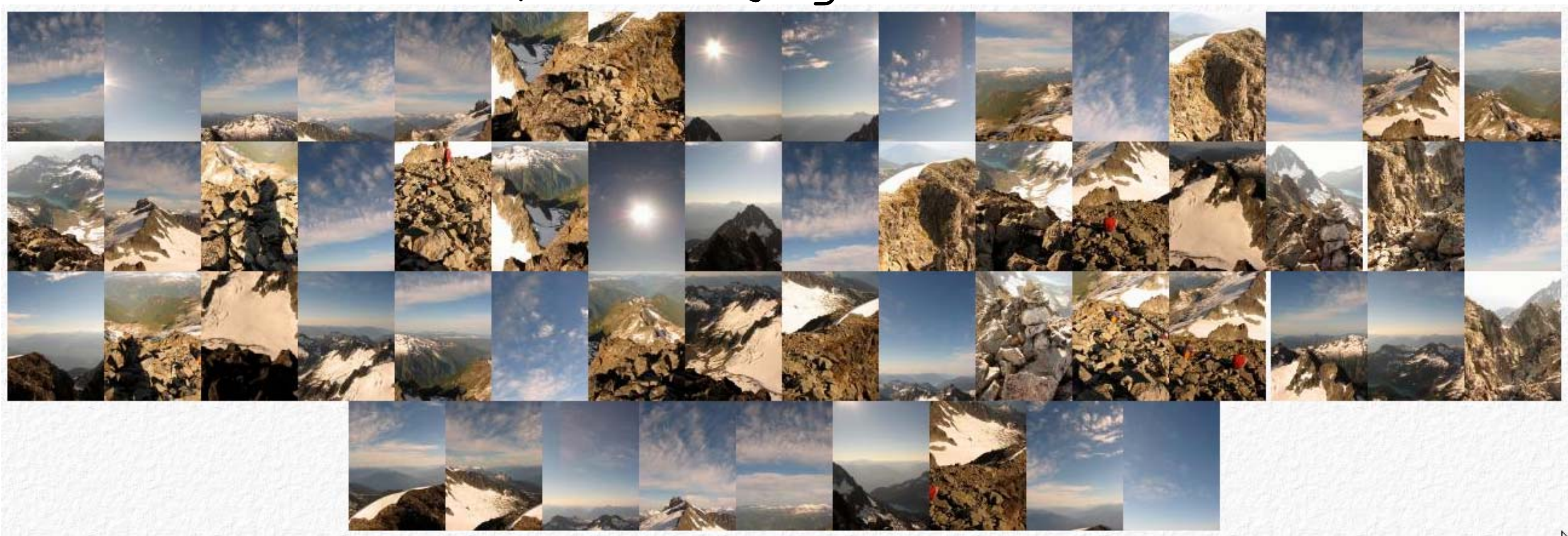
Building Panoramic Image Mosaics

Input images



automatically created mosaic

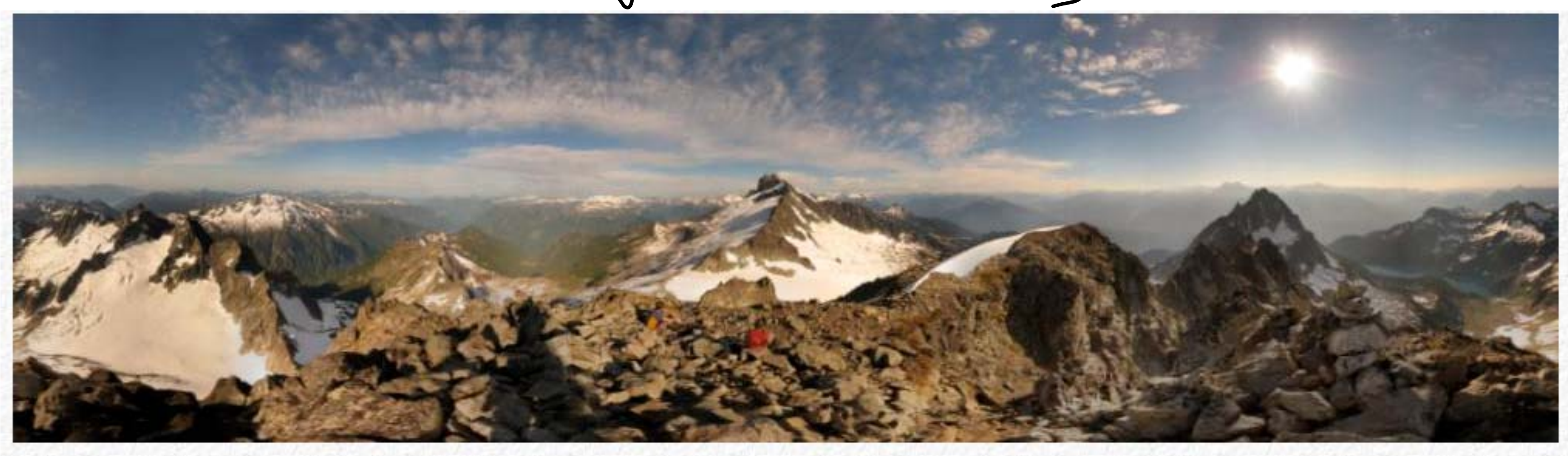
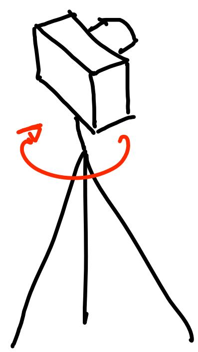


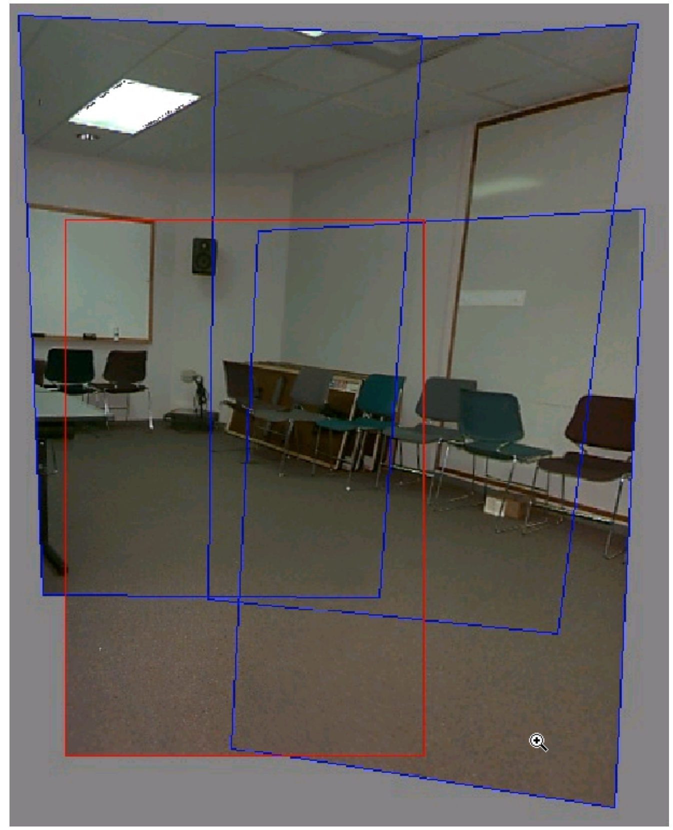
Image Mosaicing

Technique:

1. Take multiple photos while rotating camera on a tripod (or by hand)

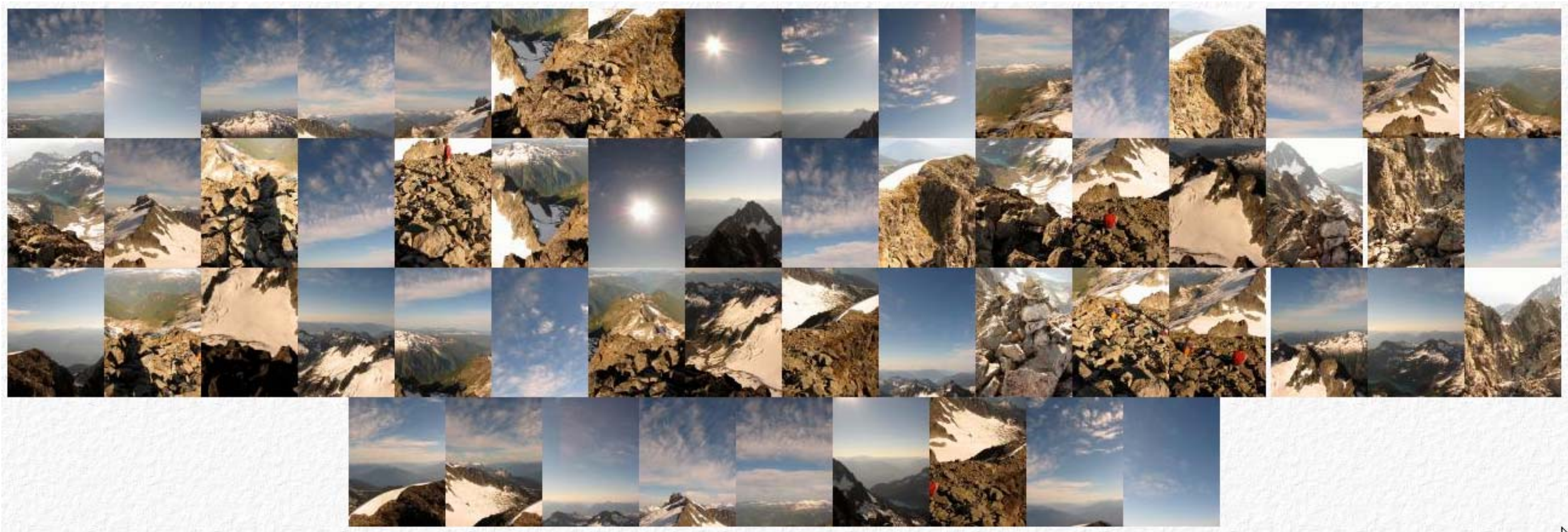


2. Warp & align the photos
3. Blend photos to compute final mosaic



* In general, photos must be warped to align their contents!

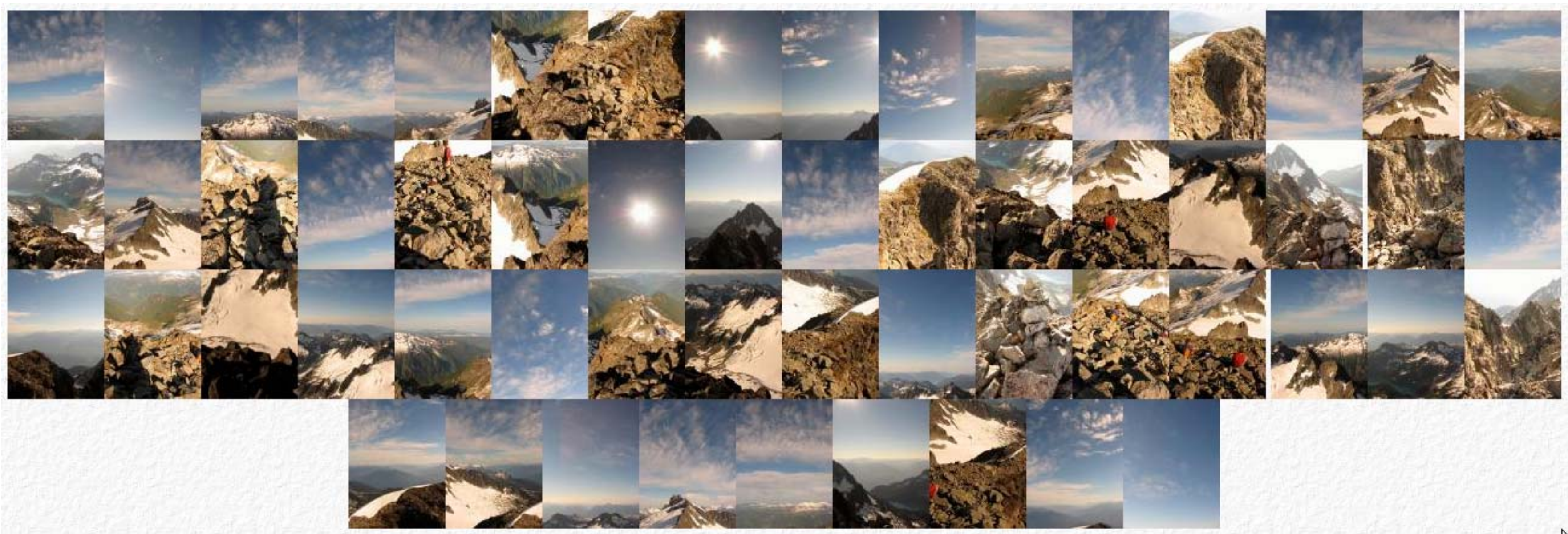
Step 1: Capture



Important:

- Camera should change orientation, not position
- Keep camera settings (gain, focus, speed, aperture) fixed if possible

Step 2: Warp & Align



⇓ 28/57 images aligned

Step 2: Warp & Align (Continued)

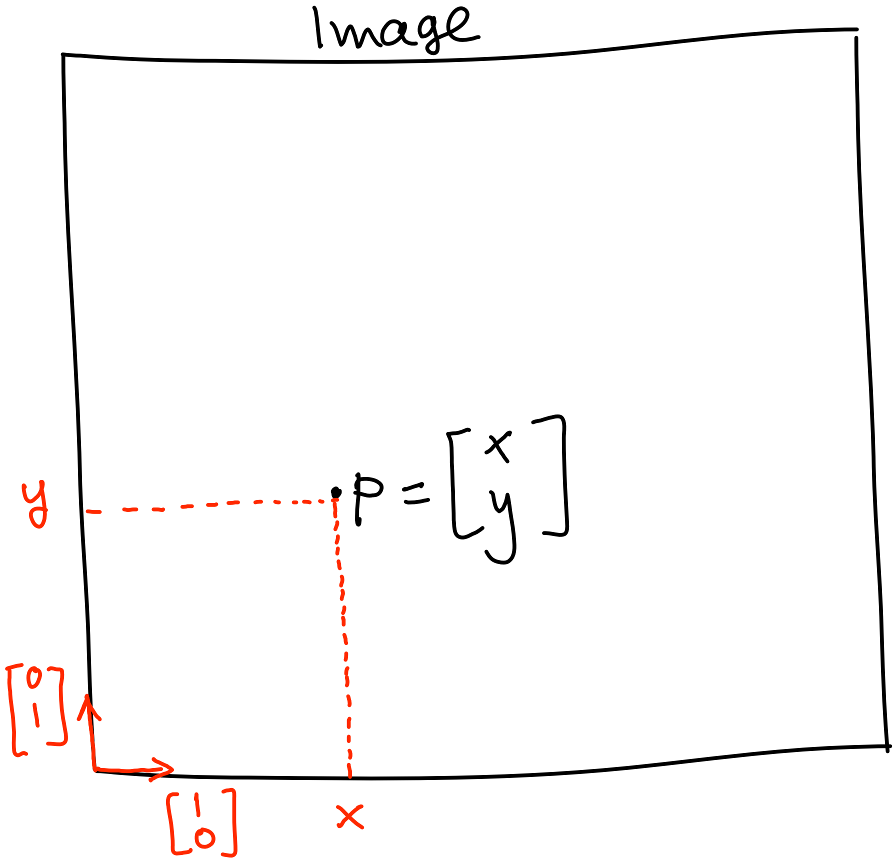
57/57 images aligned

Step 3: Blend

Laplacian Pyramid Blending ⇓ seams not visible anymore

(Brown & Lowe ; ICCV 2003) google "Lowe Brown Autostitch"

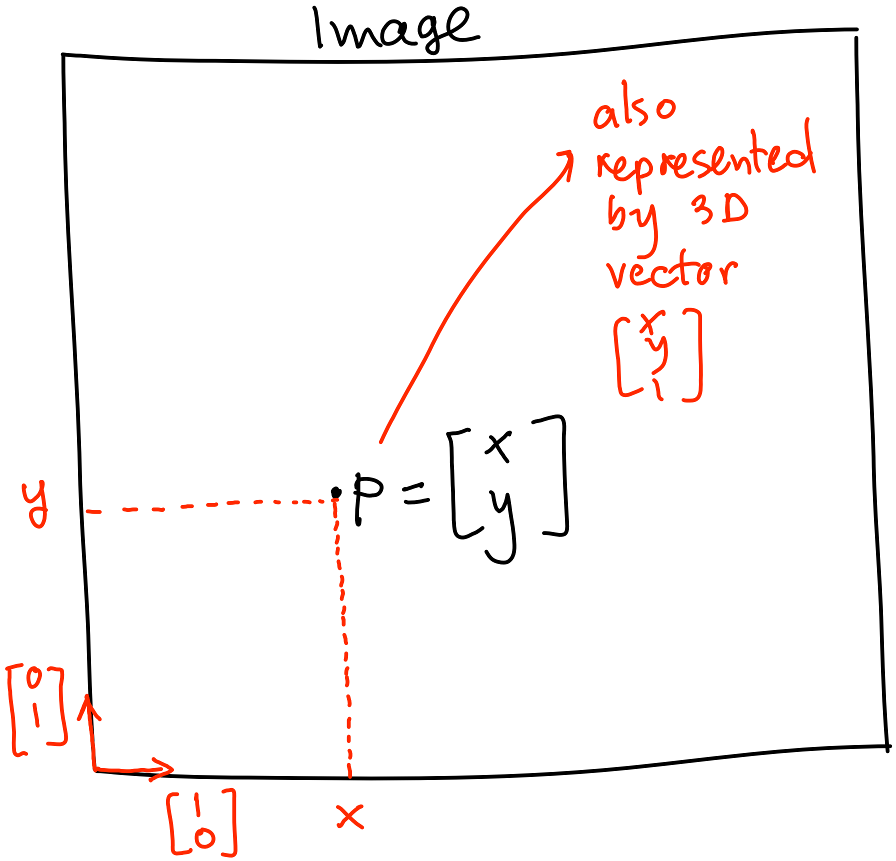
Representing Pixels by Euclidean 2D Coordinates



- "Standard" (Euclidean) representation of an image point p:

$$p = x \cdot \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\text{basis vectors}} = \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\text{Euclidean coordinates}}$$

Euclidean Coordinates ⇒ Homogeneous Coordinates



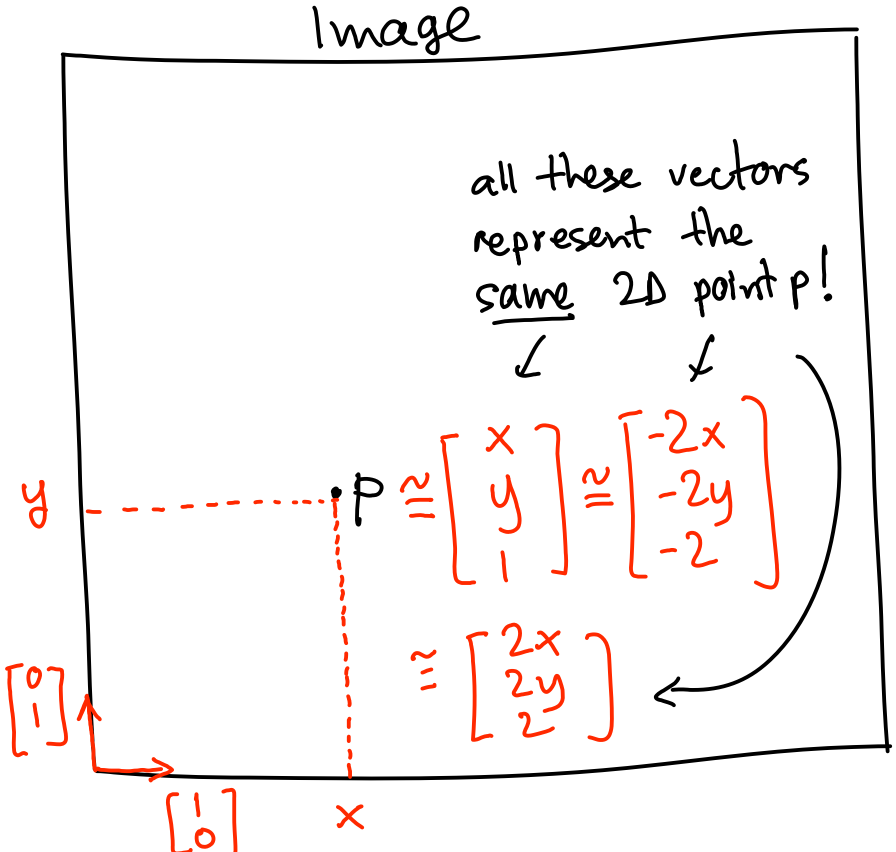
- "Standard" (Euclidean) representation of an image point p:

$$p = x \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

- Homogeneous (a.k.a. Projective) representation of p

image coordinates		homogeneous 2D coordinates
$\begin{bmatrix} x \\ y \end{bmatrix}$	$\longrightarrow$	$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

2D Homogeneous Coordinates: Definition



- For <u>any</u> $\lambda \neq 0$, the numbers $\lambda x, \lambda y, \lambda$ are called the homogeneous coordinates of point p

Definition:

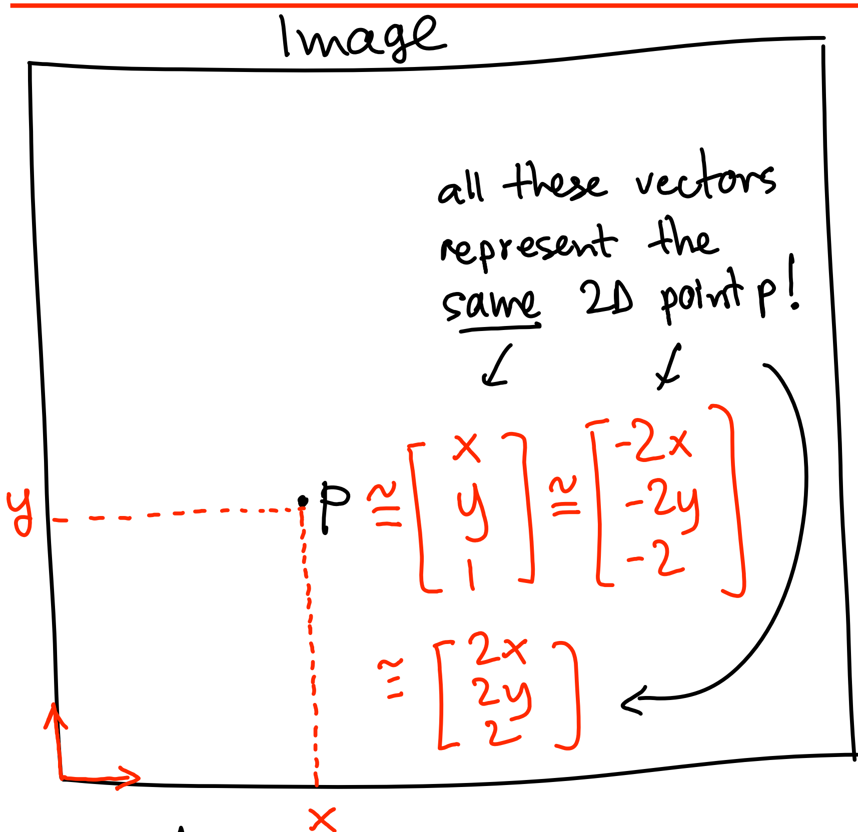
Homogeneous representation of p

> p represented by <u>any</u> 3D vector $\begin{bmatrix} \lambda x \\ \lambda y \\ \lambda \end{bmatrix}$ with $\lambda \neq 0$

- Homogeneous (a.k.a. Projective) representation of p

image coordinates		homogeneous 2D coordinates	
$\begin{bmatrix} x \\ y \end{bmatrix}$	$\longrightarrow$	$\begin{bmatrix} \lambda \cdot x \\ \lambda \cdot y \\ \lambda \cdot 1 \end{bmatrix}$	$\lambda \neq 0$

2D Homogeneous Coordinates: Equality



$$p \cong \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \cong \begin{bmatrix} -2x \\ -2y \\ -2 \end{bmatrix} \cong \begin{bmatrix} 2x \\ 2y \\ 2 \end{bmatrix}$$

Definition (Homogeneous Equality)

Two vectors of homogeneous coords $v_1 = \begin{bmatrix} x \\ y \\ w \end{bmatrix}$ and $v_2 = \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix}$ are called <u>equal</u> if they represent the <u>same</u> 2D point:

$v_1 \cong v_2$ denotes homog. equality

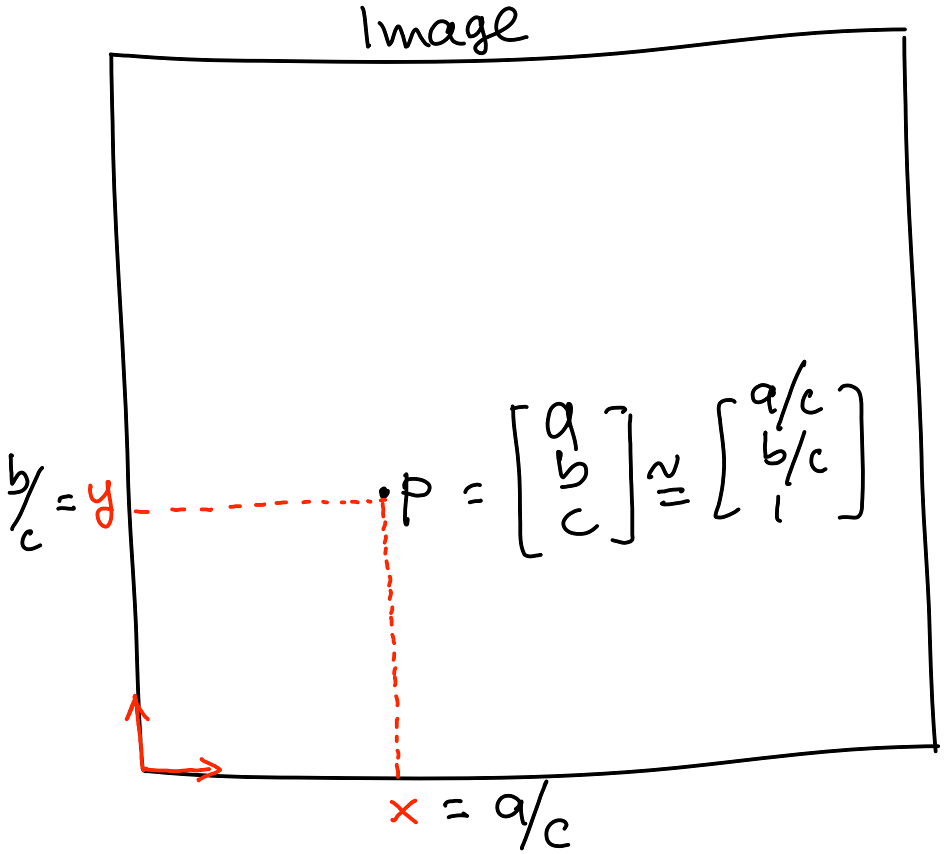
$\Longleftrightarrow$ there is a $\lambda \neq 0$ such that

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \lambda \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix}$$

Examples:

- Is $\begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix} \cong \begin{bmatrix} 6 \\ 8 \\ 12 \end{bmatrix}$? yes (take $\lambda=2$)
- Is $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cong \begin{bmatrix} 0 \\ 0 \\ 30 \end{bmatrix}$? yes (take $\lambda=30$)
- Is $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \cong \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}$? <u>no</u>!

Homogeneous Coordinates ⇒ Euclidean Coordinates



Converting from homogeneous to Euclidean coordinates:

$\begin{bmatrix} a \\ b \\ c \end{bmatrix}, \begin{bmatrix} a/c \\ b/c \\ 1 \end{bmatrix}$ represent the same 2D point

$\Longleftrightarrow$ 2D coordinates are $\begin{bmatrix} a/c \\ b/c \end{bmatrix}$

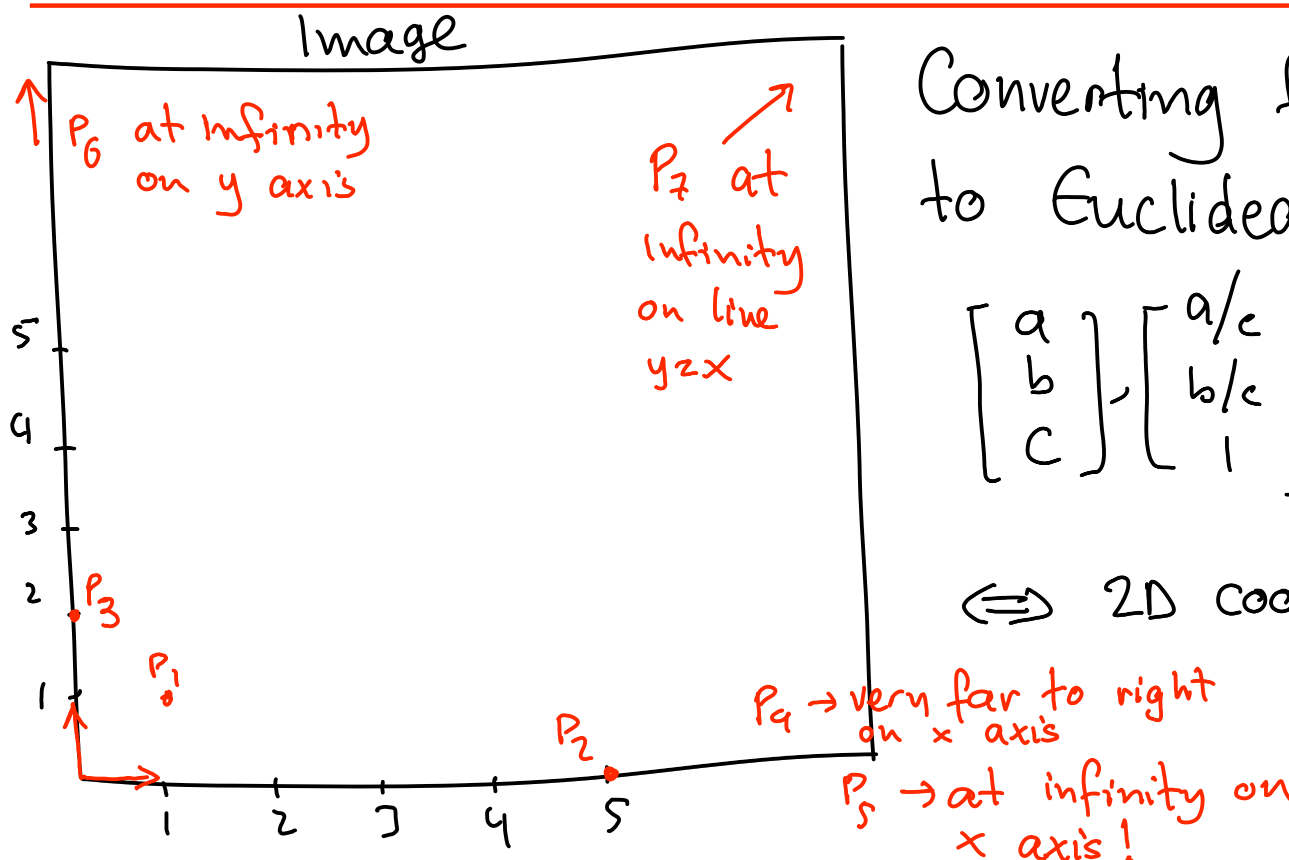
$$v_1 \cong v_2$$

$$\Longleftrightarrow$$

there is a $\lambda \neq 0$ such that

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \lambda \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix}$$

Homogeneous Coordinates ⇒ Euclidean Coordinates



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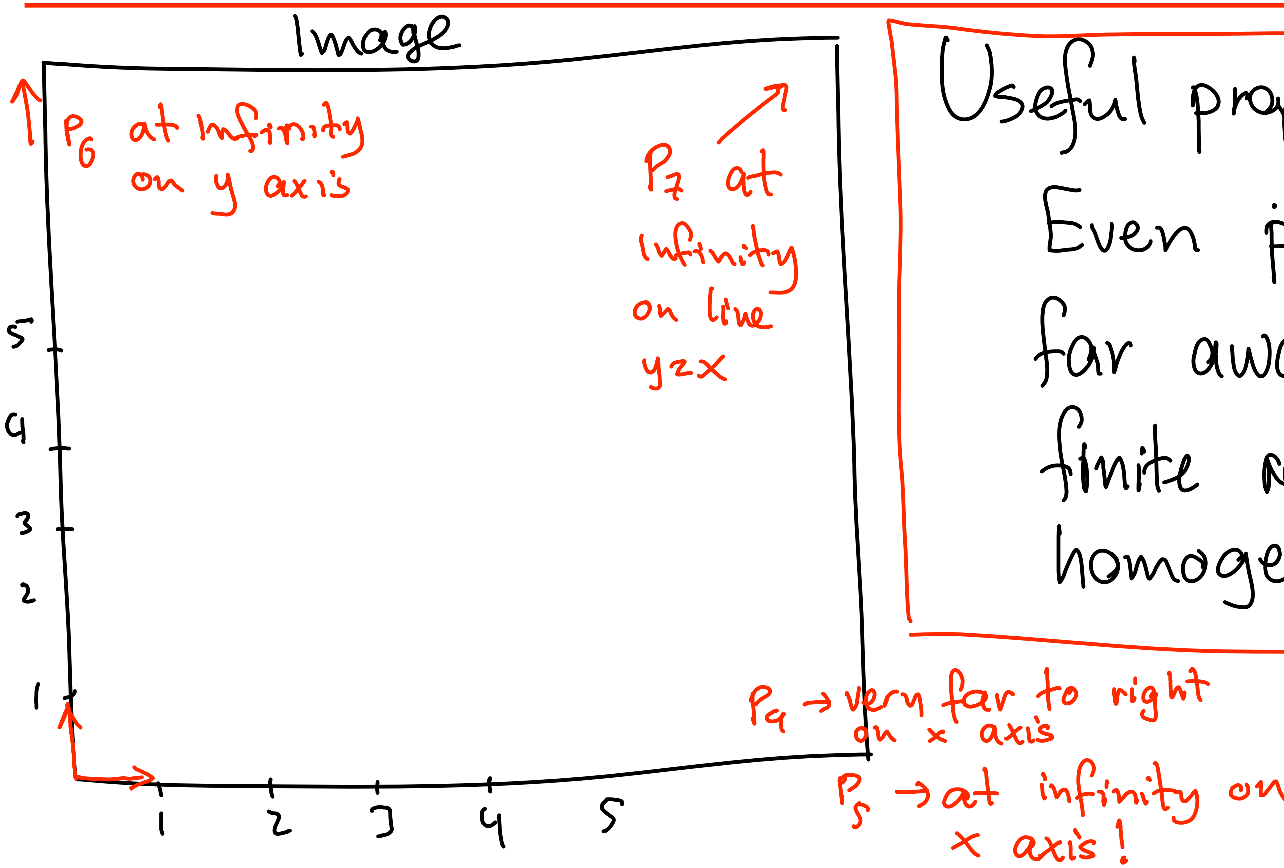
$\Longleftrightarrow$ 2D coordinates are $\begin{bmatrix} a/c \\ b/c \end{bmatrix}$

Practice exercise: Plot positions of the following points

$P_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $P_7 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$P_1 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ $P_2 = \begin{bmatrix} 10 \\ 0 \\ 2 \end{bmatrix}$ $P_3 = \begin{bmatrix} 0 \\ 8 \\ 4 \end{bmatrix}$ $P_4 = \begin{bmatrix} 1 \\ 0 \\ 0.0001 \end{bmatrix}$ $P_5 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Points at ∞ in Homogeneous Coordinates



Useful property #1:

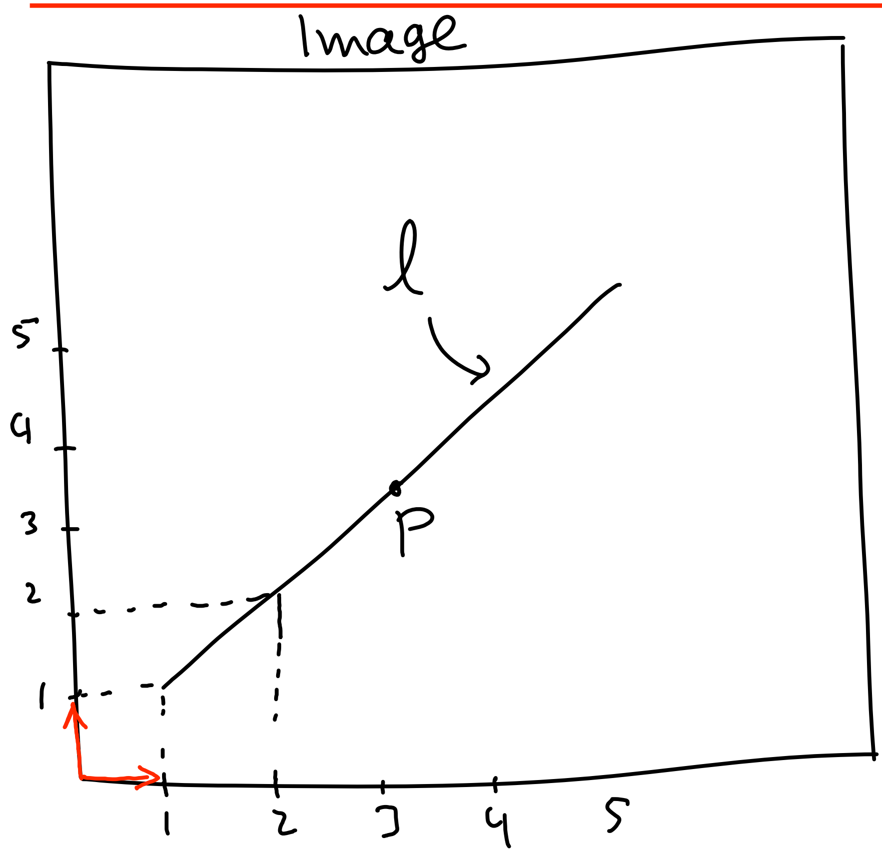
Even points infinitely far away have a finite representation in homogeneous coords!

leads to very stable geometric computations

$$P_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad P_7 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$P_4 = \begin{bmatrix} 1 \\ 0 \\ 0.0001 \end{bmatrix} \quad P_5 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Line Equations in Homogeneous Coordinates



- The equation of a line

$$ax + by + c = 0$$

line parameters

- In homogeneous coordinates

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

or $l^T \cdot p = 0$

vector holding line parameters

vector holding homogeneous coordinates of a point

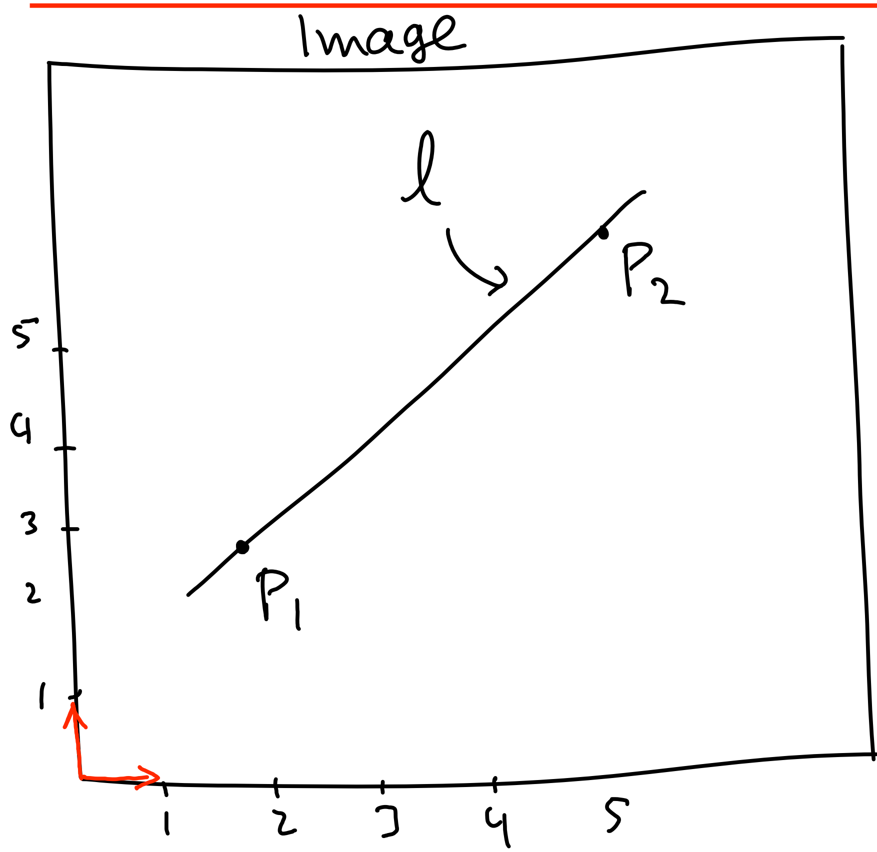
Example: line $y = x$ in homogeneous coords:

$$1 \cdot x - 1 \cdot y + 0 \cdot 1 = 0$$

line parameters of l

$$\begin{bmatrix} 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

The Line Passing Through 2 Points



- Calculating the parameters of a line through two points with homogeneous coordinates p_1, p_2

$$\boxed{l = p_1 \times p_2}$$

cross product of two 3D vectors

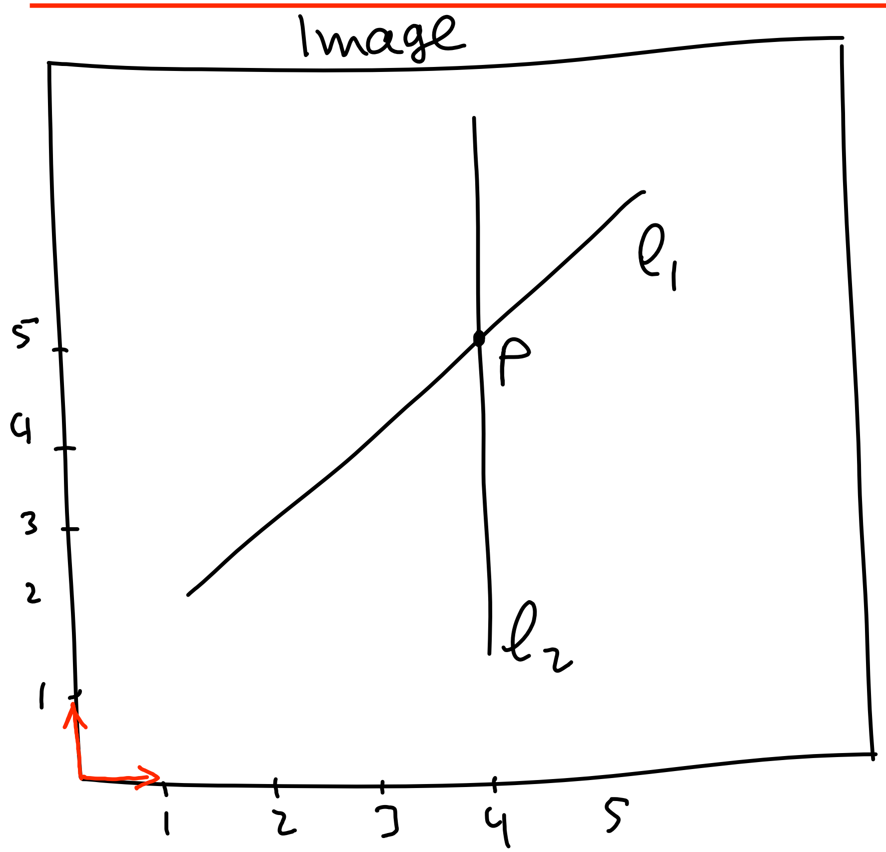
- l must satisfy $l^T \cdot p_1 = 0$, $l^T \cdot p_2 = 0$
- taken as 3D vectors, l is perpendicular to both p_1 and p_2
$\Rightarrow$ it is along the cross product, $p_1 \times p_2$

- In homogeneous coordinates

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

or $\quad l^T \cdot p = 0$

The Point of Intersection of Two Lines



- Calculating the homogeneous coordinates of the intersection of two lines ℓ_1, ℓ_2

$$\boxed{p = \ell_1 \times \ell_2}$$

cross product of two 3D vectors

- In homogeneous coordinates

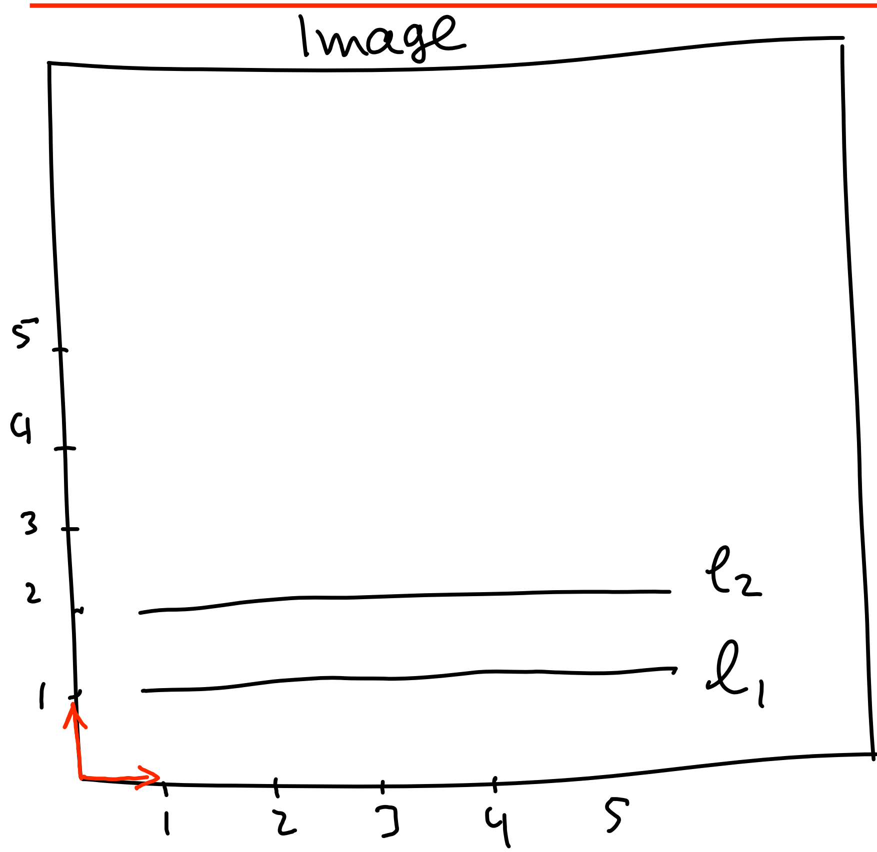
$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

or $\ell^T \cdot p = 0$

- p must satisfy $\ell_1^T p = 0$, $\ell_2^T p = 0$
- taken as 3D vectors, p is perpendicular to both ℓ_1 and ℓ_2

$\Rightarrow$ it is along the cross product, $\ell_1 \times \ell_2$

Computing the Intersection of Parallel Lines



- Calculating the homogeneous coordinates of the intersection of two lines ℓ_1, ℓ_2

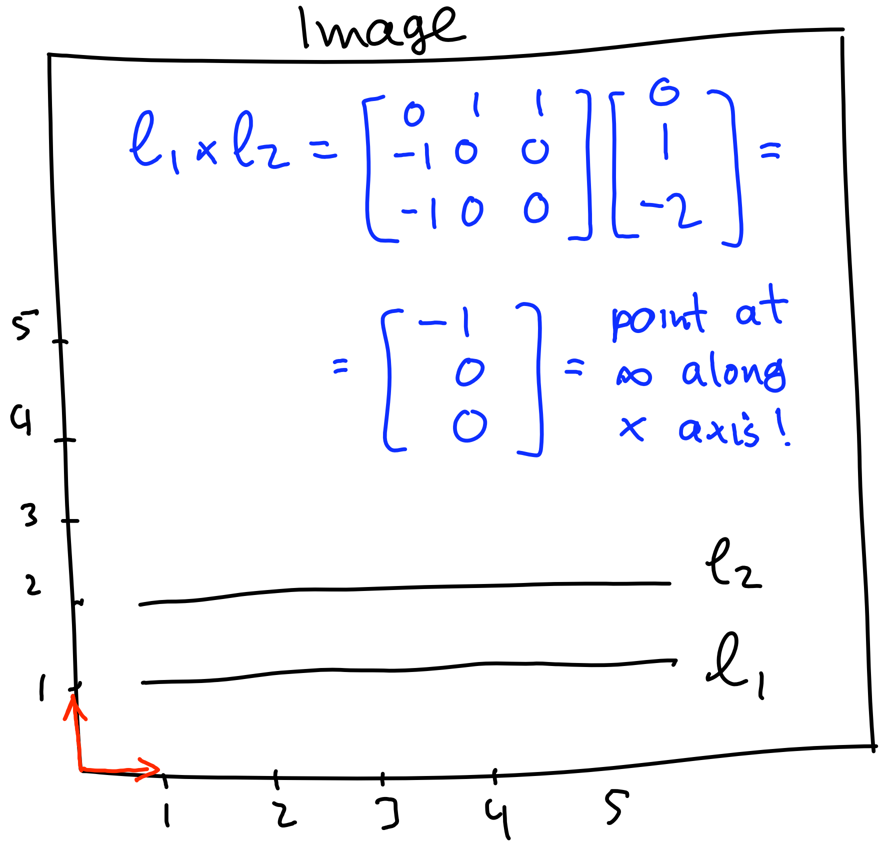
$$\boxed{p = \ell_1 \times \ell_2}$$

cross product of two 3D vectors

This calculation works even when ℓ_1, ℓ_2 are parallel!

(no floating point exceptions or divide-by-zero errors!)

Computing the Intersection of Parallel Lines



$$l_1 \times l_2 = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} =$$

$$= \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} =$$ point at ∞ along x axis!

- Calculating the homogeneous coordinates of the intersection of two lines l_1, l_2

$$p = l_1 \times l_2$$

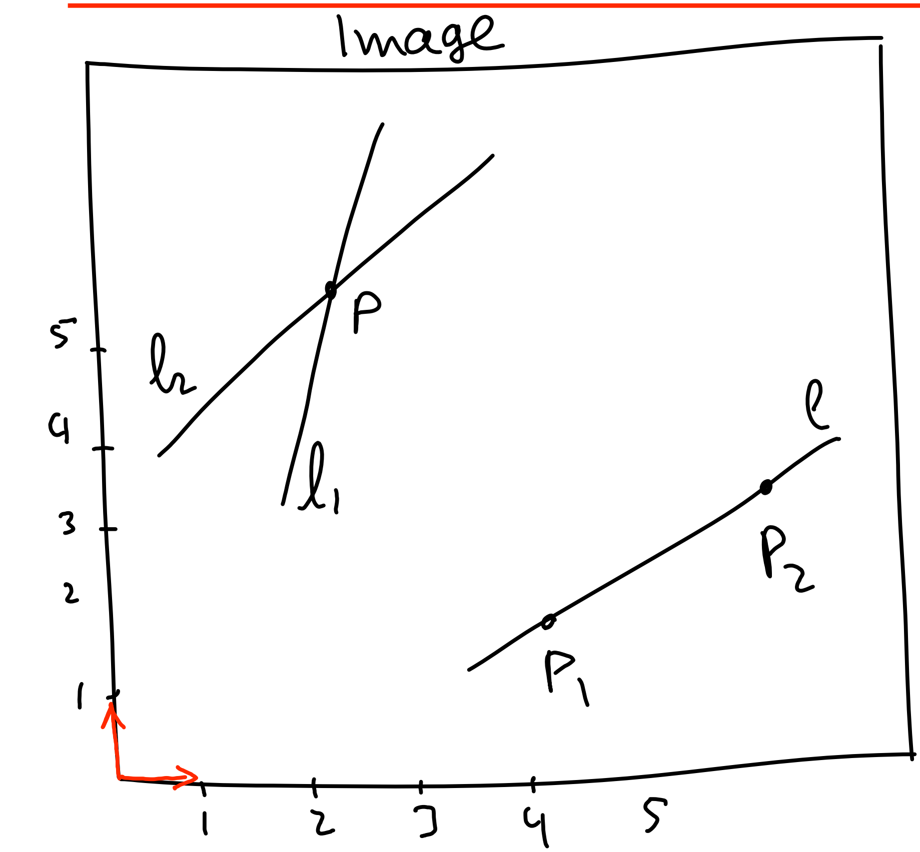
cross product of two 3D vectors

- Line eq. of l_1 is $y=1$. Also written as $0 \cdot x + 1 \cdot y - 1 = 0$. So $l_1 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$
- Similarly $l_2 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$

Aside (calculating cross products): If $l_1 = (a, b, c)$ then

$$l_1 \times l_2 = \begin{bmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{bmatrix} l_2$$

Lines from Points & Points from Lines



Useful property #2

- Very simple way of computing & intersecting lines
- Numerical stability even when result is at ∞

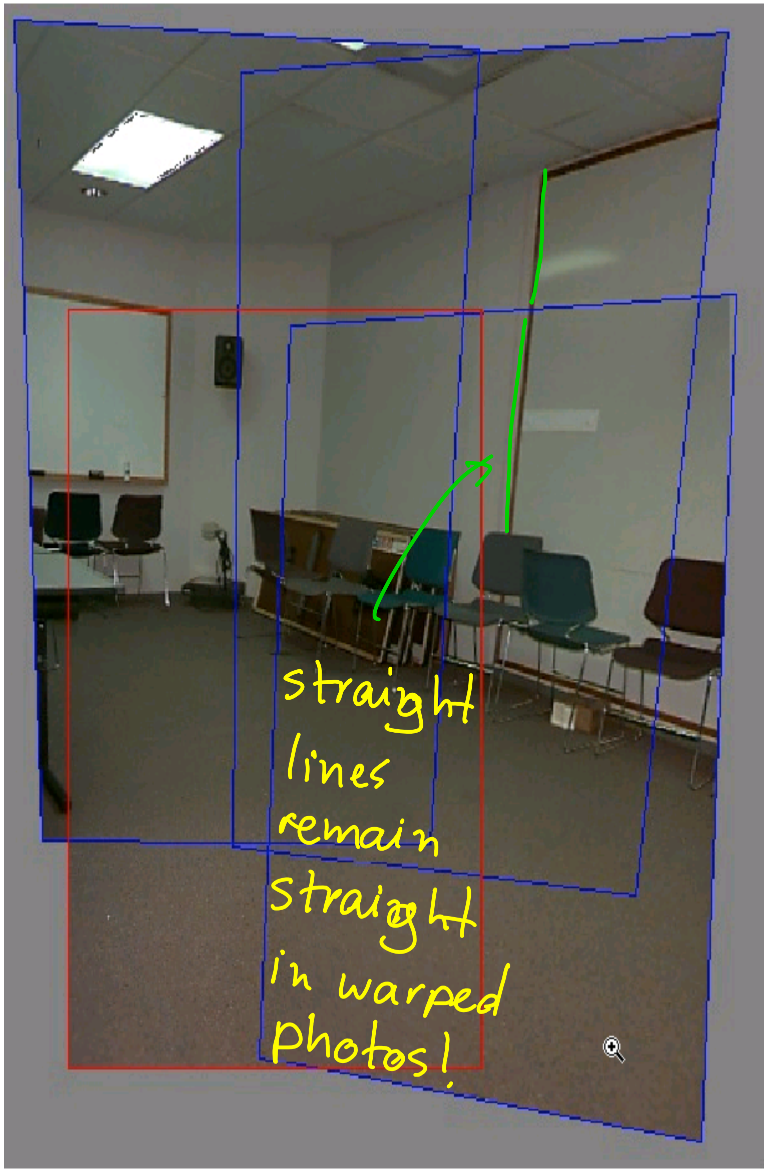
Line through 2 points

$$l = p_1 \times p_2 = \begin{bmatrix} 0 & -p_1^z & p_1^y \\ p_1^z & 0 & -p_1^x \\ -p_1^y & p_1^x & 0 \end{bmatrix} \begin{bmatrix} p_2^x \\ p_2^y \\ p_2^z \end{bmatrix}$$

Intersection of 2 lines

$$p = l_1 \times l_2 = \begin{bmatrix} 0 & -l_1^z & l_1^y \\ l_1^z & 0 & -l_1^x \\ -l_1^y & l_1^x & 0 \end{bmatrix} \begin{bmatrix} l_2^x \\ l_2^y \\ l_2^z \end{bmatrix}$$

Linear Image Warps

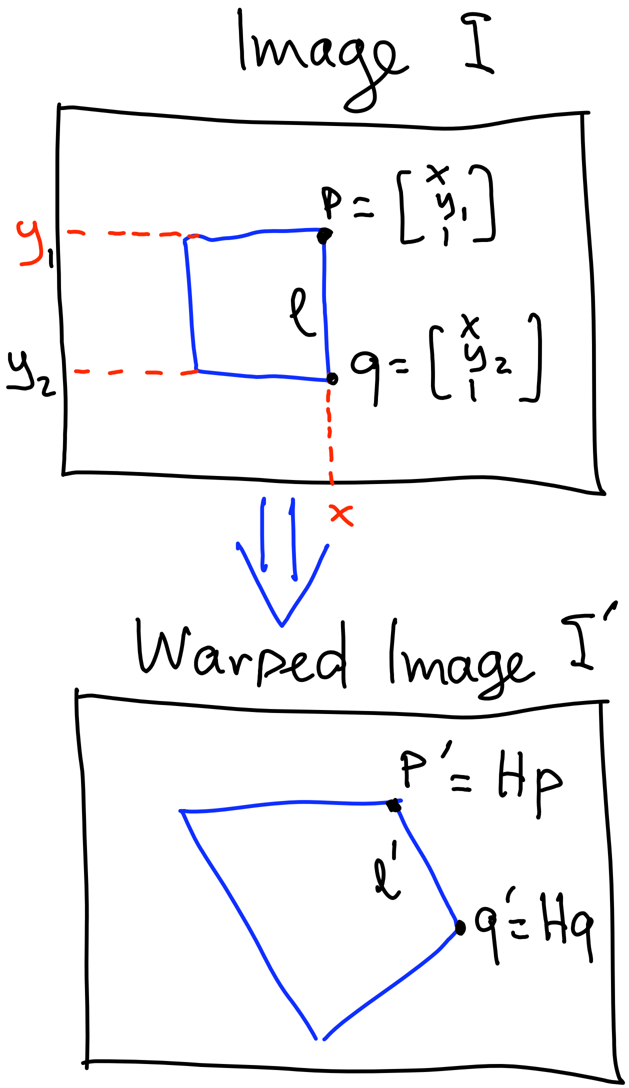


Basic Insight:

To align multiple photos for mosaicing we must warp them in a way that preserves all lines

(i.e. lines before warping remain lines after warping)

Linear Image Warps & Homographies



The matrix H is called a **Homography**

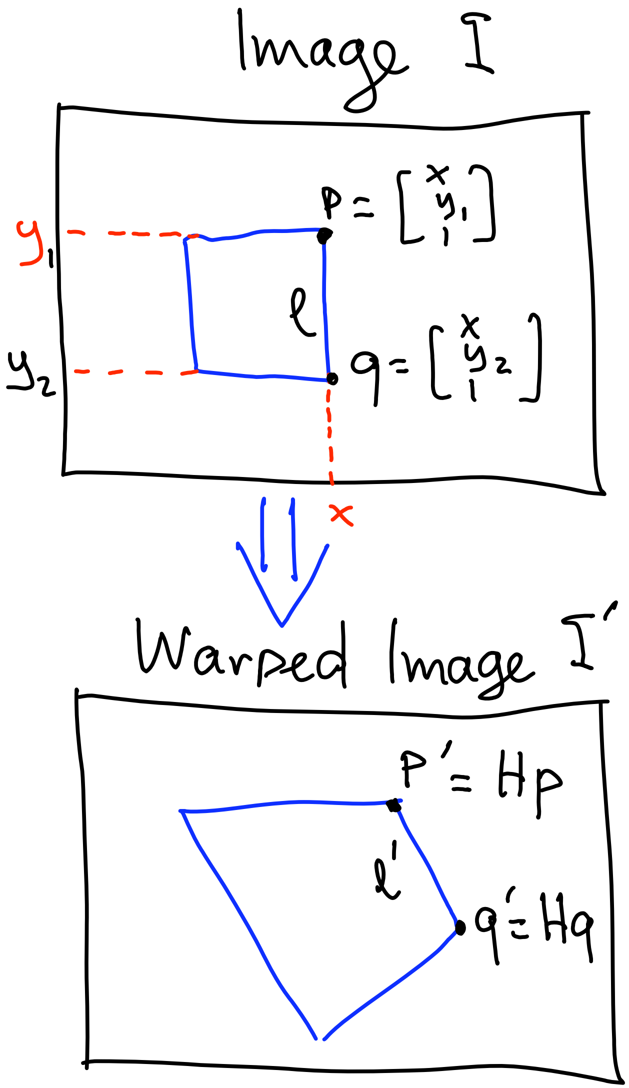
- **Definition (Linear Image Warps)**

  An image warp is called <u>linear</u> if every 2D line l in the original image is transformed into a line l' in the warped image (i.e. the warp preserves all lines in the original photo)

- **Property (w/out proof)**

  Every linear warp can be expressed as a 3x3 matrix H that transforms homogeneous image coordinates

Warping Images Using a Homography



The matrix H is called a Homography

- Linear warping equation

$$\underbrace{I(p)}_{\text{intensity at pixel in source image with homogeneous coordinates } p} = \underbrace{I'(Hp)}_{\text{intensity at pixel in warped image with homogeneous coordinates } p' = Hp}$$

- Property (w/out proof)

Every linear warp can be expressed as a 3x3 matrix H that transforms homogeneous image coordinates