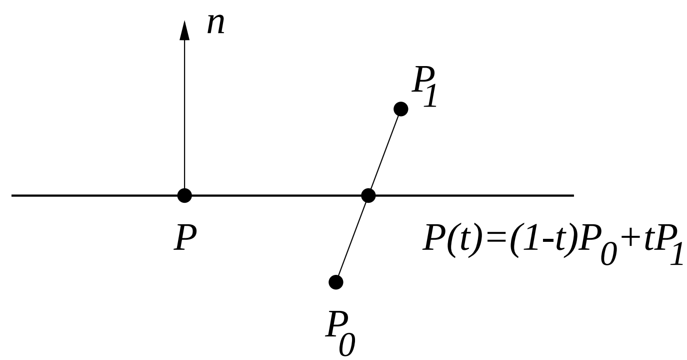
- Particularly if the trivial case is the most common one.
- Particularly if we can reuse the computation for the non-trivial case.

Do the hard stuff only if we have to: If line segment partially inside and partially outside, need to clip it:

- Represent the segment from P_0 to P_1 in parametric form:

$$P(t) = (1 - t)P_0 + tP_1 = P_0 + t(P_1 - P_0)$$

- When $t = 0$, $P(t) = P_0$. When $t = 1$, $P(t) = P_1$.
- We now have the following:



- We want t such that $P(t)$ is on ℓ:

$$\begin{aligned}(P(t) - P) \cdot \vec{n} &= (P_0 + t(P_1 - P_0) - P) \cdot \vec{n} \\ &= (P_0 - P) \cdot \vec{n} t(P_1 - P_0) \cdot \vec{n} \\ &= 0\end{aligned}$$

- Solving for t gives us

$$t = \frac{(P_0 - P) \cdot \vec{n}}{(P_0 - P_1) \cdot \vec{n}}$$

- NOTE: The values we use for our simple test can be used to compute t:

$$t = \frac{(P_0 - P) \cdot \vec{n}}{(P_0 - P) \cdot \vec{n} - (P_1 - P) \cdot \vec{n}}$$

Clipping a line segment to a window: Just clip to each of four halfspaces.

Pseudo-code:

```
given:  P, n defining a window edge
for each edge (A,B) = (P0,P1)
    wecA = (A-P) . n
    wecB = (B-P) . n
    if ( wecA < 0 AND wecB < 0 ) then reject
    if ( wecA >= 0 AND wecB >= 0 ) then next
    t = wecA / (wecA - wecB)
    if (wecA < 0 ) then
        A = A + t*(B-A)
    else
        B = B + t*(B-A)
    endif
endfor
```

NOTE:

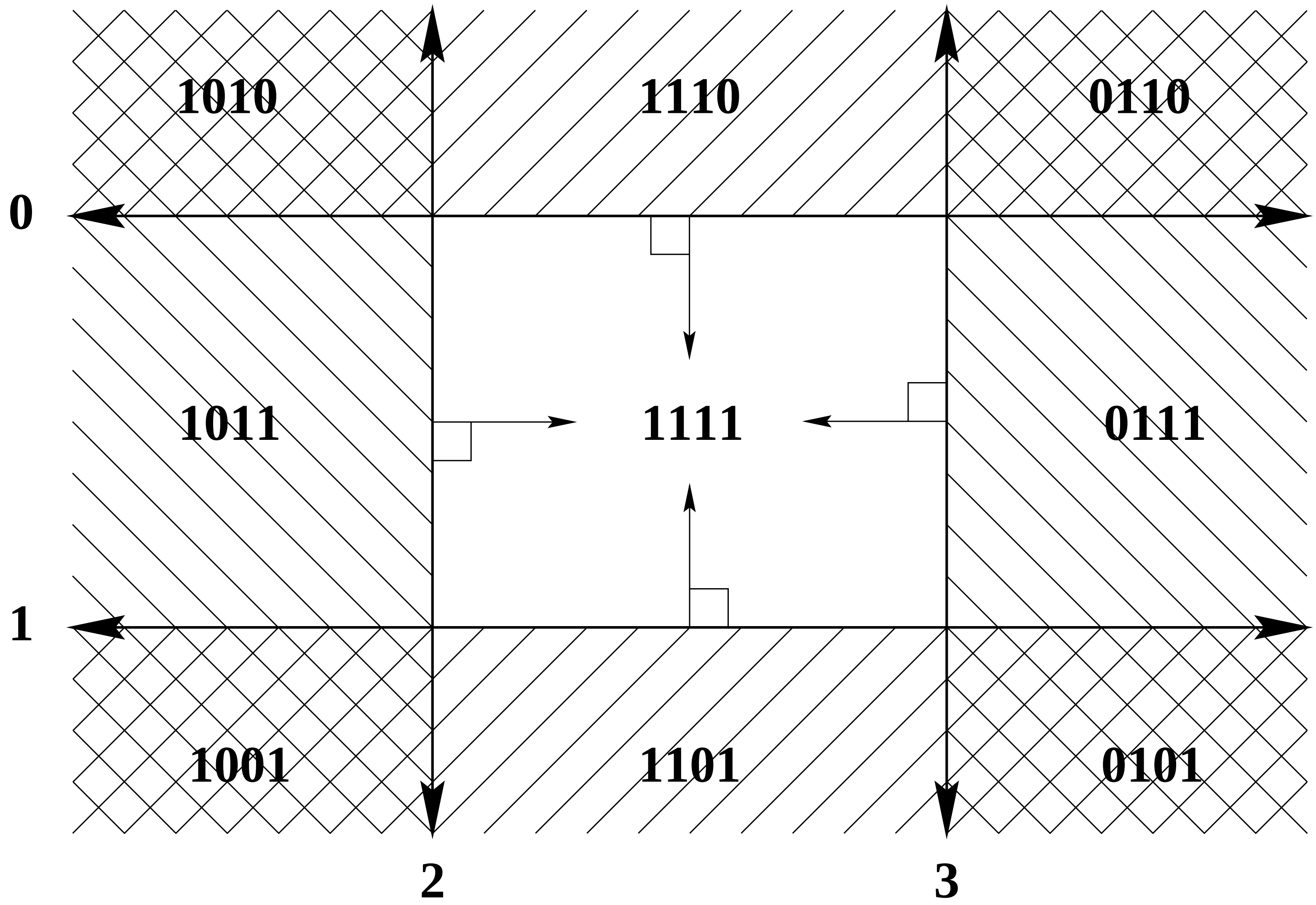
- Liang-Barsky Algorithm lets us clip lines to arbitrary convex windows.

- Optimizations can be made for the special case of horizontal and vertical window edges.

Q: Can we short-circuit evaluation of a clip on one edge if we know line segment is out on another?

A: Yes. Use *outcodes*.

- Do all trivial tests first.
- Combine results using Boolean operations to determine
  - Trivial accept on window: all edges have trivial accept.
  - Trivial reject on window: any edge has trivial reject.
- Do Boolean AND, OR operations on bits in a packed integer.
- Do full clip only if no trivial accept/reject on *window*.


1010
1110
0110
0
1011
1111
0111
1
1001
1101
0101
2
3

Line-clip Algorithm generalizes to 3D:

- Half-space now lies on one side of a *plane*.
- The implicit formula for a plane in 3D is the same as that for a line in 2D.
- The parametric formula for the line to be clipped is unchanged.

3D Clipping

- When do we clip in 3D? We should clip to the near plane *before* we project. Otherwise, we might map z to 0 and the x/z and y/z are undefined.
- We could clip to all 6 sides of the truncated viewing pyramid. but the plane equations are simpler if we clip after projection, because all sides of volume are parallel to coordinate plane.
- Clipping to a plane in 3D is identical to clipping to a line in 2D.
- We can also clip in homogeneous coordinates.

Polygon Clipping

Polygon Clipping (Sutherland-Hodgeman):

- Window must be a convex polygon.
- Polygon to be clipped can be convex or not.

Approach:

- Polygon to be clipped is given as $v_1, \ldots, v_n$
- Each polygon edge is a pair $[v_i, v_{i+1}] i = 1, \ldots, n$
  - Don't forget wraparound; $[v_n, v_1]$ is also an edge
- Process all polygon edges in succession against a window edge
  - Polygon in – polygon out
  - $v_1, \ldots, v_n \rightarrow w_1, \ldots, w_m$
- Repeat on resulting polygon with next sequential window edge.

Contrast with Line Clipping

- Line clipping:
  - Use outcodes to check all window edges before any clip
  - Clip only against possible intersecting window edges
  - Deal with window edges in any order
  - Deal with line segment endpoints in either order
- Polygon clipping:
  - Each window edge must be used
  - Polygon edges must be handled in sequence
  - Polygon edge endpoints have a given order
  - Stripped-down line-segment/window-edge clip is a subtask

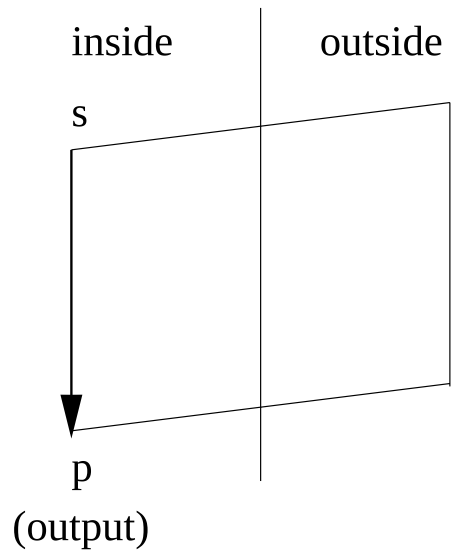
There are four cases to consider.

Four Cases

- $s = v_i$ is the polygon edge starting vertex
- $p = v_{i+1}$ is the polygon edge ending vertex
- i is a polygon-edge/window-edge intersection point
- w_j is the next polygon vertex to be output

Case 1: Polygon edge is entirely inside the window edge

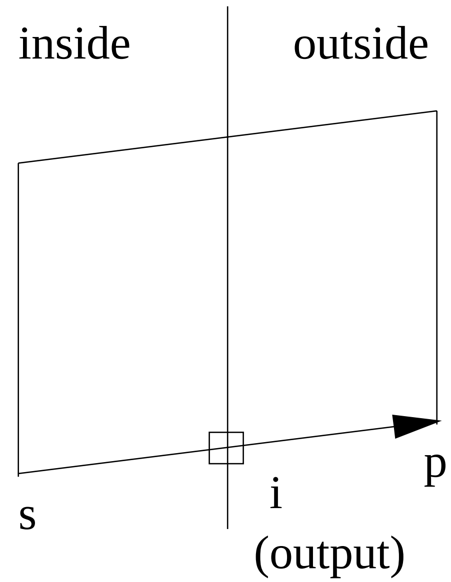
- p is next vertex of resulting polygon
- $p \rightarrow w_j$ and $j + 1 \rightarrow j$



Case 1

Case 2: Polygon edge crosses window edge going out

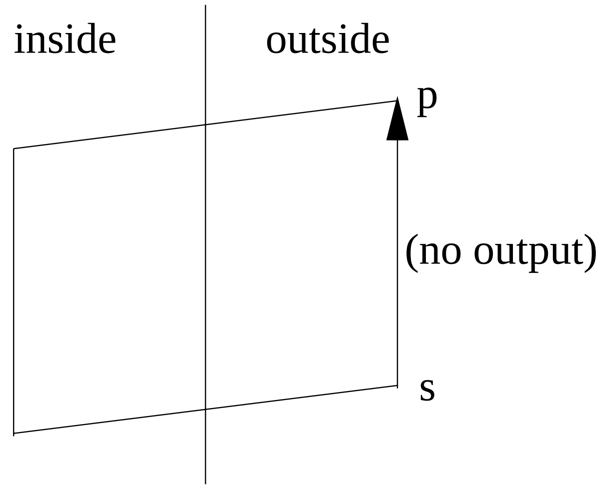
- Intersection point i is next vertex of resulting polygon
- $i \rightarrow w_j$ and $j + 1 \rightarrow j$



Case 2

Case 3: Polygon edge is entirely outside the window edge

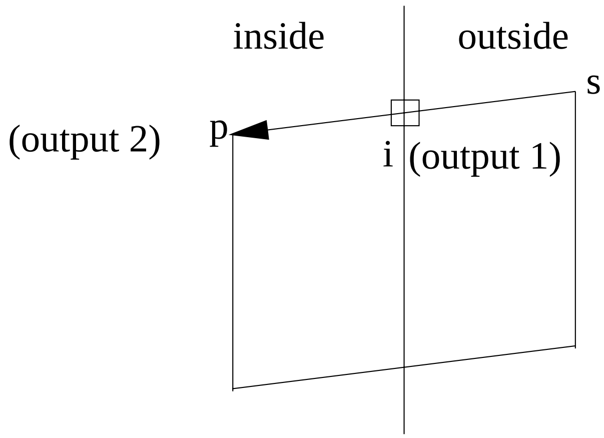
- No output



Case 3

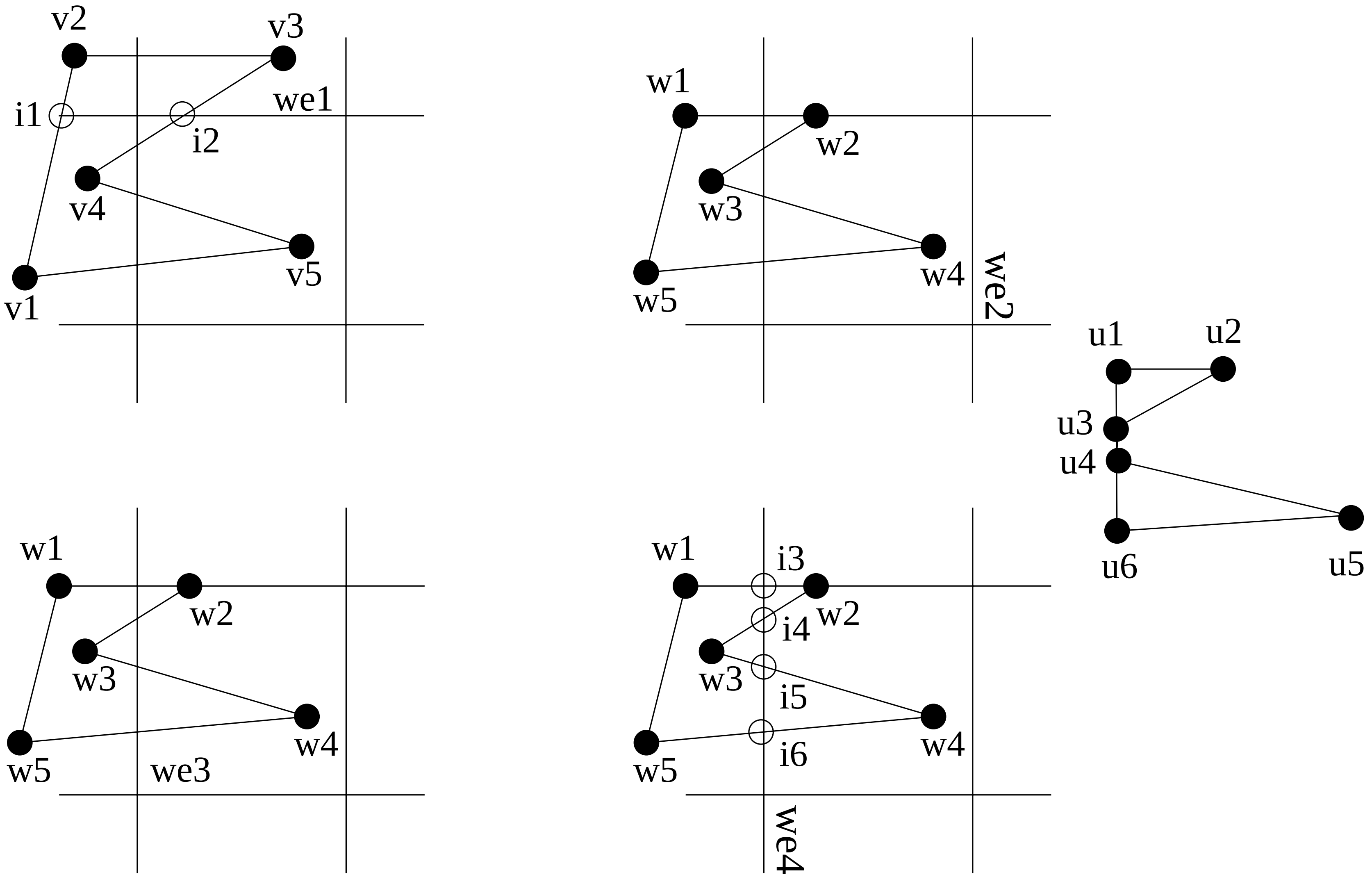
Case 4: Polygon edge crosses window edge going in

- Intersection point i and p are next two vertices of resulting polygon
- $i \rightarrow w_j$ and $p \rightarrow w_{j+1}$ and $j + 2 \rightarrow j$



Case 4

An Example with a Non-convex Polygon


v2
v3
i1
we1
i2
v4
v5
v1
w1
w2
w3
w4
w5
we2
u1
u2
u3
u4
u5
u6
w1
w2
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i3
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w2
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i5
w4
i6
w5
we4