

# **Lighting and Shading**

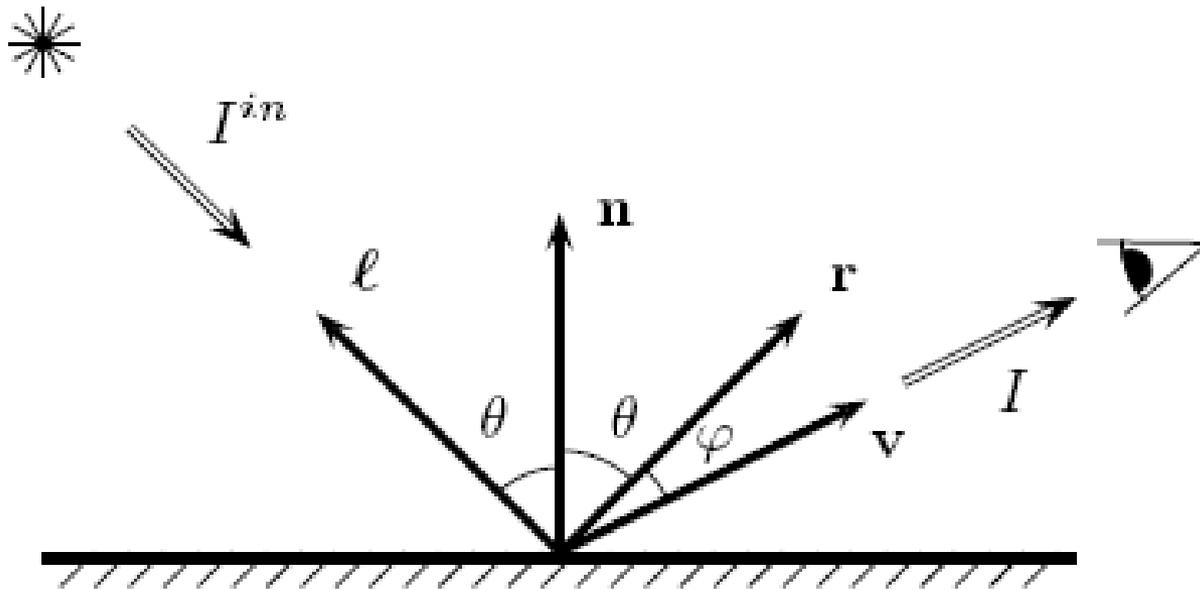
# Today: Local Illumination

Solving the rendering equation is too expensive

First do local illumination

Then “hack in” reflections and shadows

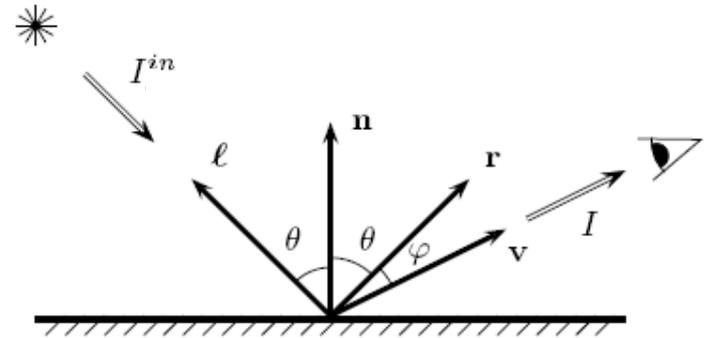
# Local Shading: Notation



$I^{in}, I$  light intensity in, light intensity out

$\hat{l}, \hat{n}, \hat{v}, \hat{r}$  vector pointing to: light, normal direction,  
eye, reflection direction

# Ambient Term

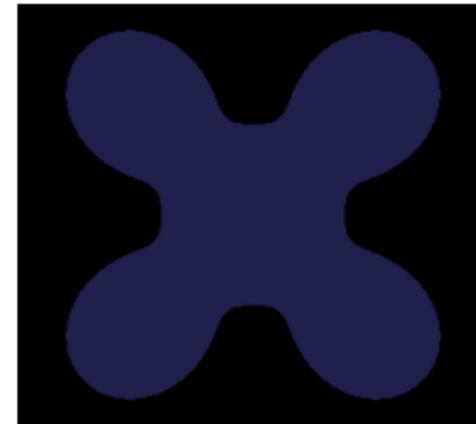


Ignore camera and light direction completely

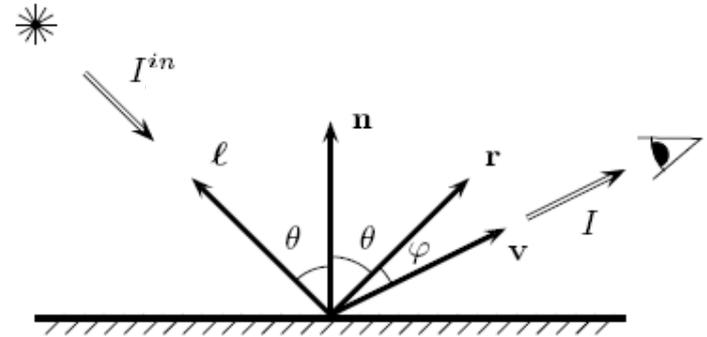
$$I_a = k_a I^{in}$$



material constant



# Ambient Term

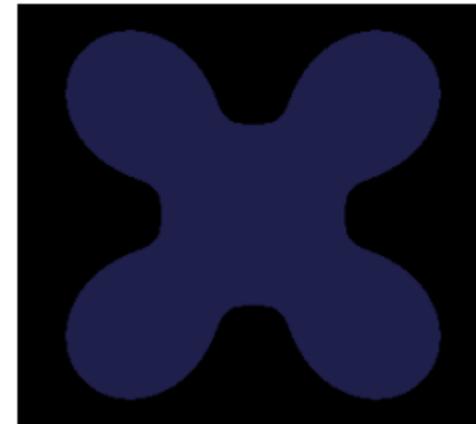


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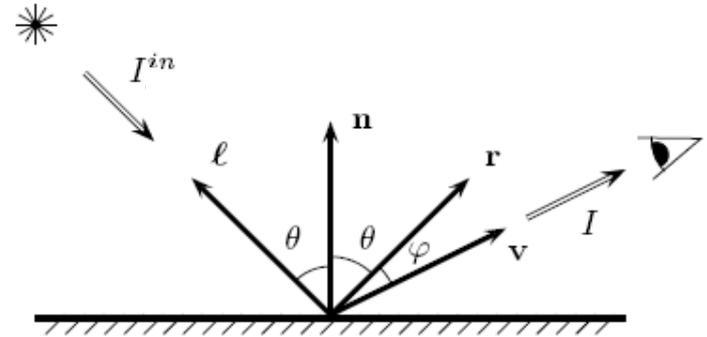
one eq.  
per color  
channel

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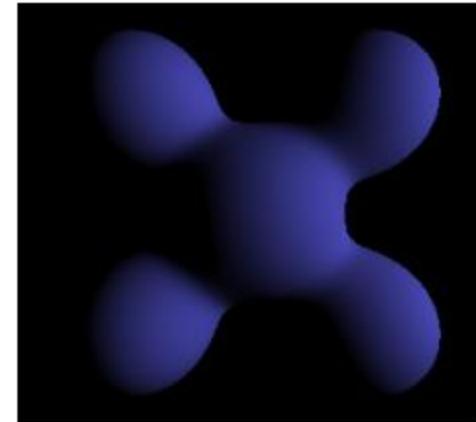


# Diffuse Term

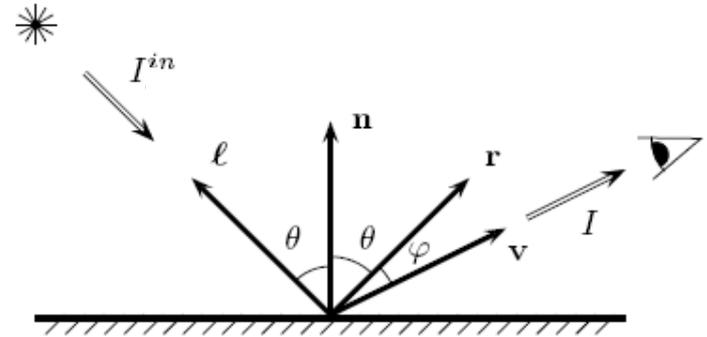


Lambertian surface – **constant BRDF**

$$I_d = k_d \max(\hat{l} \cdot \hat{n}, 0) I^{in}$$



# Diffuse Term

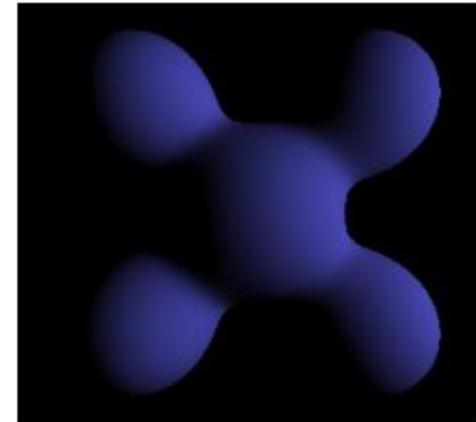


Lambertian surface – **constant BRDF**

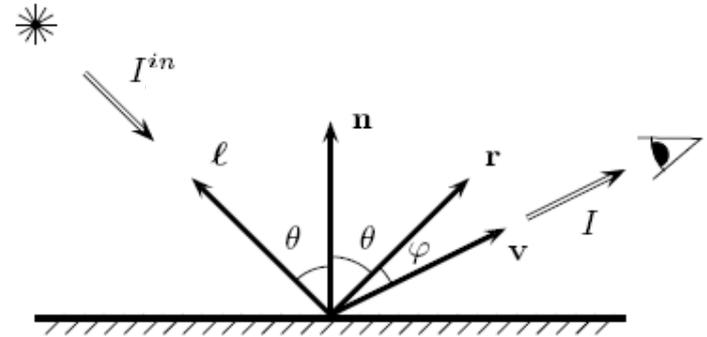
$$I_d = k_d \max(\hat{l} \cdot \hat{n}, 0) I^{in}$$



ignore back faces

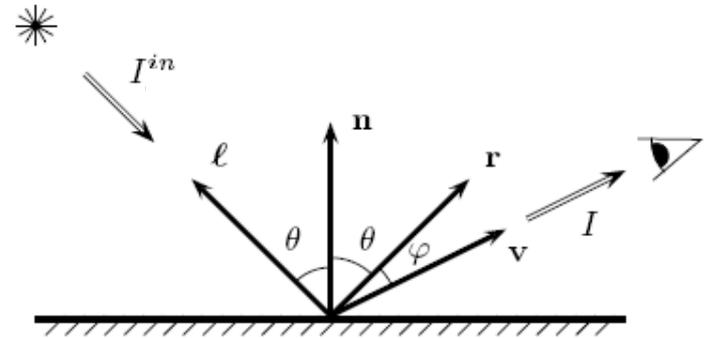


# Specular Term



Perfect specular surface doesn't work

# Specular Term

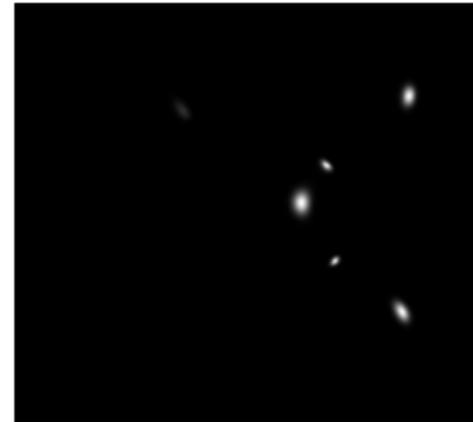


Perfect specular surface doesn't work

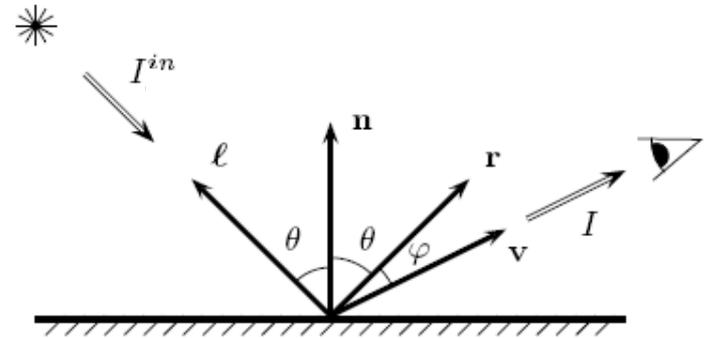
Phong model:

$$I_s = k_s \max(\hat{v} \cdot \hat{r}, 0)^\alpha I^{in}$$

specularity coefficient



# Specular Term

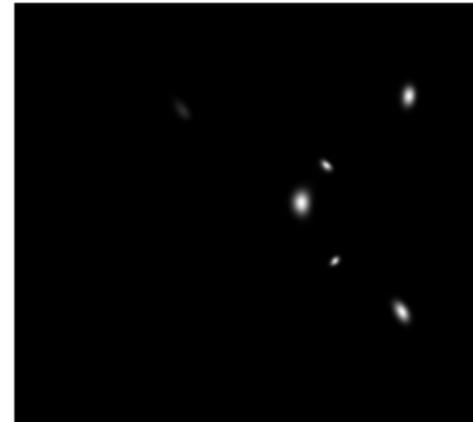


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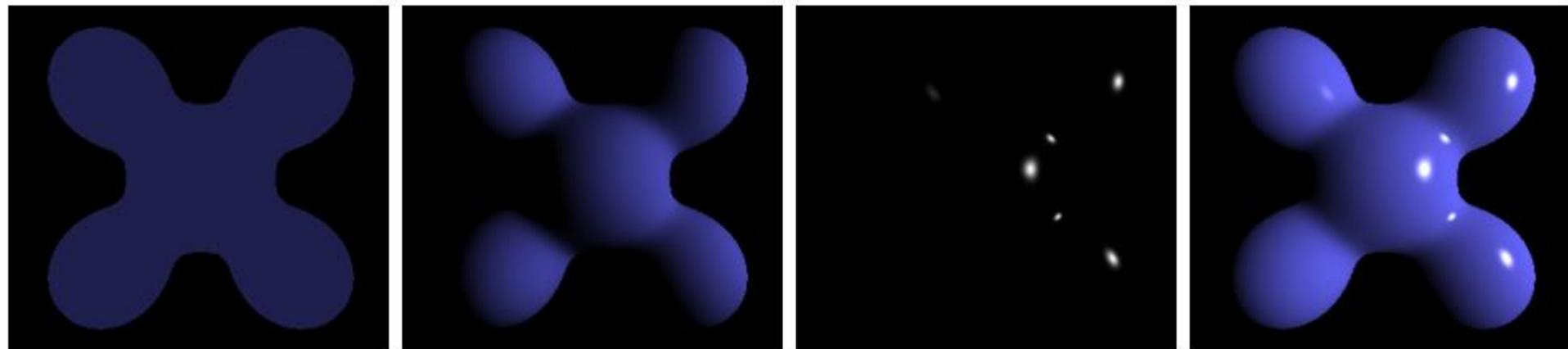
$$I_s = k_s \max(\hat{v} \cdot \hat{r}, 0)^\alpha I^{in}$$

Looks like “highlight” that  
moves with light & eye



# Putting It Together

$$I = \left[ k_a + k_d \max(\hat{l} \cdot \hat{n}) + k_s \max(\hat{v} \cdot \hat{r}, 0)^\alpha \right] I^{in}$$



Ambient

+

Diffuse

+

Specular

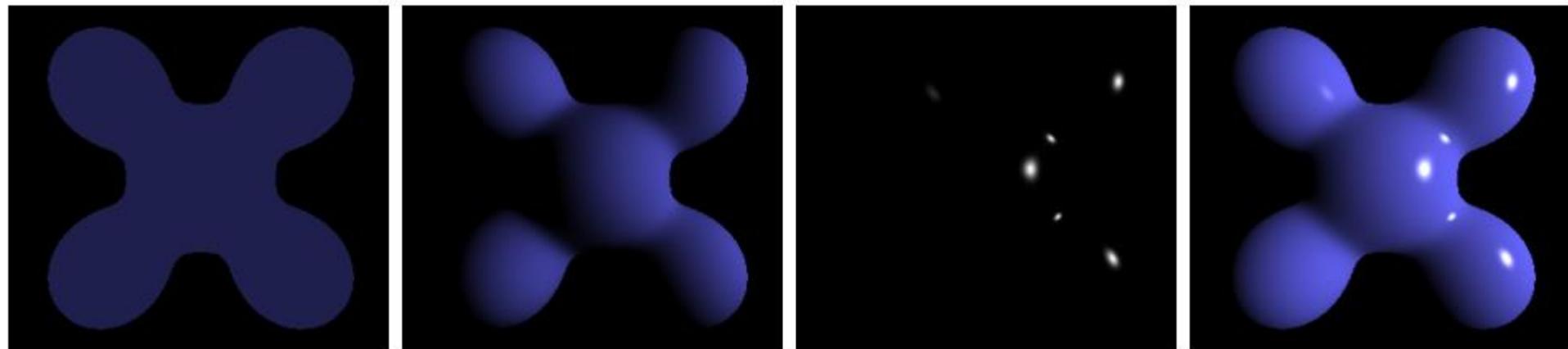
=

Phong Reflection

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typically:  $k_a = 0$ ;  $k_s = 1$



Ambient

+

Diffuse

+

Specular

=

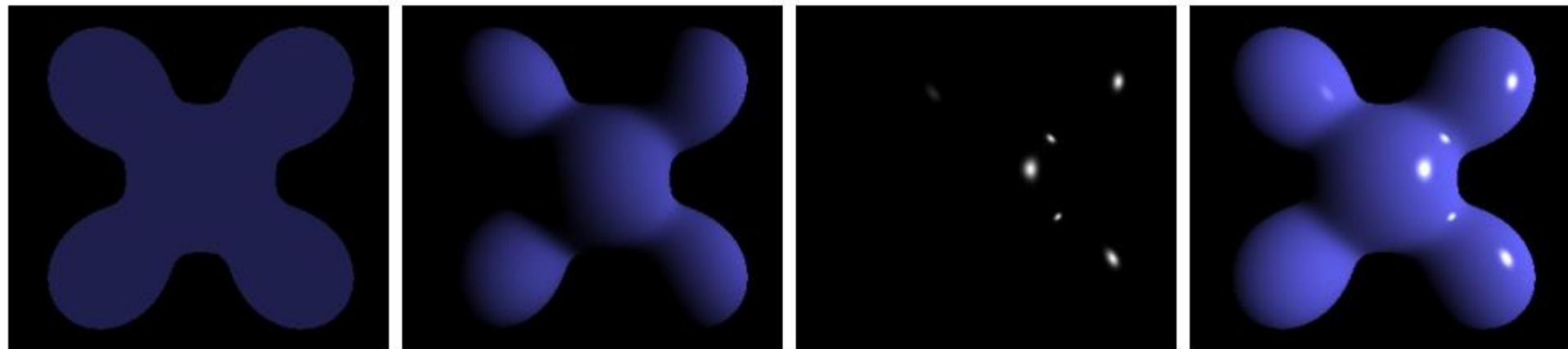
Phong Reflection

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typically:  $k_a = 0$ ;  $k_s = 1$

three copies of equation, one per color channel



Ambient

+

Diffuse

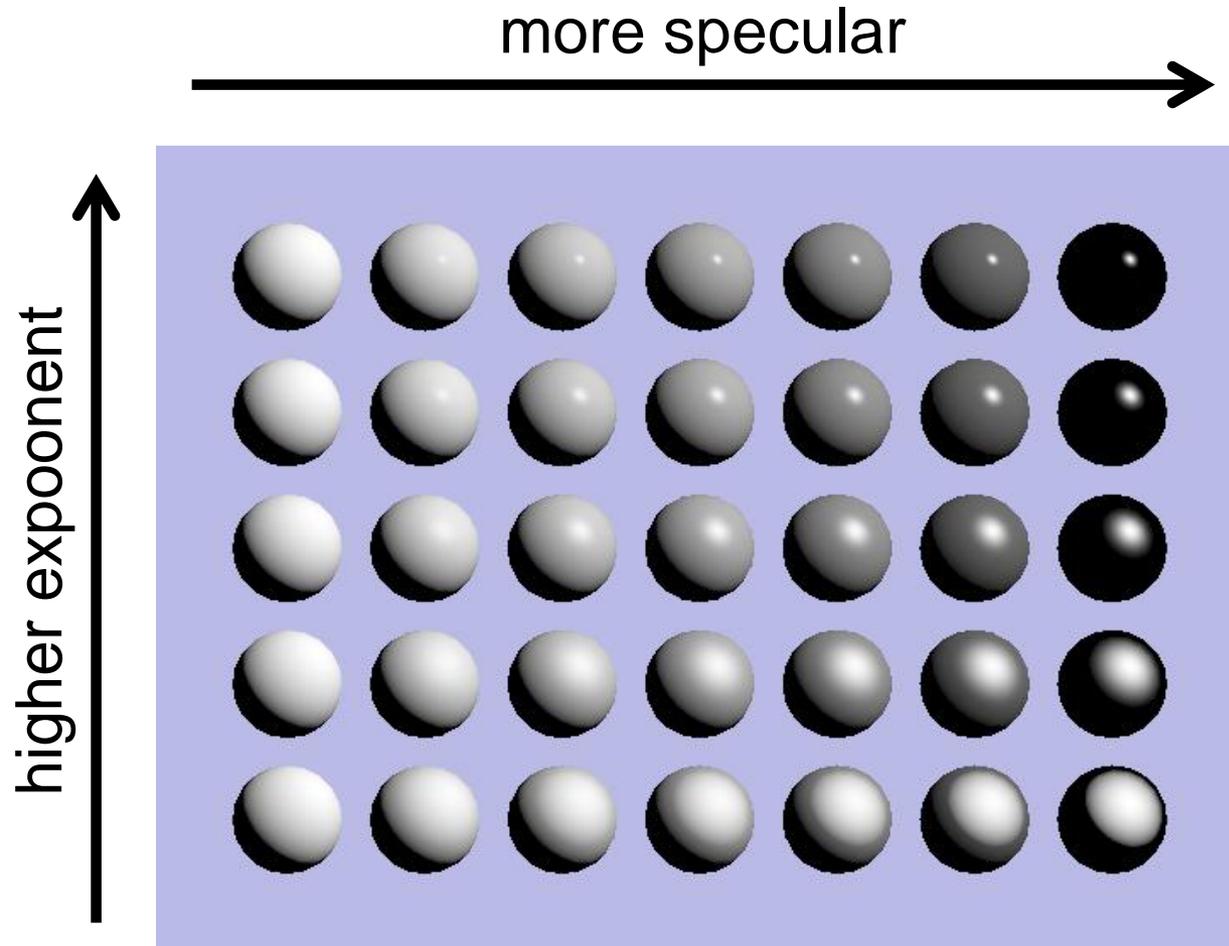
+

Specular

=

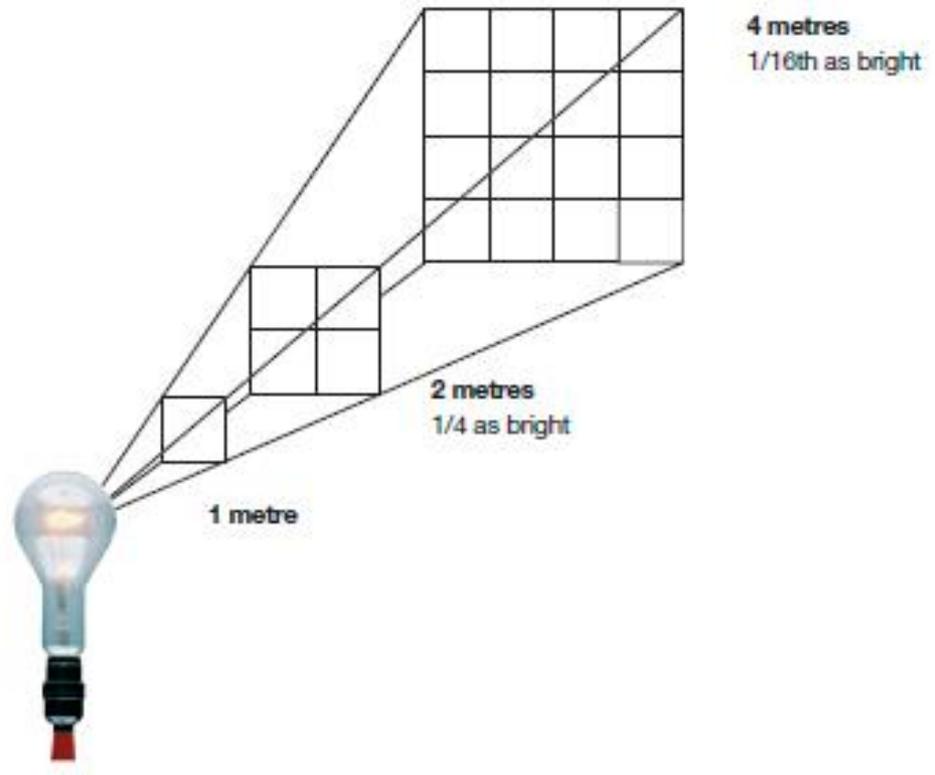
Phong Reflection

# Specularity Coefficient



# Light Attenuation

Real light attenuation: inverse square law

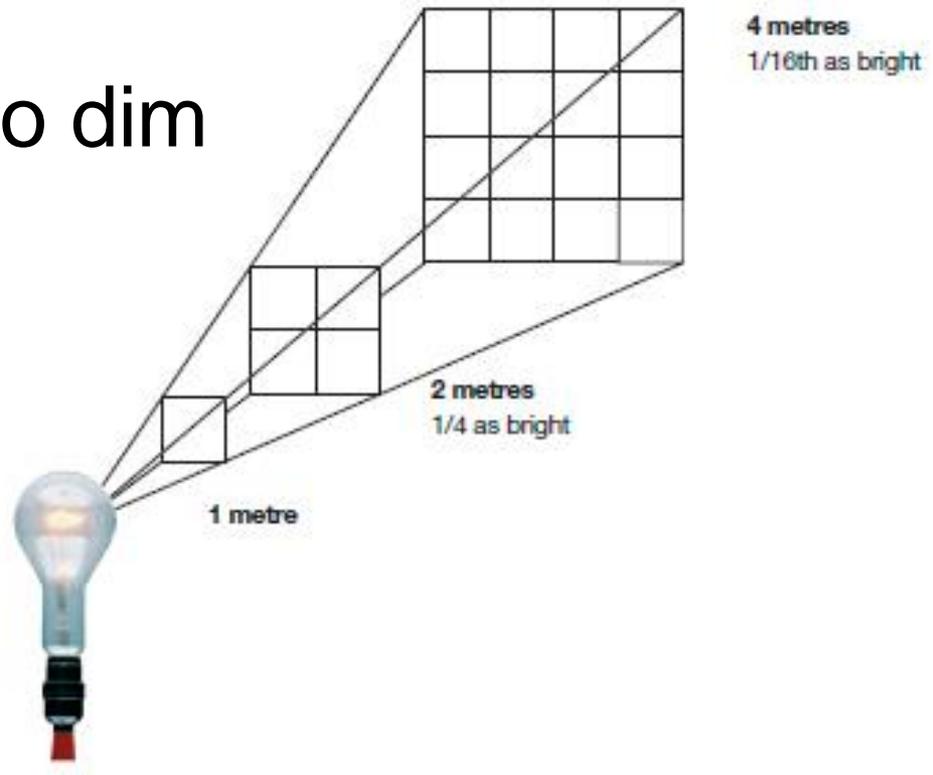


# Light Attenuation

Real light attenuation: inverse square law

Tends to look bad: too dim  
or washed out

So, we cheat



# Light Attenuation

$$I^{atten} = \frac{1}{c_0 + c_1 d + c_2 d^2} I$$

$d$  is light-to-point distance

Can tweak constant & linear term to taste

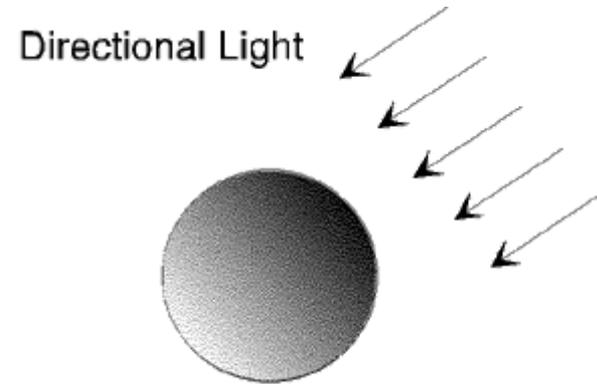
# Directional Light

“Light at infinity”

- Sun

All light rays parallel

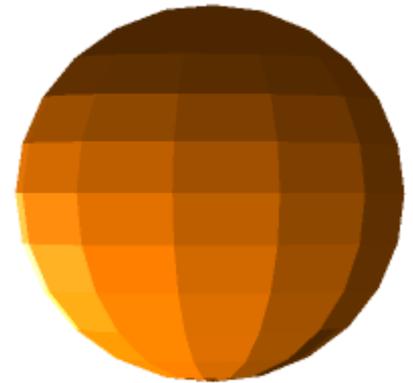
Obviously, no attenuation



# Dealing with Discrete Geometry

Flat shading: use normal per face

Very obvious discontinuities  
at edges

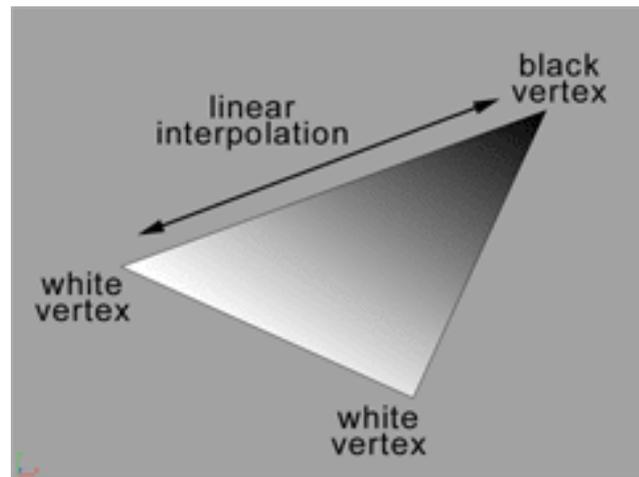
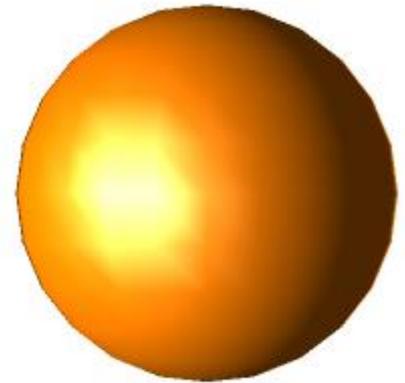


Only used for stylized “chunky” effect

# Gouraud Interpolation

First, compute color at vertices

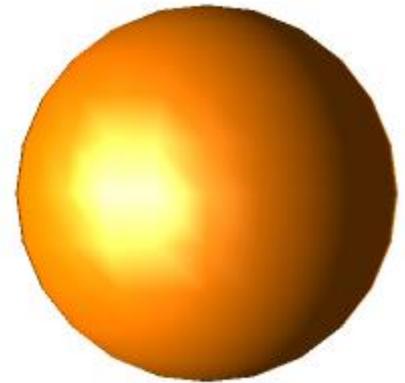
Linearly interpolate color  
over face



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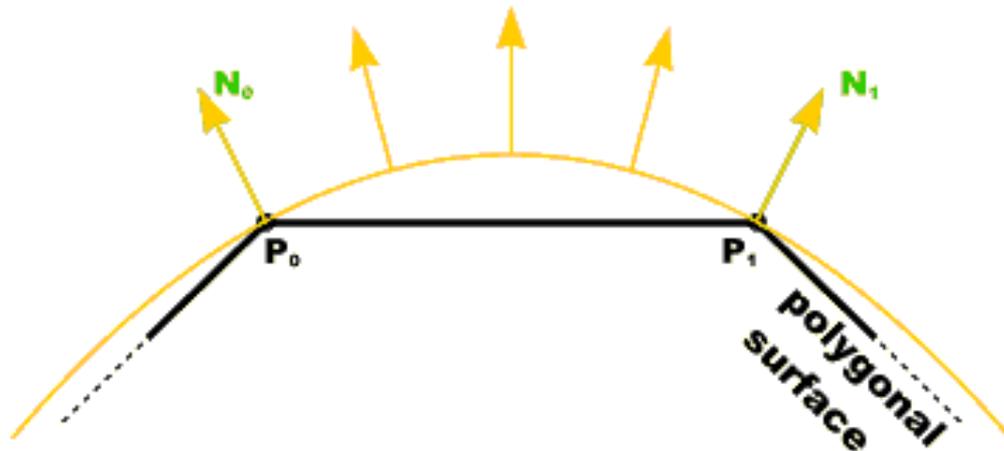
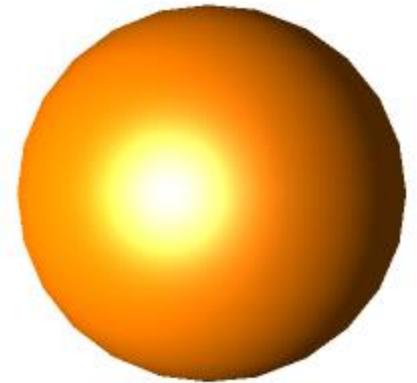
Color now continuous, but still obvious  
artifacts (nobody uses this anymore)

# Phong Interpolation

First, linearly interpolate normals

Next, renormalize normals  
(important)

Then, compute color per pixel

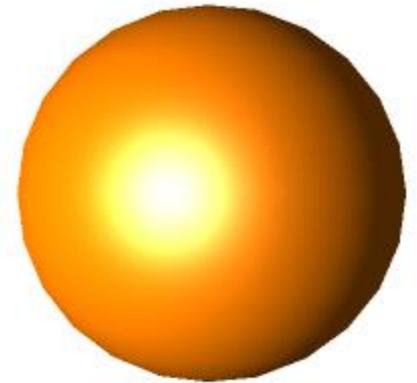


# Phong Interpolation

First, linearly interpolate normals

Next, renormalize normals  
(**important**)

Then, compute color per pixel



Because of all of the normalizations, used  
to be considered decadent;  
now **the** standard local shading scheme

# Local vs Global Illumination

Local:

- shade each object based only on itself, the eye, and the light sources

Global:

- take all other objects in scene into account also
- BRDFs and the rendering equation

# Local vs Global Illumination

Grey Area:

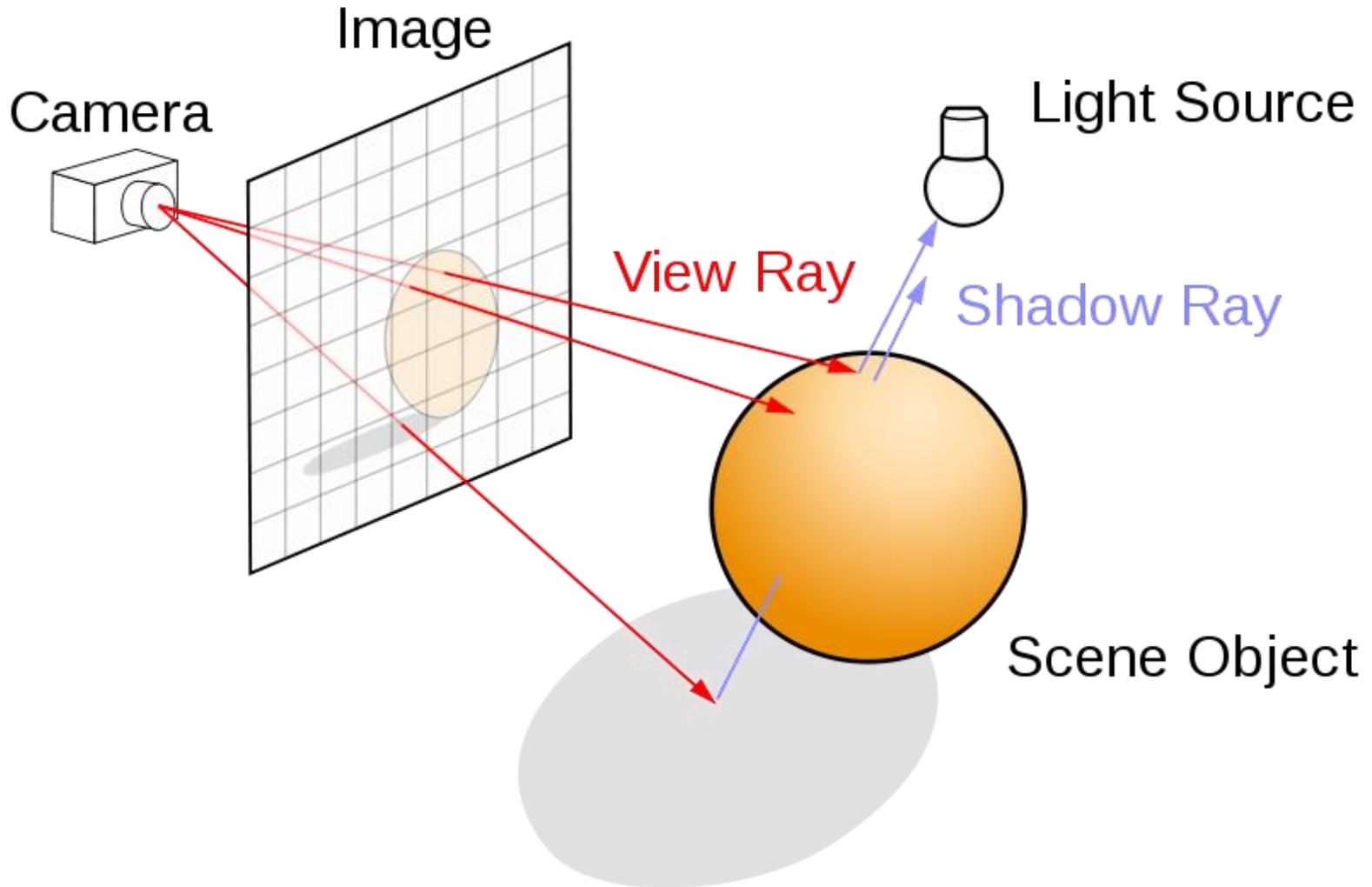
- take other objects into account, without full global illumination
- adds realism without being too slow

# Local vs Global Illumination

Grey Area:

- take other objects into account, without full global illumination
- adds realism without being too slow
- common techniques exist for
  - shadows
  - reflections
  - refractions

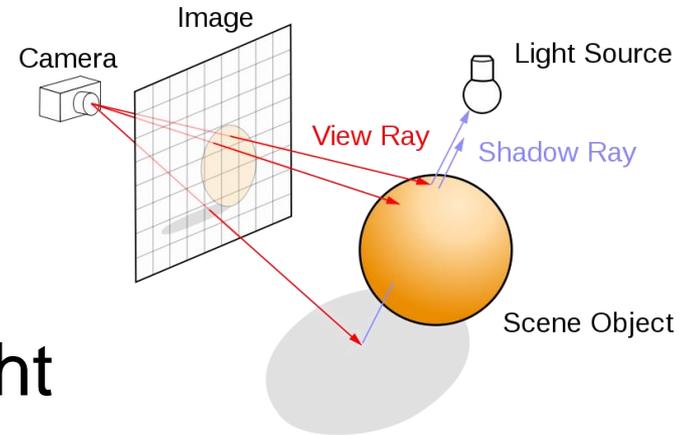
# Shooting Shadow Rays



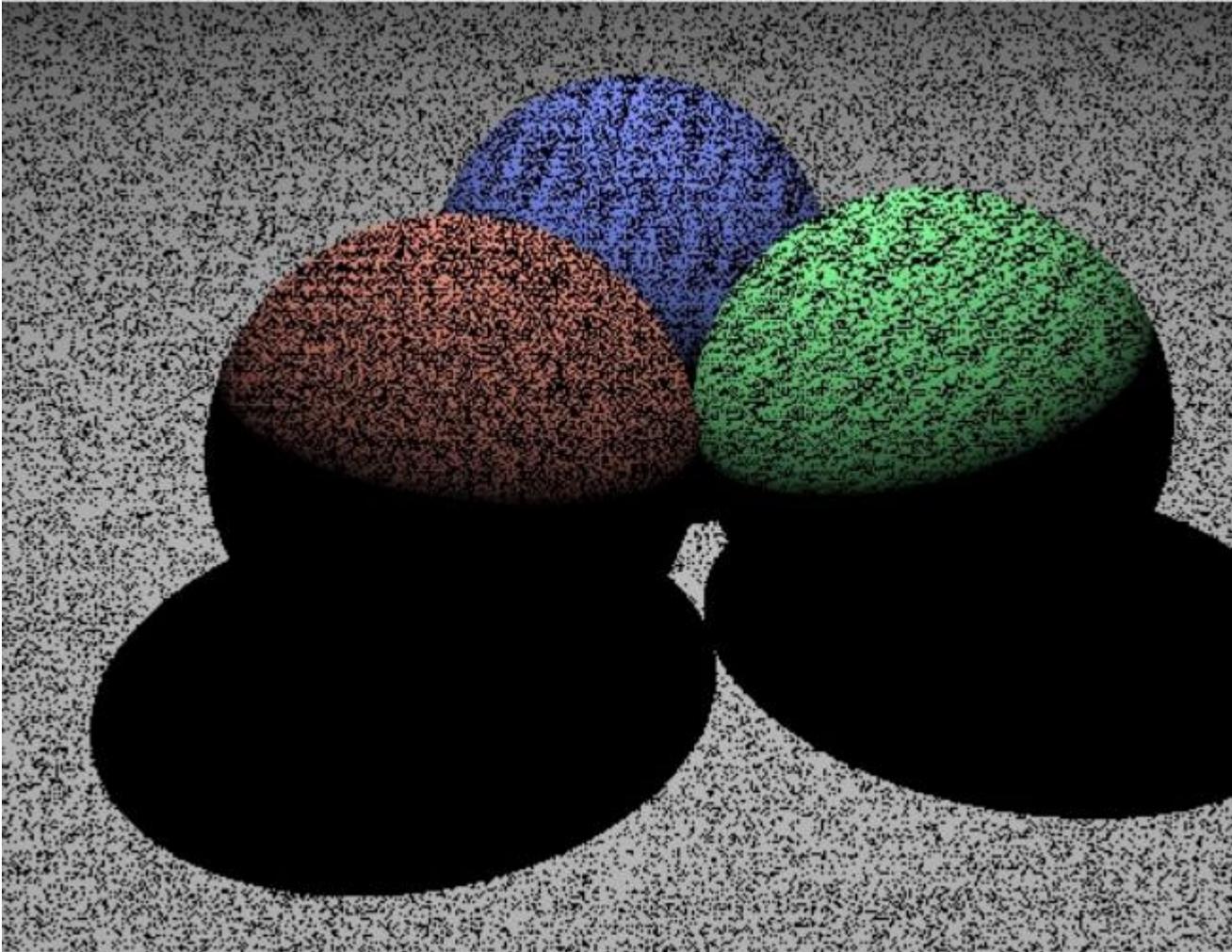
# Shooting Shadow Rays

For each intersection pt:

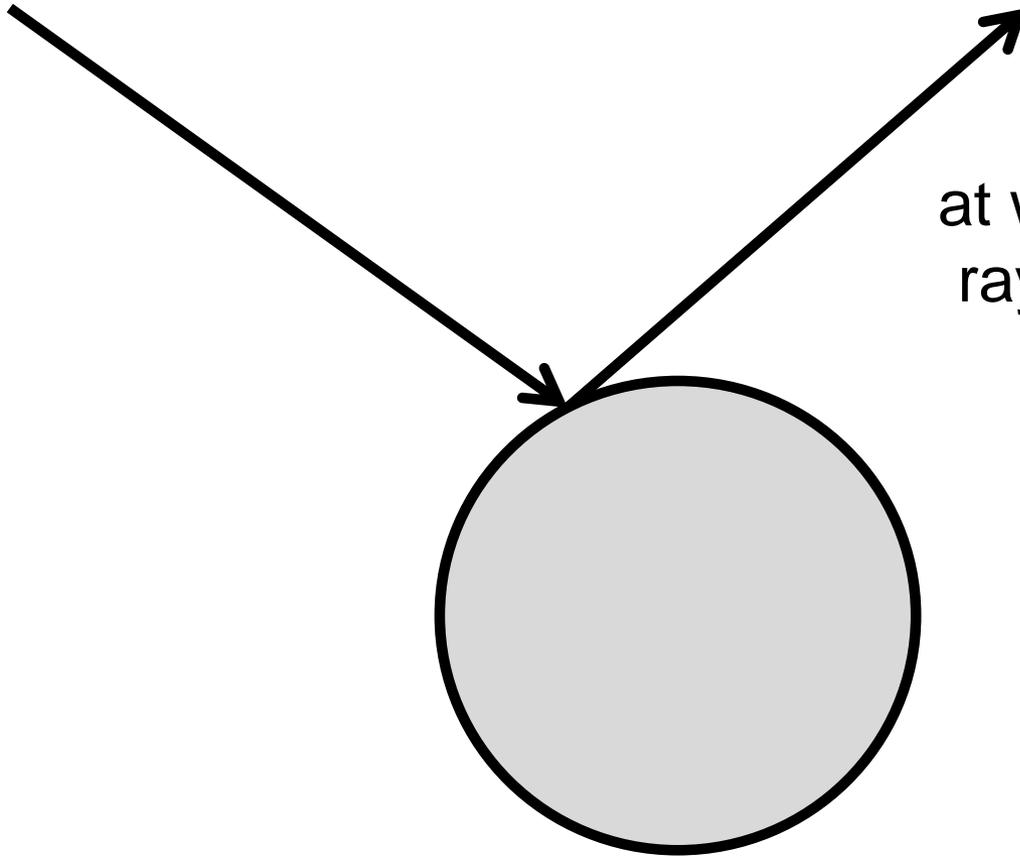
- for each light:
  - shoot ray from point to light
  - if it hits an object:
    - do nothing (shadow)
  - else add shading contribution



# Classic Bug

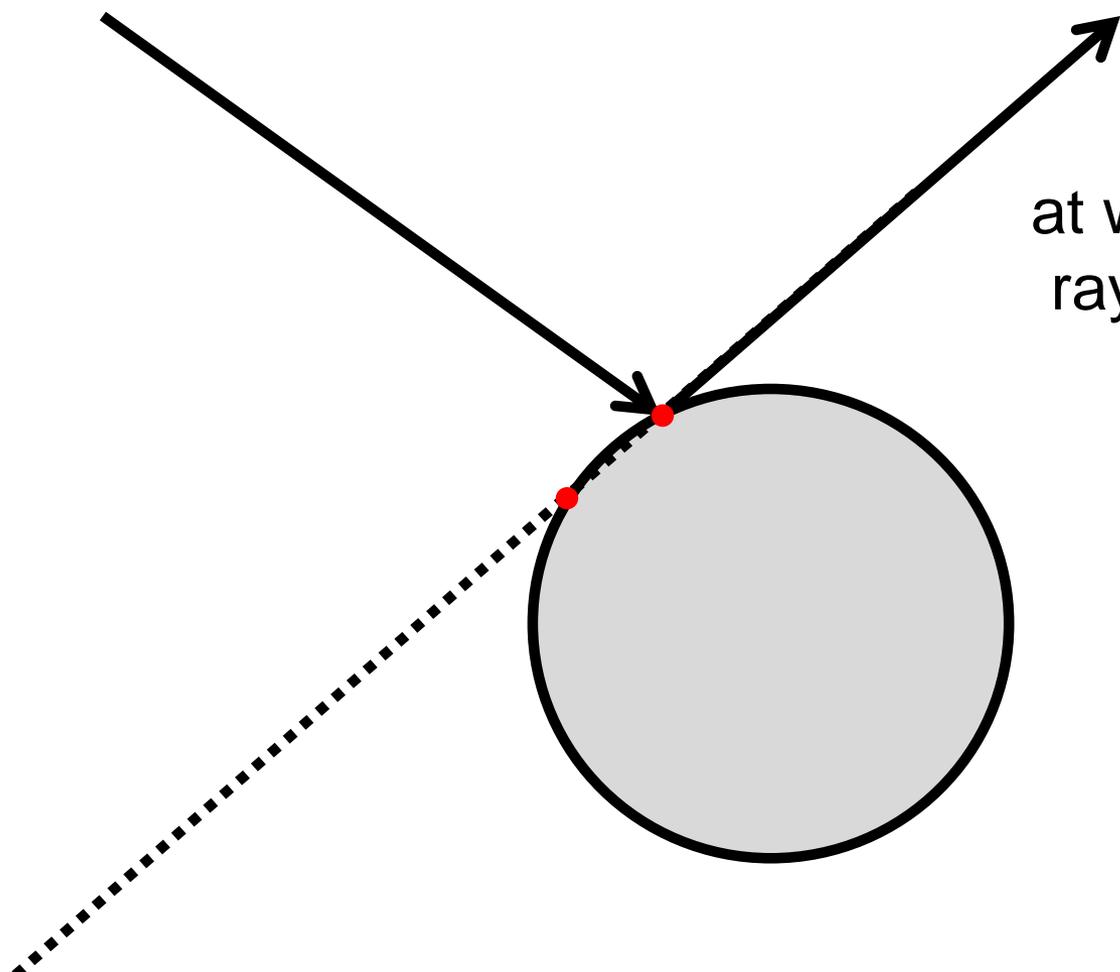


# Classic Bug



at what  $t$  does the  
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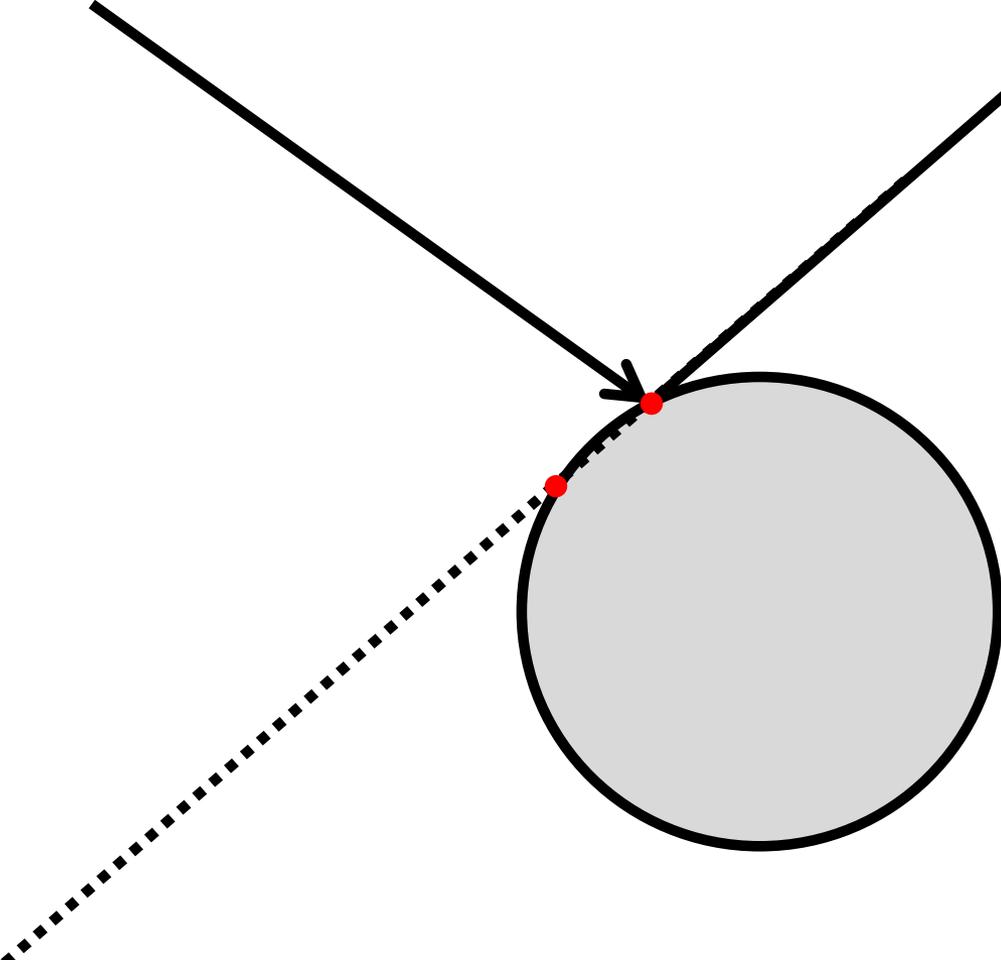
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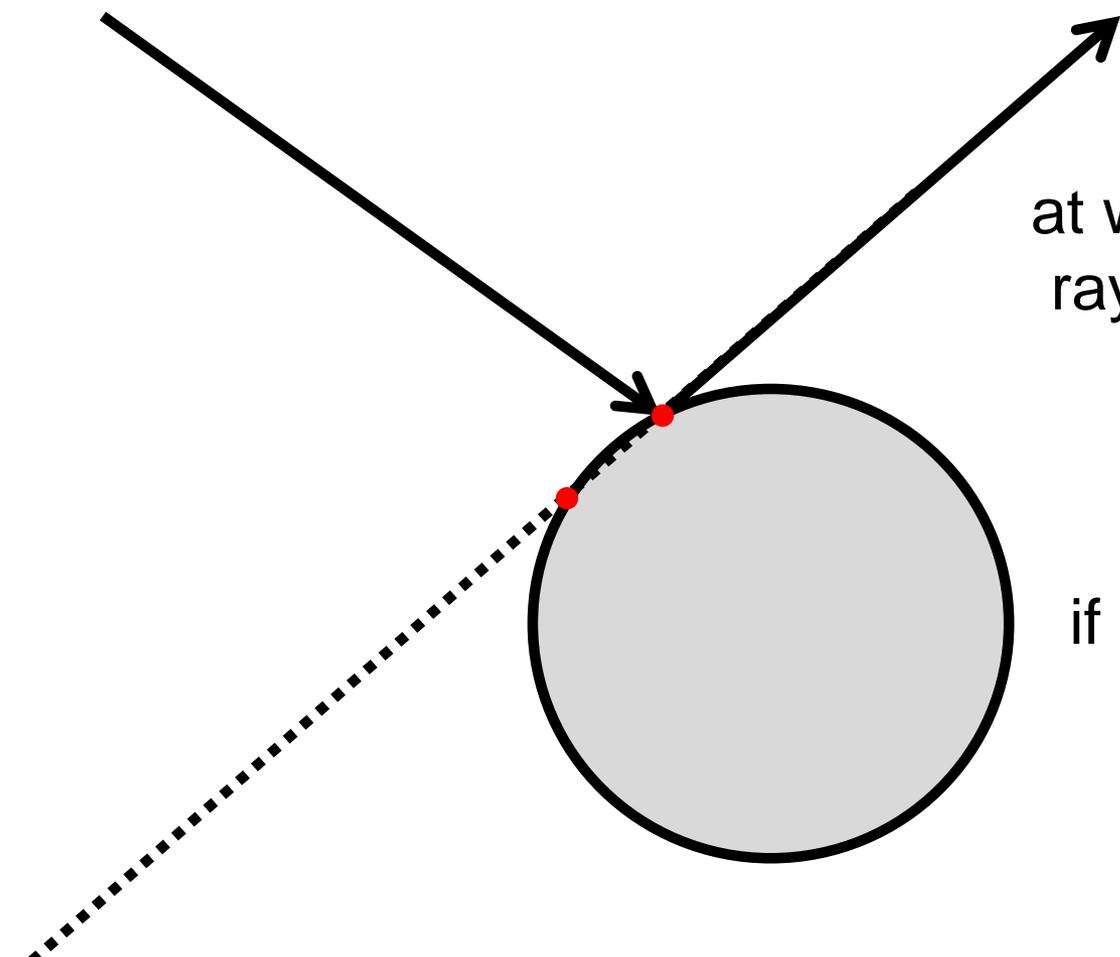


at what  $t$  does the ray hit an object?

if lucky:  $\{-1.2, 0.0\}$



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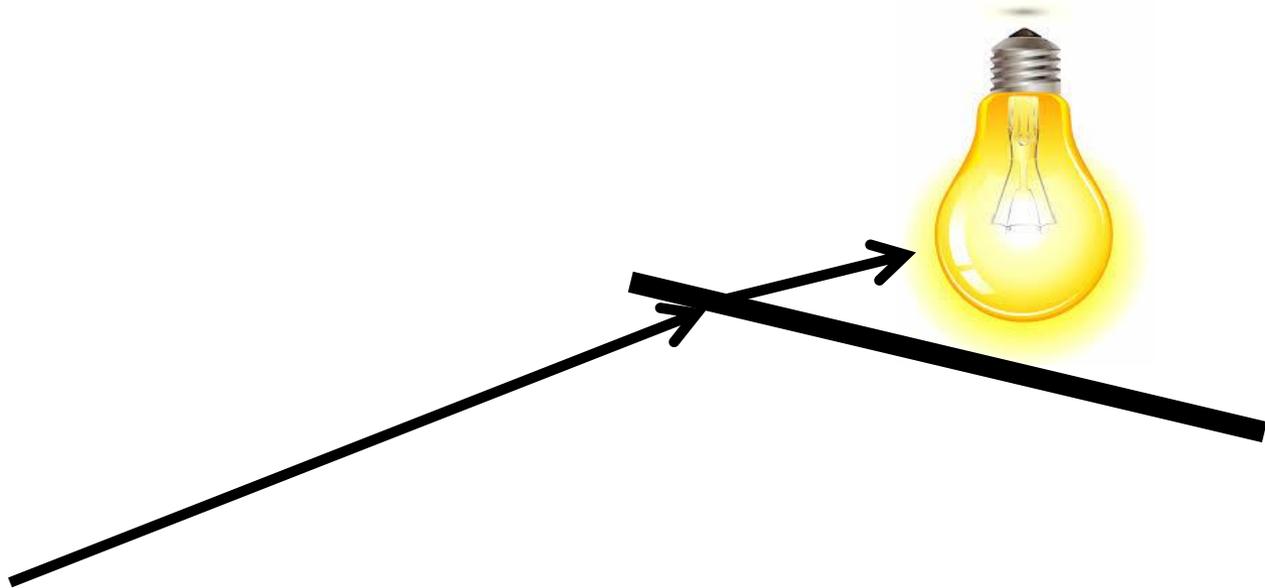
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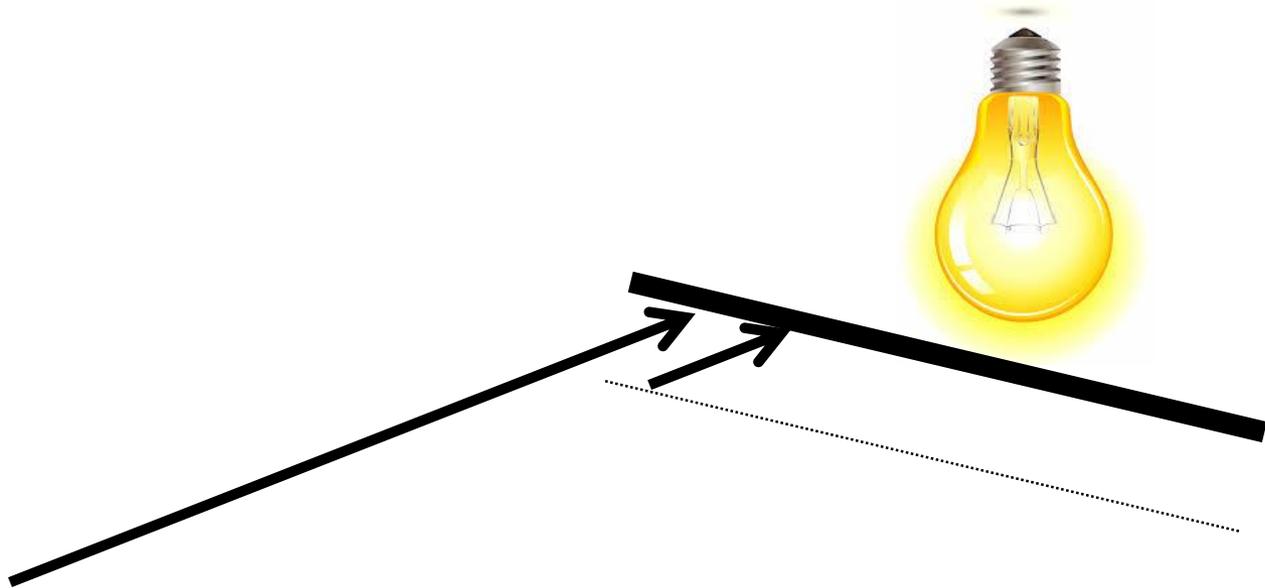
if unlucky:  $\{-1.2, 1e-12\}$

**Ignore t Near Zero?**

# Ignore t Near Zero?



# Ignore t Near Zero?



Fix: move slightly in normal direction (or backward ray direction) before shooting shadow ray

# Hard Shadows

real-world doesn't  
look like this



# Hard Shadows

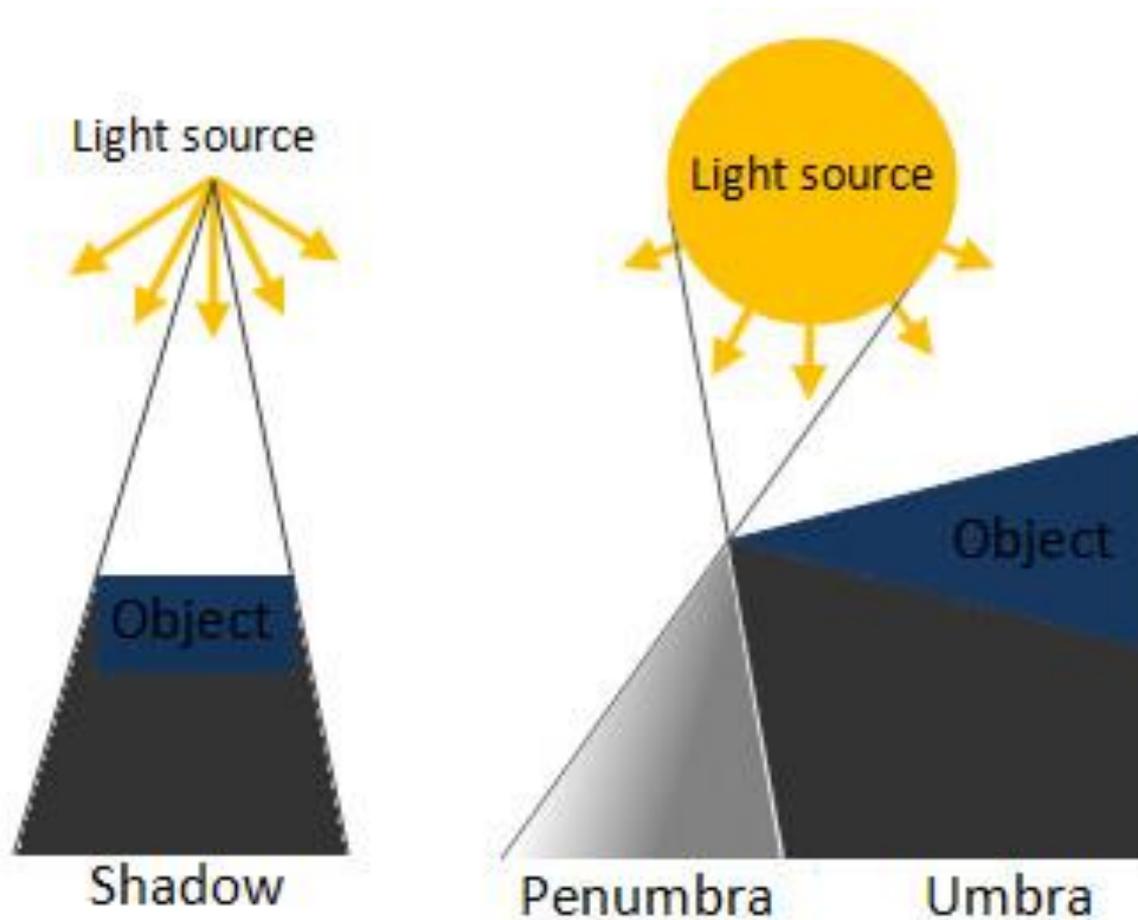
real-world doesn't  
look like this

shadows usually  
**soft**

why?

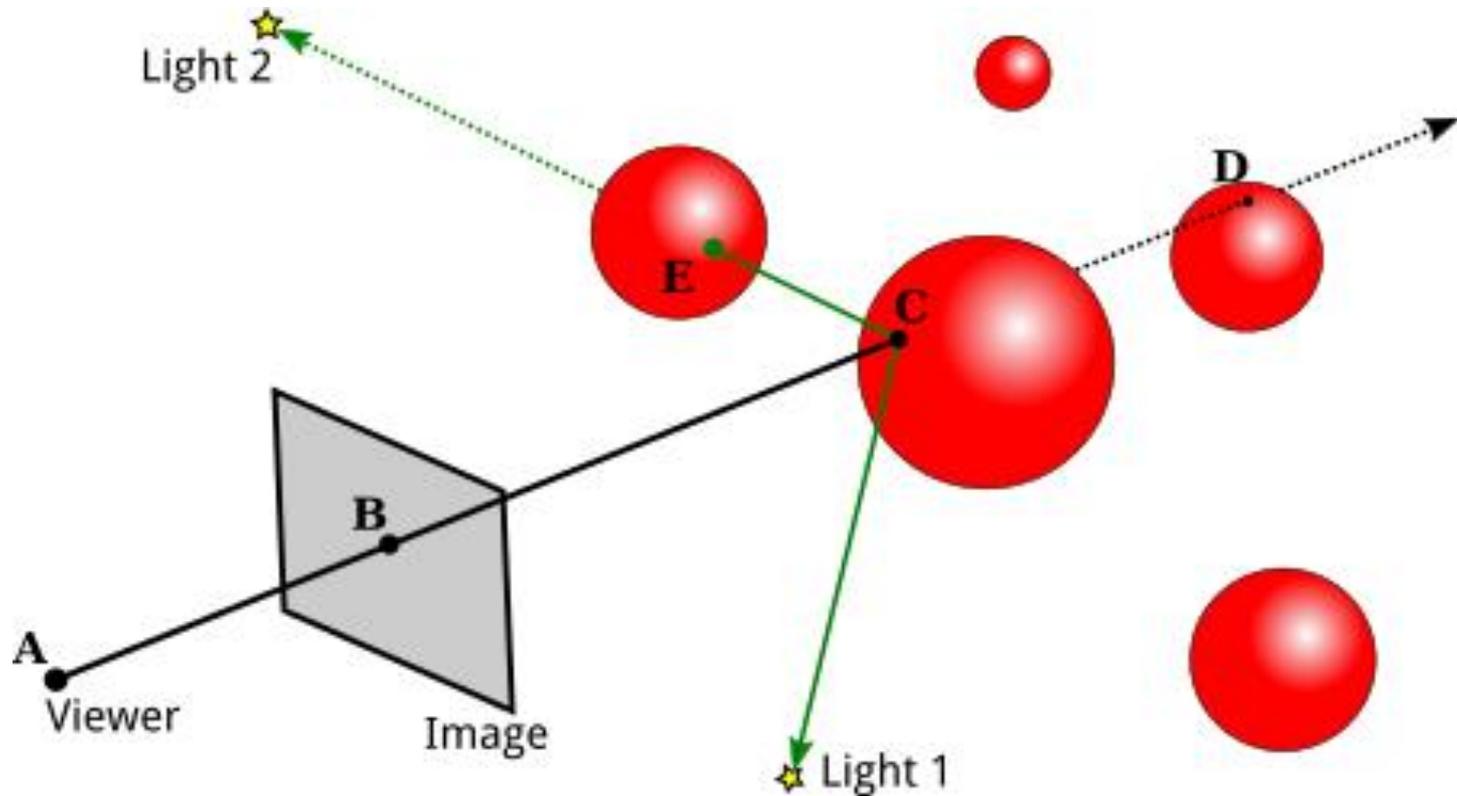


# Soft Shadows



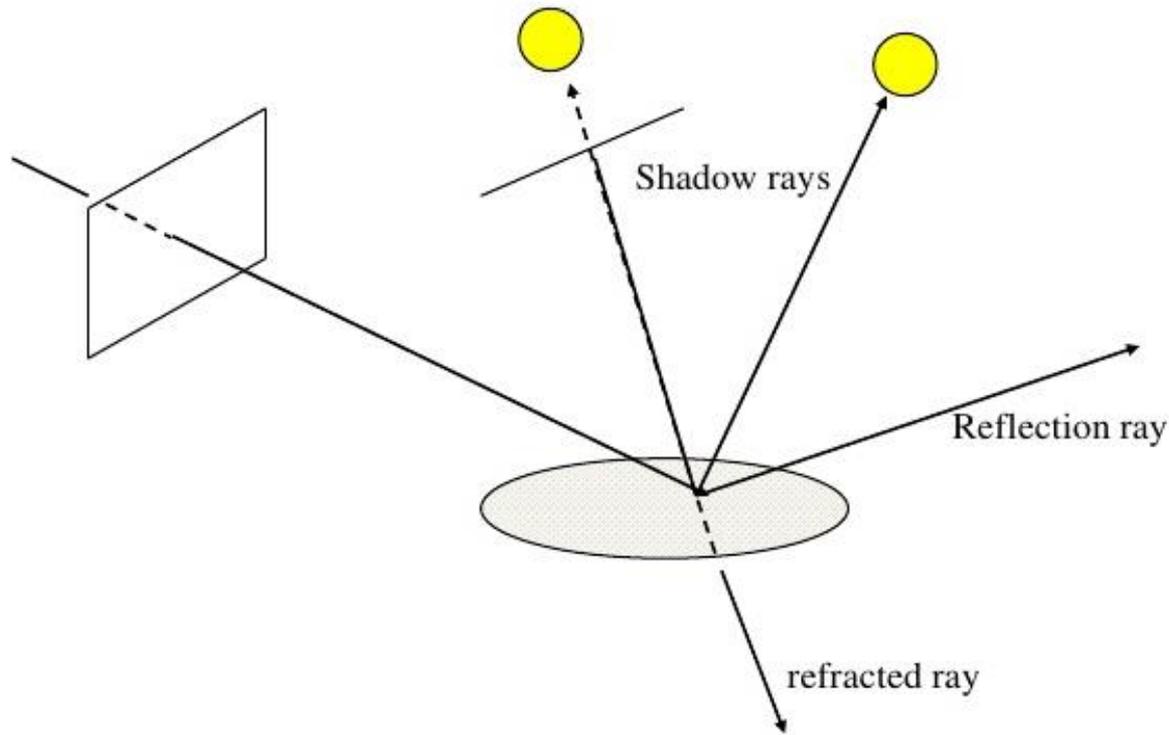
# Other Secondary Rays

Translucent objects



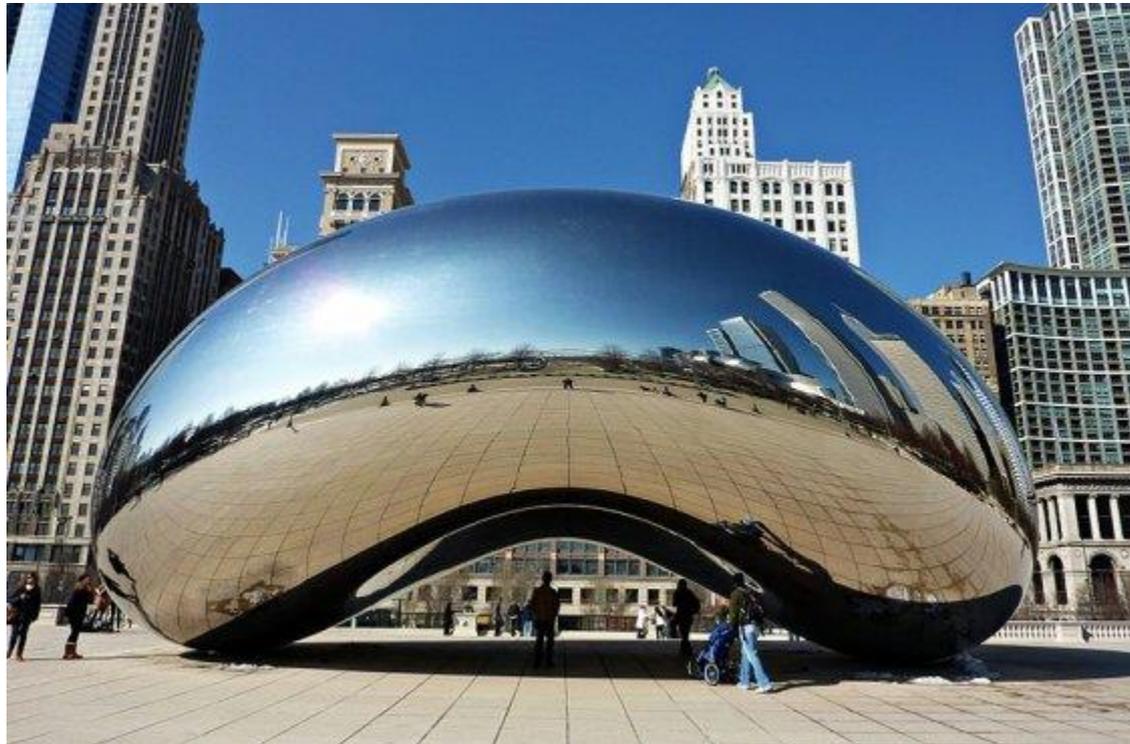
# Other Secondary Rays

Reflection & refraction



# Reflection

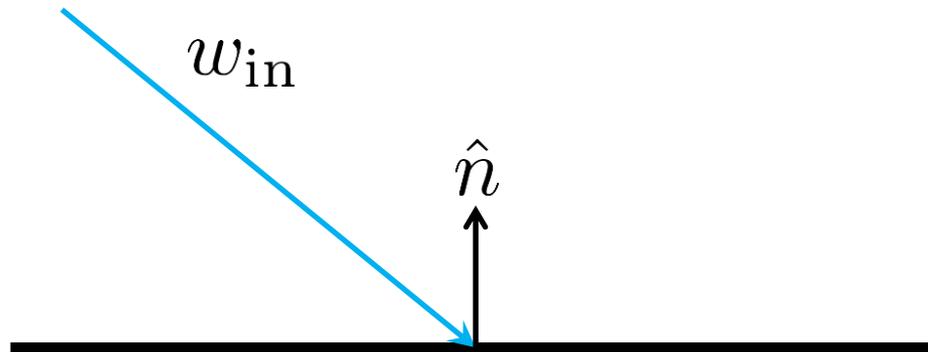
Purely specular (mirrored) surface



# Reflection

Purely specular (mirrored) surface:

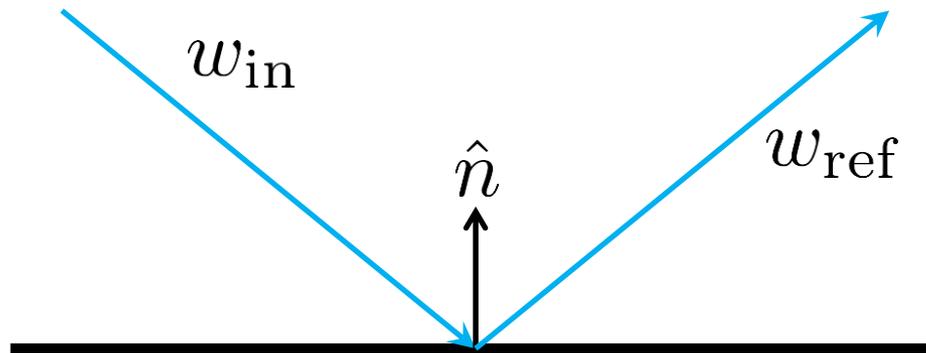
1. Incoming ray  $w_{\text{in}}$  hits surface



# Reflection

Purely specular (mirrored) surface:

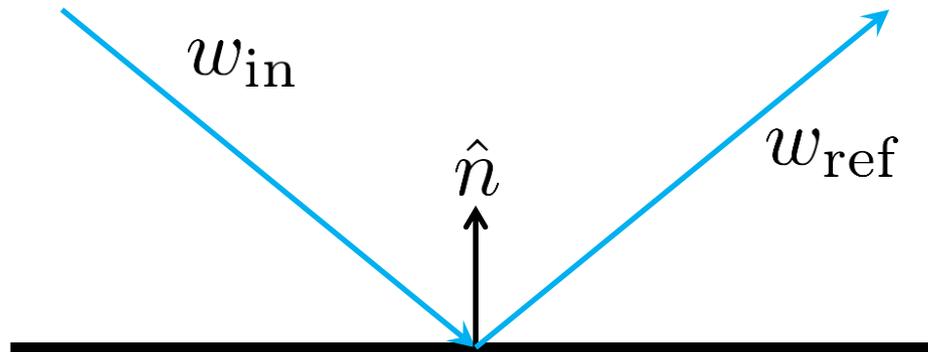
1. Incoming ray  $w_{\text{in}}$  hits surface
2. Shoot secondary **reflection ray**  $w_{\text{ref}}$



# Reflection

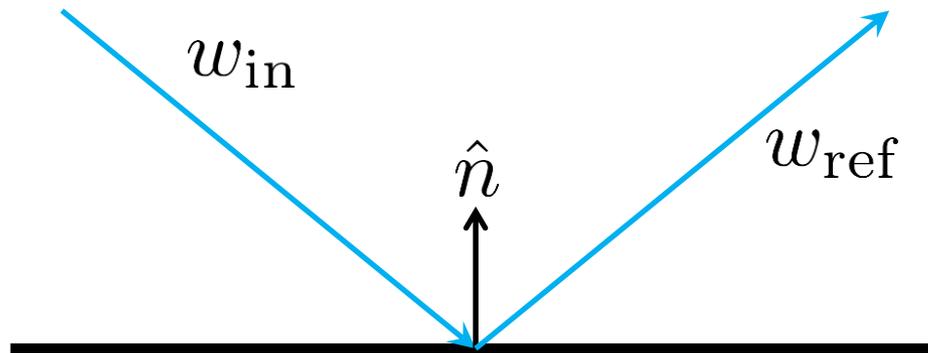
Purely specular (mirrored) surface:

1. Incoming ray  $w_{\text{in}}$  hits surface
2. Shoot secondary **reflection ray**  $w_{\text{ref}}$
3. Set pixel color to color “seen” by  $w_{\text{ref}}$



# Choosing the Reflection Ray

Angle of reflection = angle of incidence

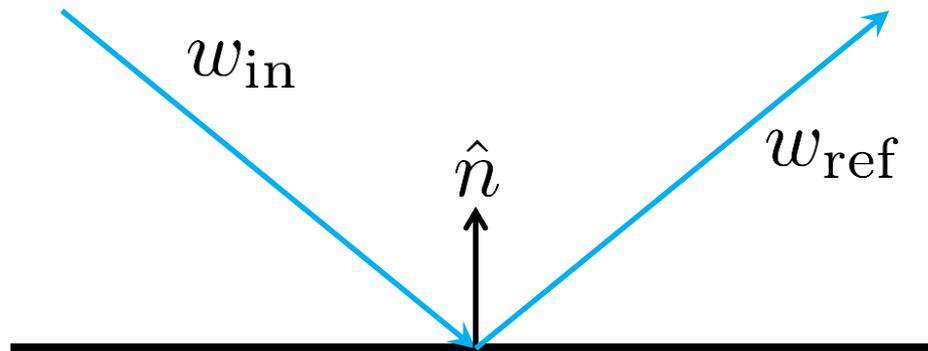


# Choosing the Reflection Ray

Angle of reflection = angle of incidence

I.e. negate component of  $w_{\text{in}}$  in normal dir

- leave **tangent** component untouched

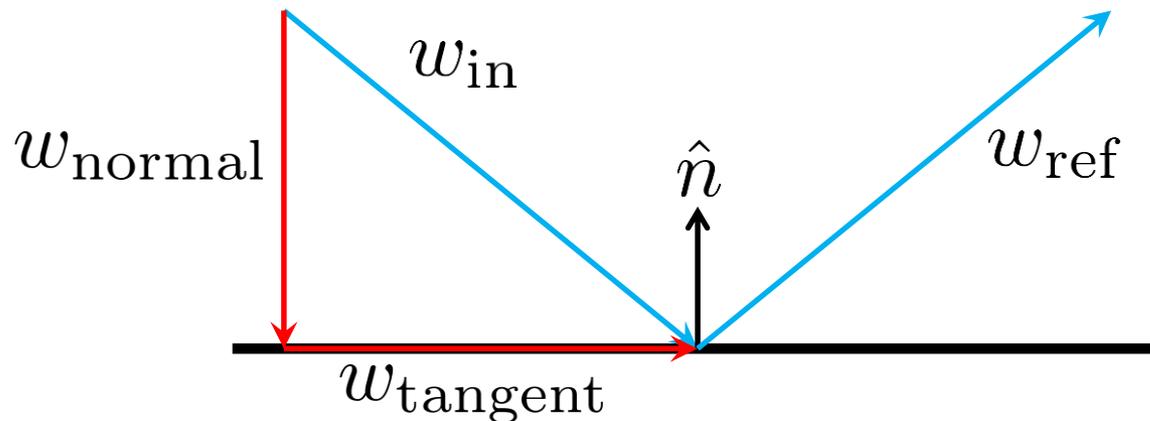


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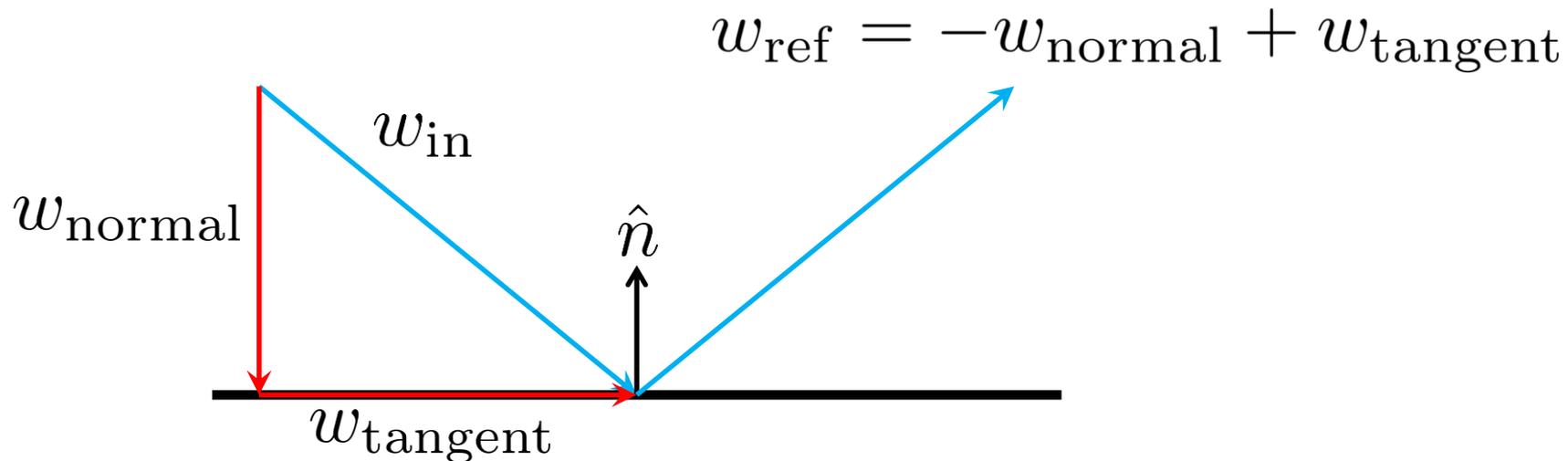


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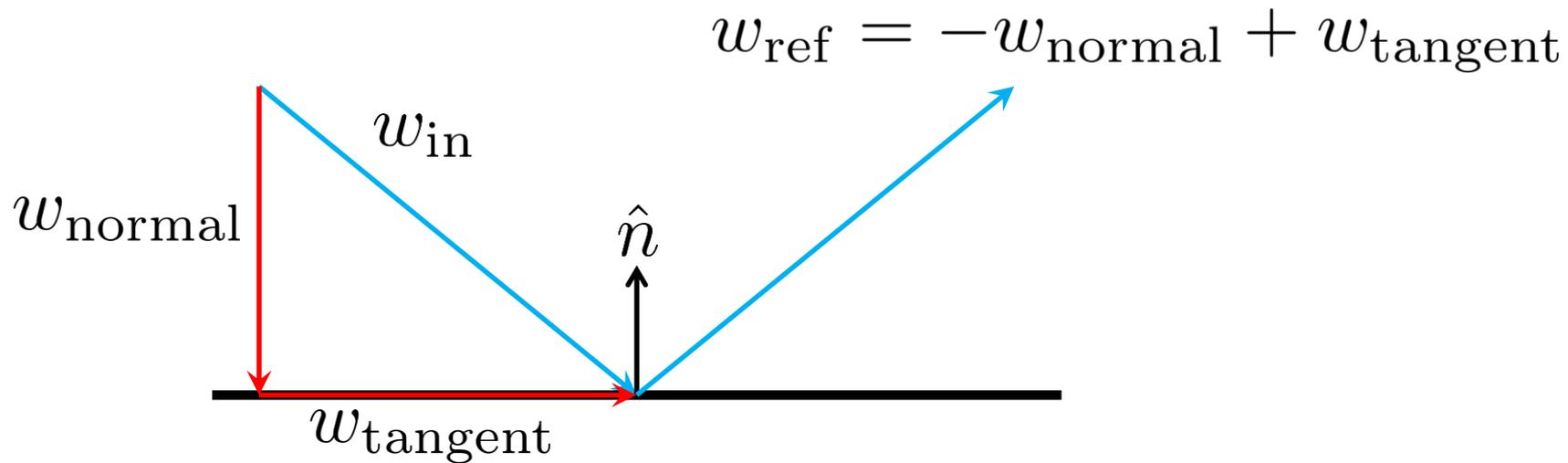


# Choosing the Reflection Ray

The math:

$$w_{\text{normal}} = (w_{\text{in}} \cdot \hat{n})\hat{n}$$

$$w_{\text{ref}} = w_{\text{in}} - 2(w_{\text{in}} \cdot \hat{n})\hat{n} = (I - 2\hat{n}\hat{n}^T)w_{\text{in}}$$



# Reflection in Practice

Objects may not be perfectly mirrored

- blend reflected color with basic shading



# Reflection in Practice

Objects may not be perfectly mirrored

- blend reflected color with basic shading

Objects have **base color**

- multiplies reflected color



# Reflections in Practice

Reflection ray might hit reflective objects

- cast recursive reflection rays...
- stop after some maximum recursion limit



# Refraction

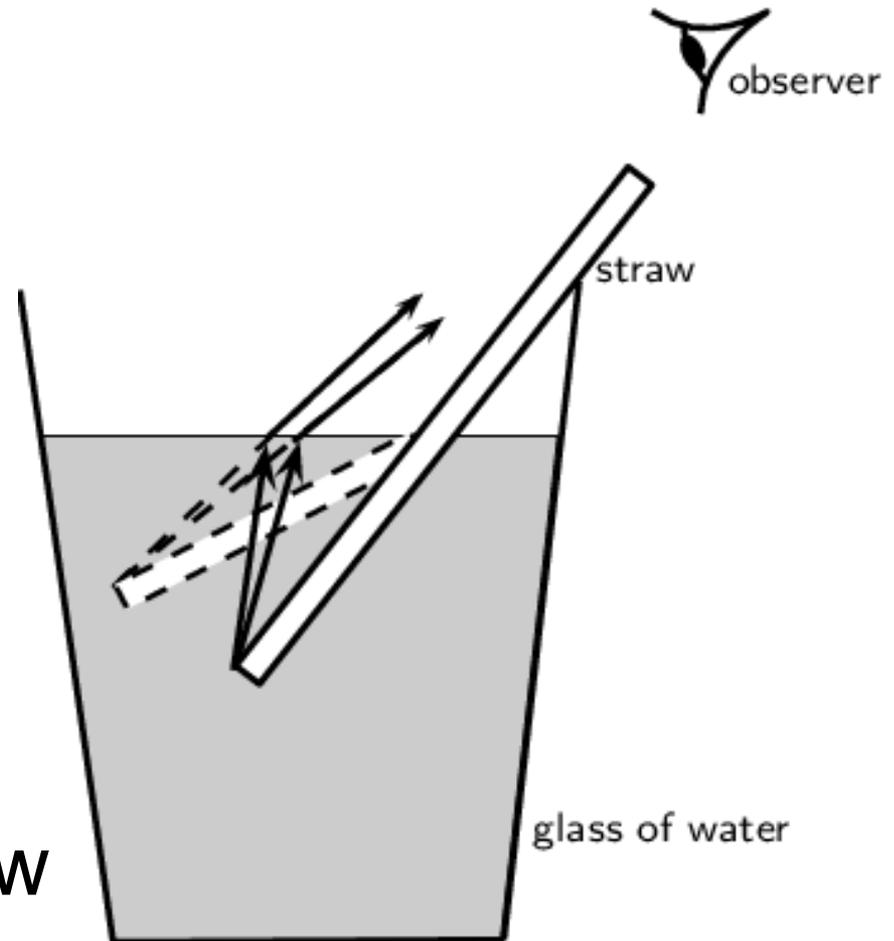


# Refraction

Light **bends** when moving between different materials

Caused by change in speed of light

We “see” dotted straw



# Index of Refraction

Measures speed of light in material

$$\text{index of refraction} = \frac{\text{speed in vacuum}}{\text{speed in material}}$$

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Measures speed of light in material

$$\text{index of refraction} = \frac{\text{speed in vacuum}}{\text{speed in material}}$$

Common values:

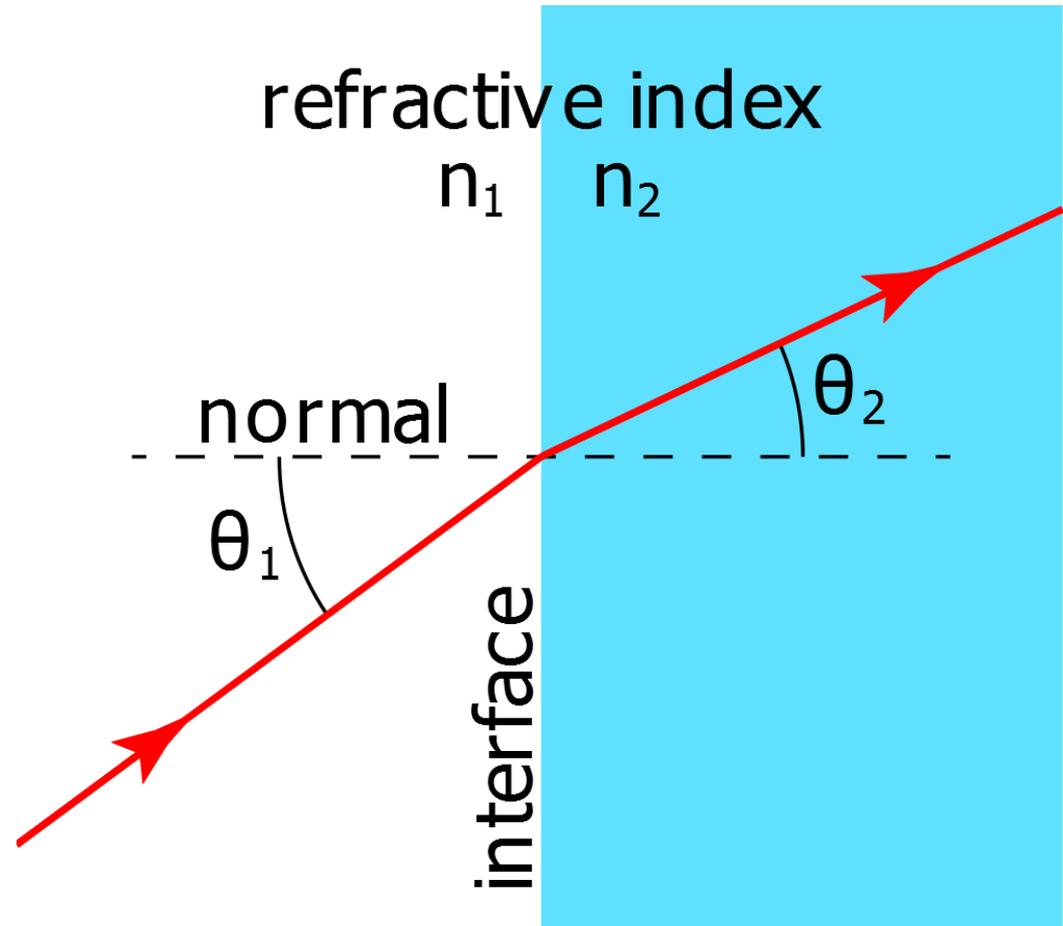
- Vacuum: 1.0
- Air: 1.0001
- Water: 1.33
- Glass: 1.5

# Snell's Law

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1}$$

Special cases:

- $n_1 = n_2$
- $\theta_1 = 0$



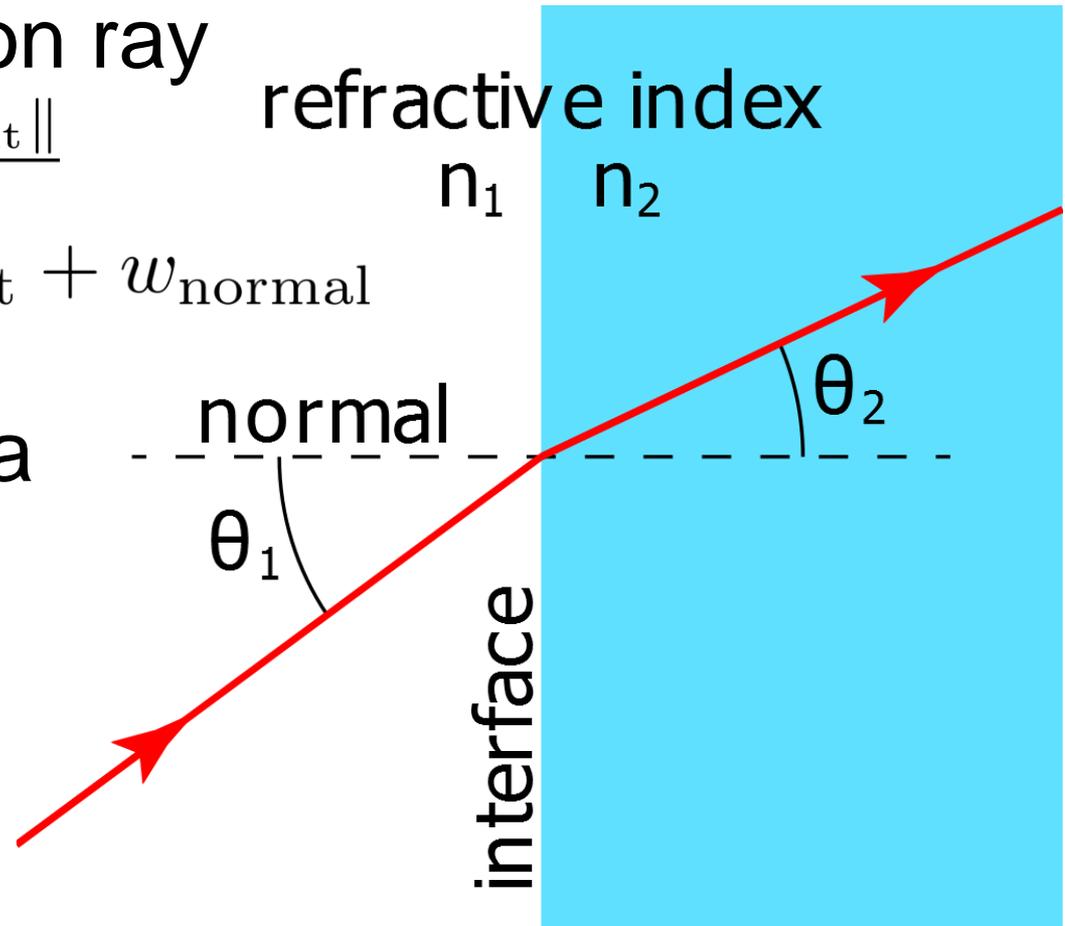
# Implementing Snell's Law

Shoot refraction ray

$$\sin \theta_1 = \frac{\|w_{\text{tangent}}\|}{\|w_{\text{in}}\|}$$

$$w_{\text{rfr}} = \alpha w_{\text{tangent}} + w_{\text{normal}}$$

Solve for alpha

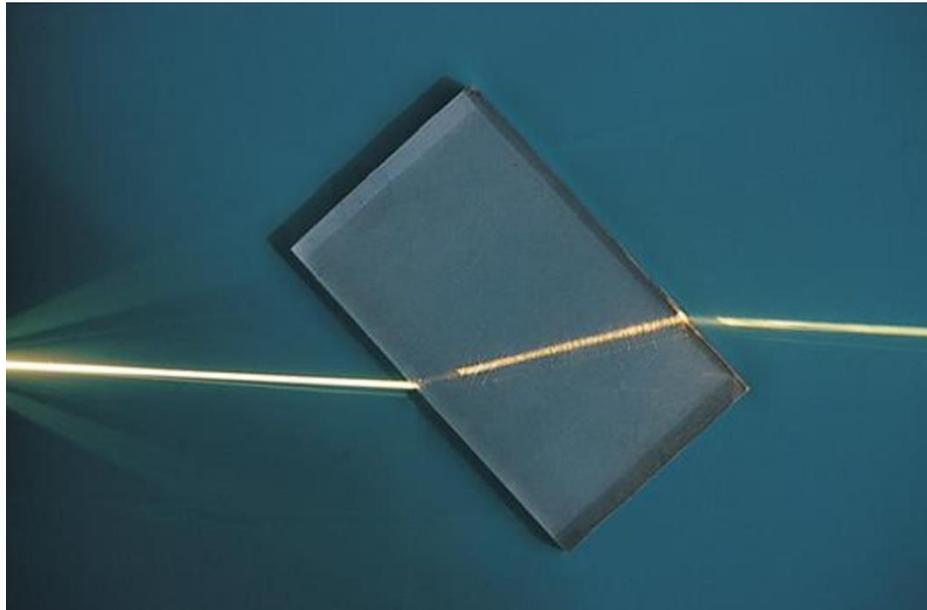


# Refractions in Practice

Again, usually multiplied by a base color

Light bends when entering **and** leaving

- must detect both when ray-tracing



# Reflection & Refraction Example

