

CS388: Natural Language Processing

Lecture 3: Multiclass Classification

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Some slides adapted from Vivek Srikumar, University of Utah



"Now! ... That should clear up
a few things around here!"



Administrivia

- ▶ P1: Code deliverable + **two** “explorations” discussed in writeup
- ▶ P1 due Thursday, January 26. After this Thursday, you’ll have seen all of the content for it



Recall: Binary Classification

Logistic regression: $P(y = 1|x) = \frac{\exp(\sum_{i=1}^n w_i x_i)}{(1 + \exp(\sum_{i=1}^n w_i x_i))}$ these sums are sparse!

Decision rule: $P(y = 1|x) \geq 0.5 \Leftrightarrow w^\top x \geq 0$

Gradient: differentiate the log likelihood: $x(y - P(y = 1|x))$

- ▶ This is the gradient of a single example. Can then apply stochastic gradient (or related optimization methods like Adagrad, etc.)
- ▶ ML pipeline: input -> feature representation, train model on labeled data (with stochastic gradient methods), then test on new data



This Lecture

- ▶ Multiclass fundamentals
- ▶ Feature extraction
- ▶ Multiclass logistic regression
- ▶ Start NNs (if time)

Multiclass Fundamentals



Text Classification

A Cancer Conundrum: Too Many Drug Trials, Too Few Patients

Breakthroughs in immunotherapy and a rush to develop profitable new treatments have brought a crush of clinical trials scrambling for patients.

By GINA KOLATA

Yankees and Mets Are on Opposite Tracks This Subway Series

As they meet for a four-game series, the Yankees are playing for a postseason spot, and the most the Mets can hope for is to play spoiler.

By FILIP BONDY



→ Health



→ Sports

~20 classes



Image Classification



→ Dog



→ Car

- ▶ Thousands of classes (ImageNet)



Entailment

- ▶ Three-class task over sentence pairs
- ▶ Not clear how to do this with simple bag-of-words features

A soccer game with multiple males playing.

ENTAILS

Some men are playing a sport.

A black race car starts up in front of a crowd of people.

CONTRADICTS

A man is driving down a lonely road

A smiling costumed woman is holding an umbrella.

NEUTRAL

A happy woman in a fairy costume holds an umbrella.



Entity Linking

Although he originally won the event, the United States Anti-Doping Agency announced in August 2012 that they had disqualified **Armstrong** from his seven consecutive Tour de France wins from 1999–2005.



Lance Edward Armstrong is an American former professional road cyclist



Armstrong County is a county in Pennsylvania...

?

?

- ▶ 4,500,000 classes (all articles in Wikipedia)



Reading Comprehension

One day, James thought he would go into town and see what kind of trouble he could get into. He went to the grocery store and pulled all the pudding off the shelves and ate two jars. Then he walked to the fast food restaurant and ordered 15 bags of fries. He didn't pay, and instead headed home.

3) Where did James go after he went to the grocery store?

- A) his deck
- B) his freezer
- ☒ C) a fast food restaurant
- D) his room

After about a month, and after getting into lots of trouble, James finally made up his mind to be a better turtle.

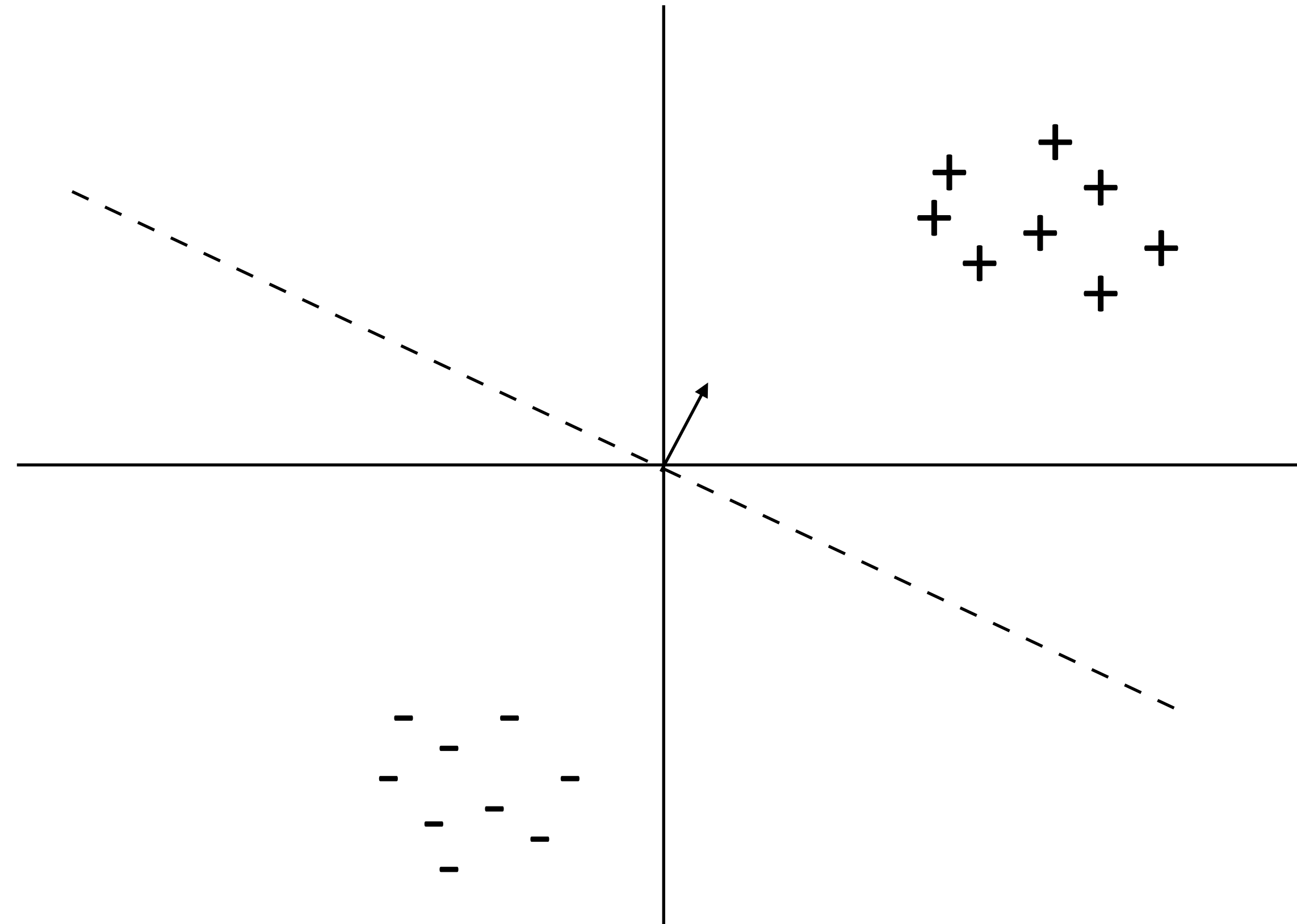


- ▶ Multiple choice questions, 4 classes
- ▶ Classes change per example



Binary Classification

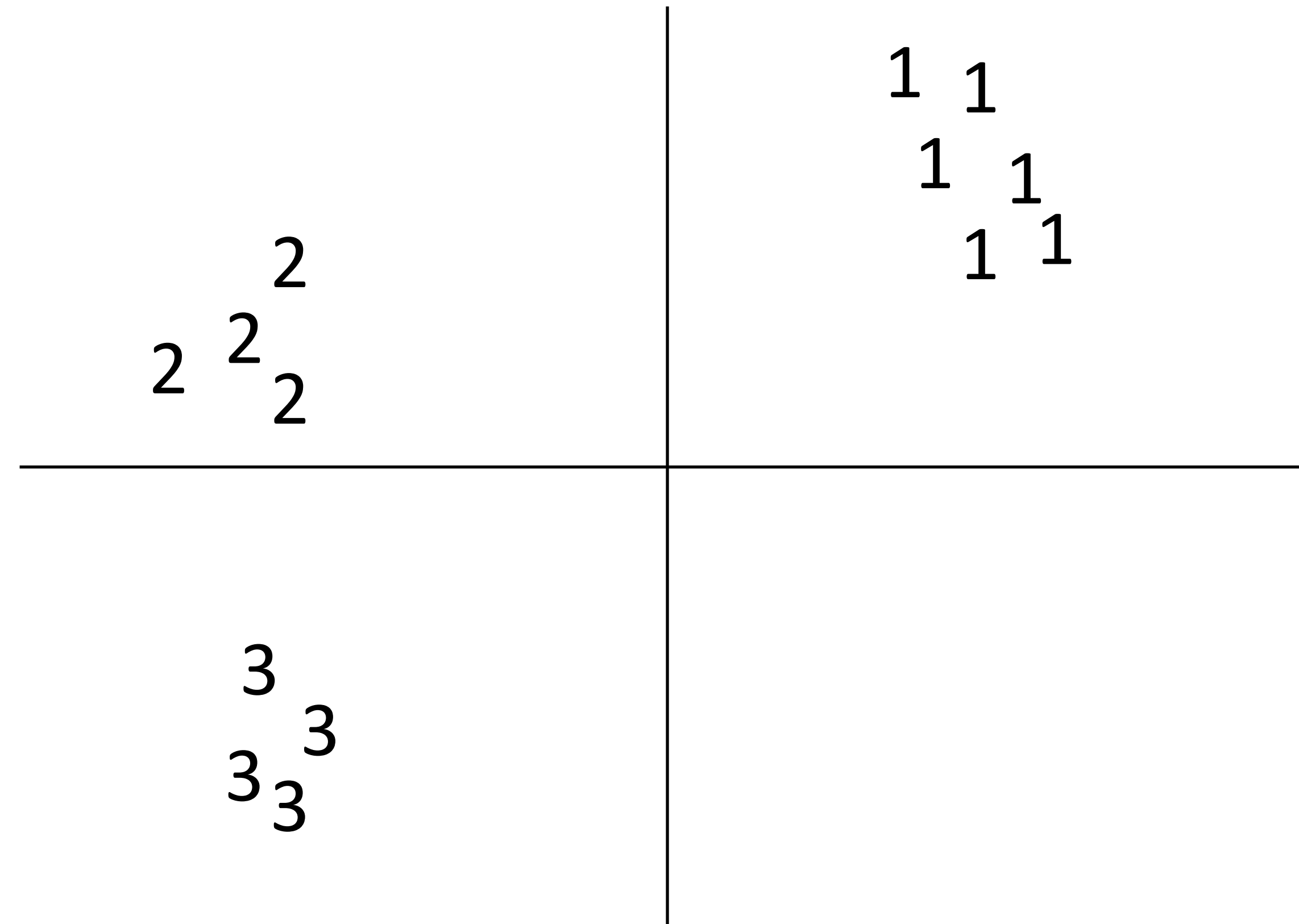
- ▶ Binary classification: one weight vector defines positive and negative classes





Multiclass Classification

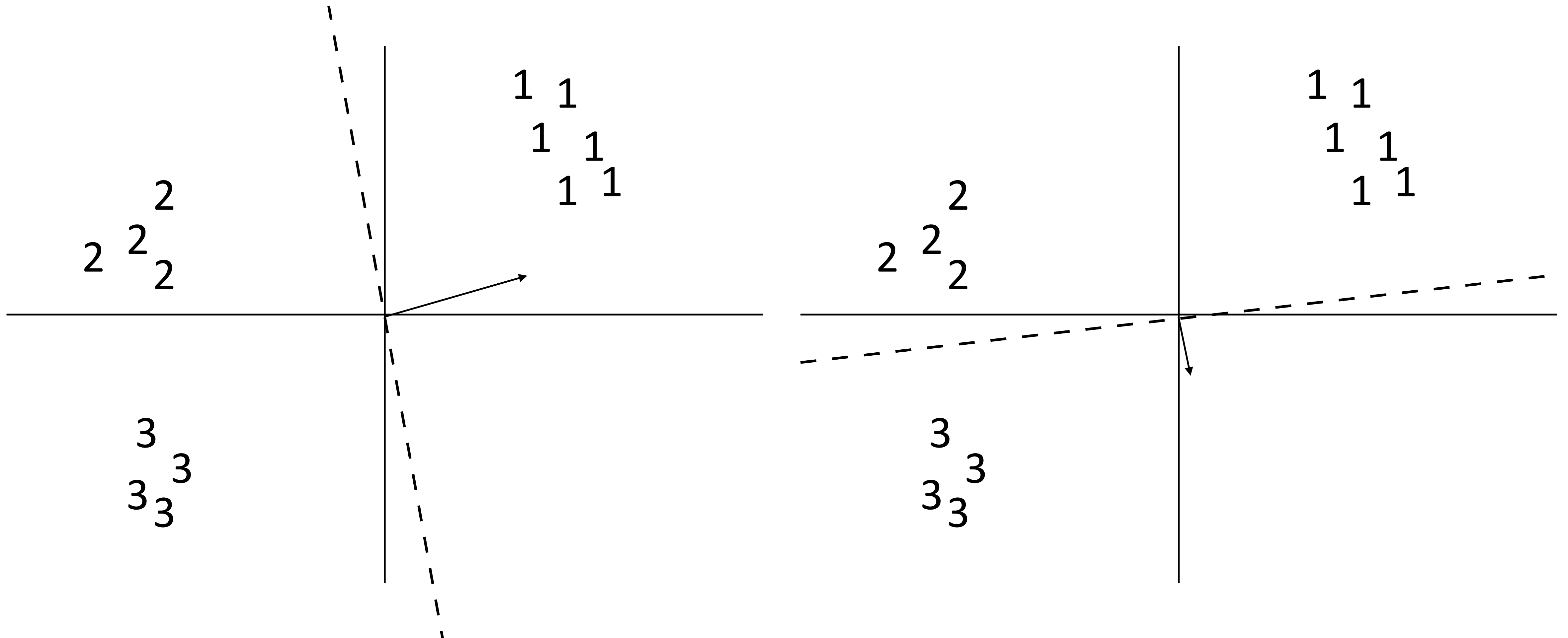
- Can we just use binary classifiers here?





Multiclass Classification

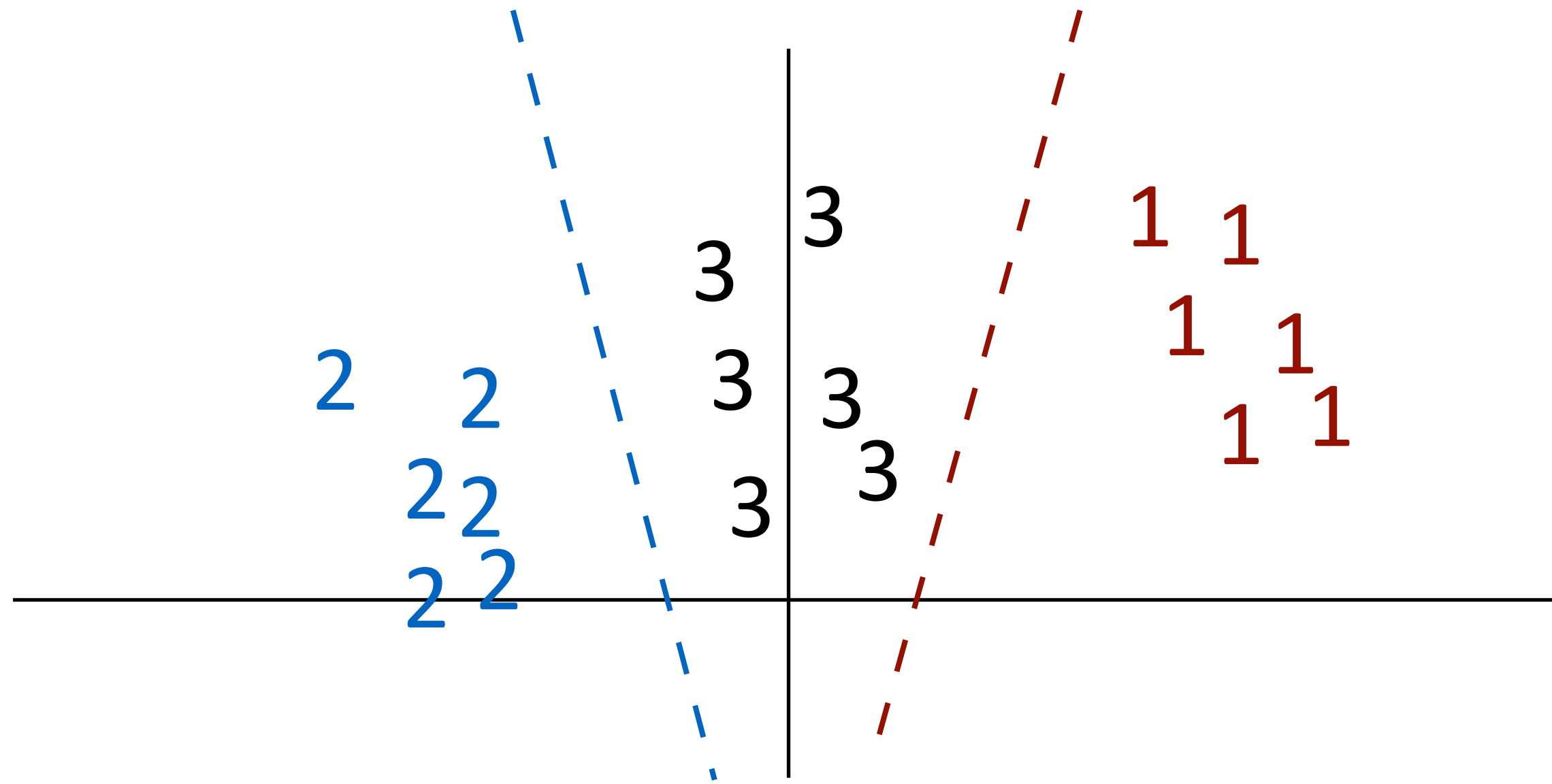
- ▶ One-vs-all: train k classifiers, one to distinguish each class from all the rest
- ▶ How do we reconcile multiple positive predictions? Highest score?





Multiclass Classification

- ▶ Not all classes may even be separable using this approach

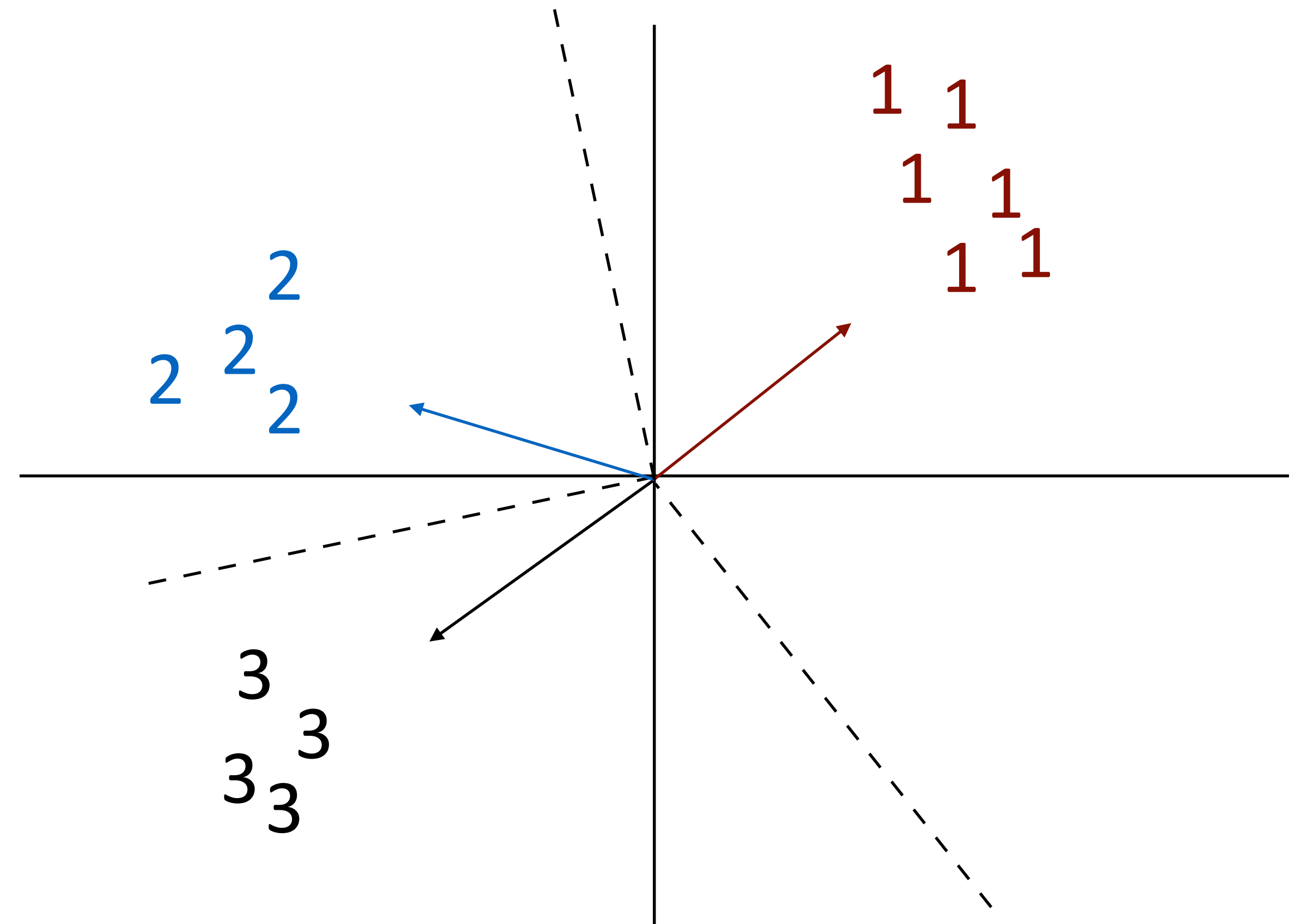
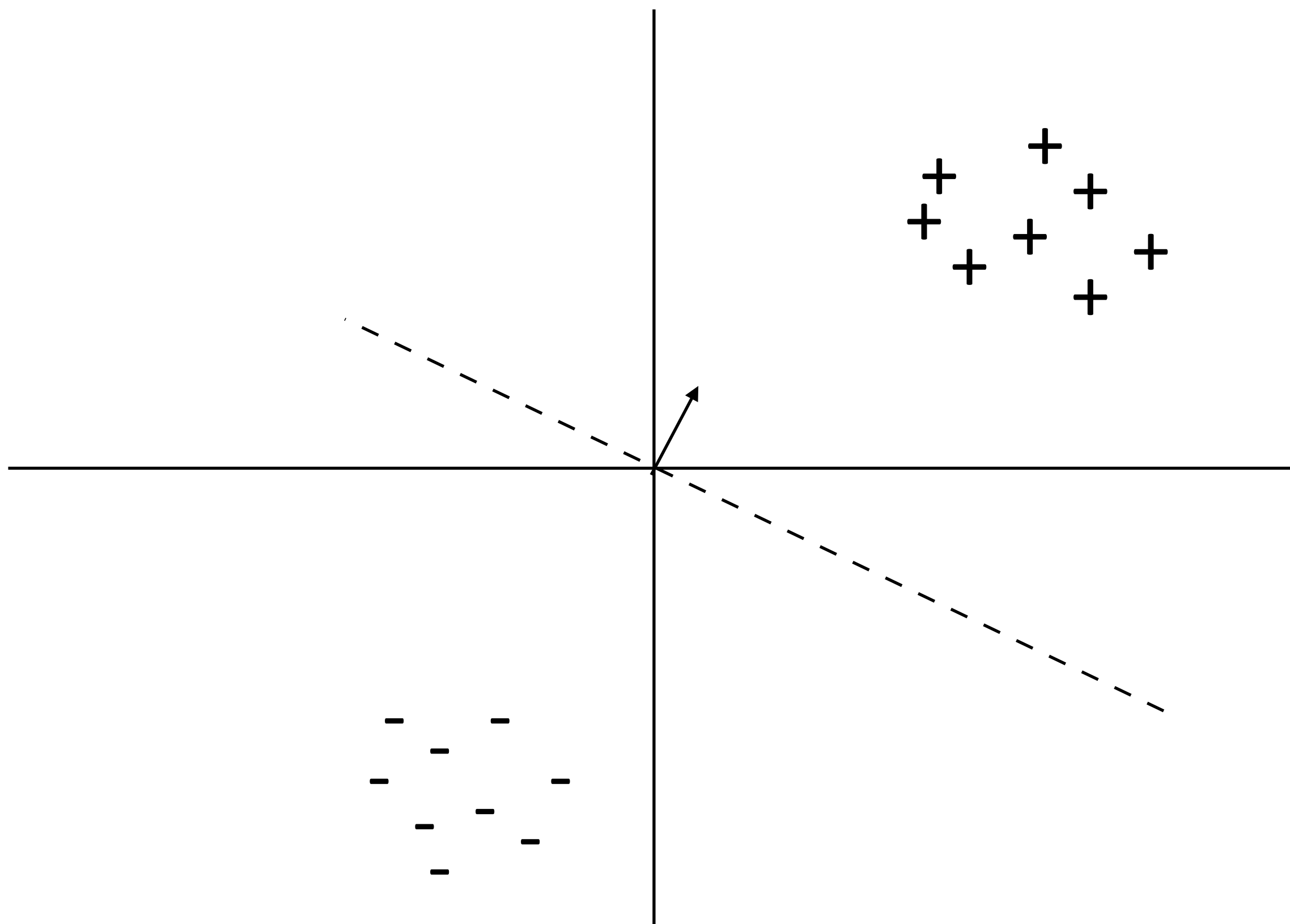


- ▶ Can separate 1 from 2+3 and 2 from 1+3 but not 3 from the others (with these features)



Multiclass Classification

- ▶ Binary classification: one weight vector defines both classes
- ▶ Multiclass classification: different weights and/or features per class





Multiclass Classification

- ▶ Formally: instead of two labels, we have an output space \mathcal{Y} containing a number of possible classes
 - ▶ Same machinery that we'll use later for exponentially large output spaces, including sequences and trees
- ▶ One weight vector per class: $\operatorname{argmax}_{y \in \mathcal{Y}} \mathbf{w}_y^\top \mathbf{f}(\mathbf{x})$
- ▶ Can also view it as a feature vector per class: $\operatorname{argmax}_{y \in \mathcal{Y}} \mathbf{w}^\top \mathbf{f}(\mathbf{x}, y)$
 - ▶ Multiple feature vectors, one weight vector

↑
features depend on choice
of label now! note: this
isn't the gold label



Different Weights vs. Different Features

- ▶ Different weights: $\operatorname{argmax}_{y \in \mathcal{Y}} \mathbf{w}_y^\top \mathbf{f}(\mathbf{x})$
 - ▶ Generalizes to neural networks: $f(x)$ is the first $n-1$ layers of the network, then you apply a final linear layer at the end
- ▶ Different features: $\operatorname{argmax}_{y \in \mathcal{Y}} \mathbf{w}^\top \mathbf{f}(\mathbf{x}, y)$
 - ▶ Suppose \mathcal{Y} is a structured label space (part-of-speech tags for each word in a sentence). $f(x, y)$ extracts features over shared parts of these
- ▶ For linear multiclass classification with discrete classes, these are identical

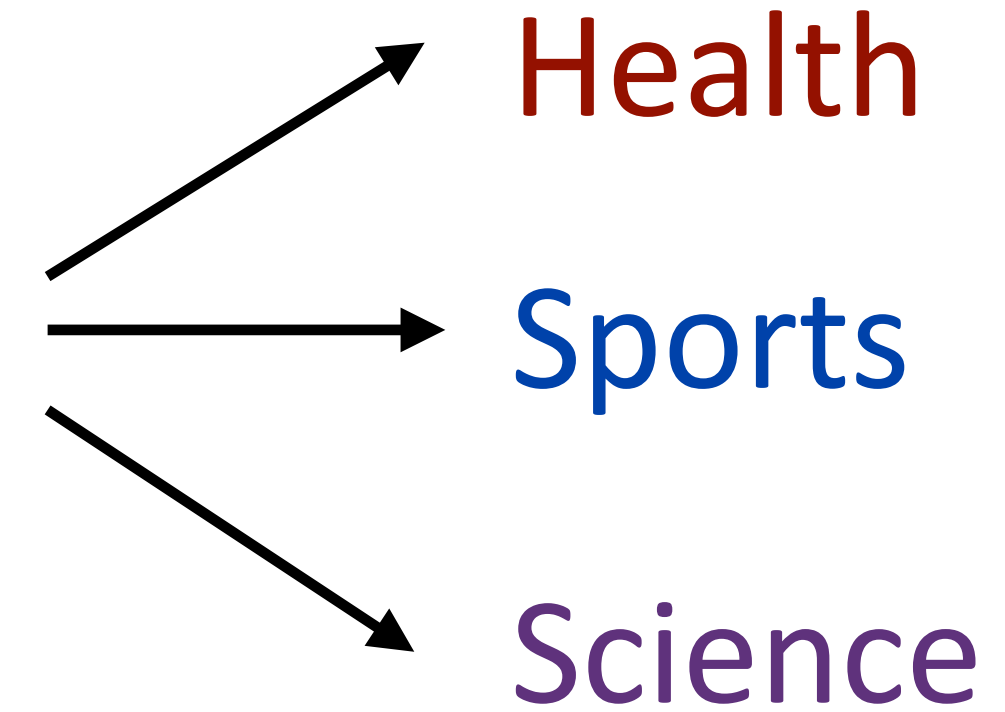
Feature Extraction: Multiclass, Token Tagging Tasks



Multiclass Bag-of-words

- Decision rule: $\operatorname{argmax}_{y \in \mathcal{Y}} \mathbf{w}_y^\top \mathbf{f}(\mathbf{x})$

too many drug trials, too few patients



- Feature extraction:

$$\mathbf{f}(\mathbf{x}) = \mathbb{I}[\text{contains } \textit{drug}], \mathbb{I}[\text{contains } \textit{patients}], \mathbb{I}[\text{contains } \textit{baseball}] = [1, 1, 0]$$

$$\mathbf{w}_{\text{health}} = [+2.1, +2.3, -5]$$

$$\mathbf{w}_{\text{sports}} = [-2.1, -3.8, +5.2]$$

$$\mathbf{w}_{\text{science}} = [+1.1, -1.7, -1.3]$$

$$\mathbf{w}_y^\top \mathbf{f}(\mathbf{x}) = \text{Health: } +4.4 \quad \text{Sports: } -5.9 \quad \text{Science: } -0.6$$

↖ argmax



Features for Tagging Tasks

DT NN VBZ DT NNS
the router blocks the packets

- ▶ Part-of-speech tagging (discussed later in the semester): make a classification decision about each word. Is “*blocks*” a verb or a noun? (~10-40 POS tags depending on the tagset, language, etc.)
- ▶ Input: sequence of words \mathbf{x} , output is a sequence of tags \mathbf{y}
- ▶ Simpler version: input is a sequence of words \mathbf{x} and one index i we care about, output is the tag y for that position

$P(y = VBZ \mid \mathbf{x} = \textit{the router blocks the packets}, i = 2)$

the router *blocks* *the ...*

NNS
VBZ
NN
DT
...



Features for Tagging Tasks

DT NN VBZ DT NNS
the router blocks the packets

- ▶ Do bag-of-words features work here?

[contains <i>the</i>]	[contains <i>router</i>]	[contains <i>is</i>]	[contains <i>packets</i>]	...
position 0	position 1	position 2	position 3	
$f(x) = [1$	1	0	1	...

- ▶ Everyone word in the sequence gets the same features — so everything gets the same label?
- ▶ Instead we need **position-sensitive features**. Let's see how this works with *different features*



Feature Extraction

DT NN VBZ DT NNS
the router blocks the packets
 $i = 0 \quad 1 \quad 2 \quad 3 \quad 4$

- ▶ Position-sensitive feature extractor: function from (sentence, position) => sparse feature vector describing that position in the sentence
 - ▶ “Current word”: what is the word at this index?
 - ▶ “Previous word”: what is the word that precedes the index?

[currWord=*router*] [currWord=*blocks*] [prevWord = *router*]

$f(x, i=2) = [\quad 0 \quad \quad \quad 1 \quad \quad \quad 1 \quad \dots]$

- ▶ Feature vector only has 2 nonzero entries out of 10k+ possible
- ▶ All features coexist in the same space! Other feats (char level, ...) possible



Different Features for Multiclass

- ▶ Classify *blocks* as one of 36 POS tags
- ▶ Example is a (sentence, index) pair ($x, i=2$): the word *blocks* in this sentence. Let's look at the **different features** view of extraction
- ▶ Different features: conjoin feats with pred label:

$$f(x, y=\text{VBZ}) = \text{I}[\text{curr_word}=\text{blocks} \ \& \ \text{tag} = \text{VBZ}], \\ \text{I}[\text{prev_word}=\text{router} \ \& \ \text{tag} = \text{VBZ}] \\ \text{I}[\text{next_word}=\text{the} \ \& \ \text{tag} = \text{VBZ}] \\ \text{I}[\text{curr_suffix}=\text{s} \ \& \ \text{tag} = \text{VBZ}]$$

- ▶ Get features for all tags, score, take the highest scoring one — but just one weight vector!

the router *blocks* *the packets*

NNS
VBZ
NN
DT
...

← not saying that *the* is tagged as VBZ! saying that *the* follows the VBZ word

Multiclass Logistic Regression



Multiclass Logistic Regression

$$P_{\mathbf{w}}(y = \hat{y} \mid \mathbf{x}) = \frac{\exp(\mathbf{w}_{\hat{y}}^{\top} \mathbf{f}(\mathbf{x}))}{\sum_{y'} \exp(\mathbf{w}_{y'}^{\top} \mathbf{f}(\mathbf{x}))}$$

sum over output
space to normalize

► exp/sum(exp): also called *softmax*

► Training: maximize $\mathcal{L}(D) = \sum_{i=1}^n \log P_{\mathbf{w}}(y^{(i)} \mid \mathbf{x}^{(i)})$

(we'll minimize the
negation of this objective)

$$= \sum_{i=1}^n \left(\mathbf{w}_{y^{(i)}}^{\top} \mathbf{f}(\mathbf{x}^{(i)}) - \log \sum_{y'} \exp(\mathbf{w}_{y'}^{\top} \mathbf{f}(\mathbf{x}^{(i)})) \right)$$

► Compare to binary:

$$P_{\mathbf{w}}(y = + \mid \mathbf{x}) = \frac{\exp(\mathbf{w}^{\top} \mathbf{f}(\mathbf{x}))}{1 + \exp(\mathbf{w}^{\top} \mathbf{f}(\mathbf{x}))}$$

negative class implicitly has
a weight vector of all zeroes



Training

- ▶ Multiclass logistic regression $P_{\mathbf{w}}(y = \hat{y} \mid \mathbf{x}) = \frac{\exp(\mathbf{w}_{\hat{y}}^\top \mathbf{f}(\mathbf{x}))}{\sum_{y'} \exp(\mathbf{w}_{y'}^\top \mathbf{f}(\mathbf{x}))}$
- ▶ Log loss:

$$\mathcal{L}(\mathbf{x}^{(i)}, y^{(i)}) = -\mathbf{w}_{y^{(i)}}^\top \mathbf{f}(\mathbf{x}^{(i)}) + \log \sum_{y'} \exp(\mathbf{w}_{y'}^\top \mathbf{f}(\mathbf{x}^{(i)}))$$

$$\frac{\partial}{\partial \mathbf{w}_{y^{(i)}}} \mathcal{L}(\mathbf{x}^{(i)}, y^{(i)}) = -\mathbf{f}(\mathbf{x}^{(i)}) + \frac{\mathbf{f}(\mathbf{x}^{(i)}) \exp(\mathbf{w}_{y^{(i)}}^\top \mathbf{f}(\mathbf{x}^{(i)}))}{\sum_{y'} \exp(\mathbf{w}_{y'}^\top \mathbf{f}(\mathbf{x}^{(i)}))}$$

$$\frac{\partial}{\partial \mathbf{w}_{y^{(i)}}} \mathcal{L}(\mathbf{x}^{(i)}, y^{(i)}) = -\mathbf{f}(\mathbf{x}^{(i)}) + \mathbf{f}(\mathbf{x}^{(i)}) P_{\mathbf{w}}(y^{(i)} \mid \mathbf{x}^{(i)})$$

- ▶ Update for other classes is the same but without the first term



Training

$$\frac{\partial}{\partial \mathbf{w}_{y^{(i)}}} \mathcal{L}(\mathbf{x}^{(i)}, y^{(i)}) = -\mathbf{f}(\mathbf{x}^{(i)}) + \mathbf{f}(\mathbf{x}^{(i)}) P_{\mathbf{w}}(y^{(i)} | \mathbf{x}^{(i)})$$

too many drug trials, too few patients

$y^* = \text{Health}$

$$\mathbf{f}(\mathbf{x}) = [1, 1, 0] \quad P_{\mathbf{w}}(y|x) = [\text{0.2}, \text{0.5}, \text{0.3}]$$

(made up values)

$$\text{gradient } \mathbf{w}_{\text{Health}} = -[1, 1, 0] + 0.2[1, 1, 0]$$

$$\text{gradient } \mathbf{w}_{\text{Sports}} = 0.5[1, 1, 0]$$

$$\text{gradient } \mathbf{w}_{\text{Science}} = 0.3[1, 1, 0]$$

When we make these updates: make Sports and Science look less like the example, make Health look more like it



Multiclass Logistic Regression: Summary

- ▶ Model: $P_{\mathbf{w}}(y = \hat{y} \mid \mathbf{x}) = \frac{\exp(\mathbf{w}_{\hat{y}}^\top \mathbf{f}(\mathbf{x}))}{\sum_{y'} \exp(\mathbf{w}_{y'}^\top \mathbf{f}(\mathbf{x}))}$
- ▶ Inference: $\operatorname{argmax}_{y \in \mathcal{Y}} \mathbf{w}_y^\top \mathbf{f}(\mathbf{x})$ (equivalent to finding most likely y)
- ▶ Learning: gradient descent on the log loss

$$\frac{\partial}{\partial \mathbf{w}_{y^{(i)}}} \mathcal{L}(\mathbf{x}^{(i)}, y^{(i)}) = \mathbf{f}(\mathbf{x}^{(i)}) (P_{\mathbf{w}}(y^{(i)} \mid \mathbf{x}^{(i)}) - 1)$$

$$\frac{\partial}{\partial \mathbf{w}_{\tilde{y}}} \mathcal{L}(\mathbf{x}^{(i)}, y^{(i)}) = \mathbf{f}(\mathbf{x}^{(i)}) P_{\mathbf{w}}(y^{(i)} \mid \mathbf{x}^{(i)})$$

“move towards $\mathbf{f}(\mathbf{x})$ in proportion to how wrong you were”

Generative vs. Discriminative Models



Learning in Probabilistic Models

- ▶ So far we have talked about discriminative classifiers (e.g., logistic regression which models $P(y|x)$)
- ▶ Cannot analytically compute optimal weights for such models, need to use gradient descent
- ▶ What about generative models? Let's briefly look at a generative classifier (naive Bayes) which will introduce useful concepts about maximum likelihood estimation



Naive Bayes

- ▶ Data point $x = (x_1, \dots, x_n)$, label $y \in \{0, 1\}$
- ▶ Formulate a probabilistic model that places a distribution $P(x, y)$
- ▶ Compute $P(y|x)$, predict $\operatorname{argmax}_y P(y|x)$ to classify

$$P(y|x) = \frac{P(y)P(x|y)}{P(x)}$$

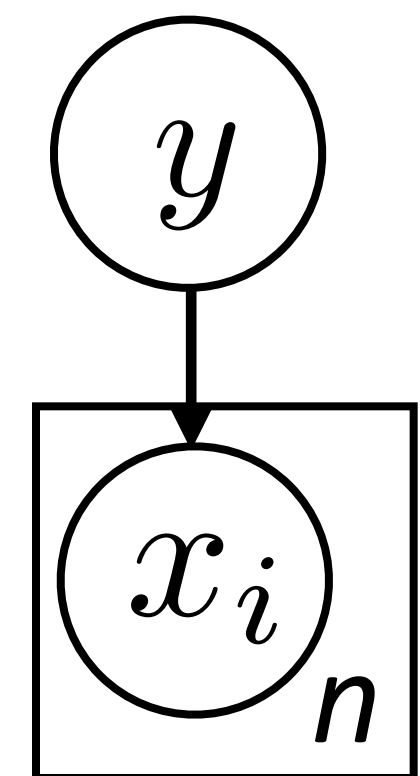
Bayes' Rule

$$\propto P(y)P(x|y)$$

constant: irrelevant
for finding the max

$$= P(y) \prod_{i=1}^n P(x_i|y)$$

“Naive” assumption:





Maximum Likelihood Estimation

- ▶ Data points (x_j, y_j) provided (j indexes over examples)
- ▶ Find values of $P(y)$, $P(x_i|y)$ that maximize data likelihood (generative):

$$\prod_{j=1}^m P(y_j, x_j) = \prod_{j=1}^m P(y_j) \left[\prod_{i=1}^n P(x_{ji}|y_j) \right]$$

data points (j) features (i) i th feature of j th example



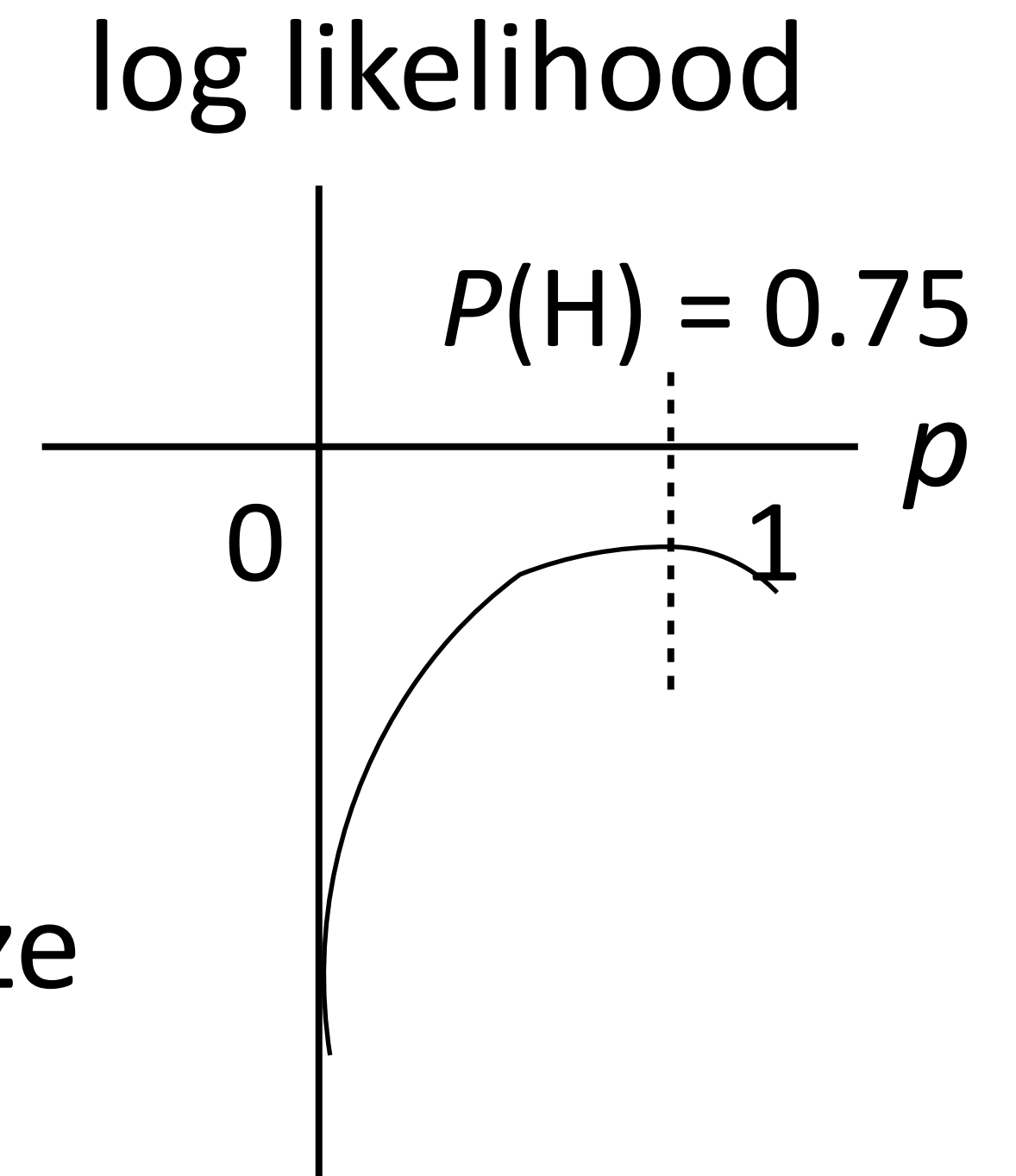
Maximum Likelihood Estimation

- ▶ Imagine a coin flip which is heads with probability p
- ▶ Observe (H, H, H, T) and maximize likelihood: $\prod_{j=1}^m P(y_j) = p^3(1 - p)$

- ▶ Easier: maximize *log* likelihood

$$\sum_{j=1}^m \log P(y_j) = 3 \log p + \log(1 - p)$$

- ▶ Maximum likelihood parameters for binomial/
multinomial = read counts off of the data + normalize





Maximum Likelihood Estimation

- ▶ Data points (x_j, y_j) provided (j indexes over examples)
- ▶ Find values of $P(y)$, $P(x_i|y)$ that maximize data likelihood (generative):

$$\prod_{j=1}^m P(y_j, x_j) = \prod_{j=1}^m P(y_j) \left[\prod_{i=1}^n P(x_{ji}|y_j) \right]$$

data points (j) features (i) i th feature of j th example

- ▶ Equivalent to maximizing log of data likelihood:

$$\sum_{j=1}^m \log P(y_j, x_j) = \sum_{j=1}^m \left[\log P(y_j) + \sum_{i=1}^n \log P(x_{ji}|y_j) \right]$$

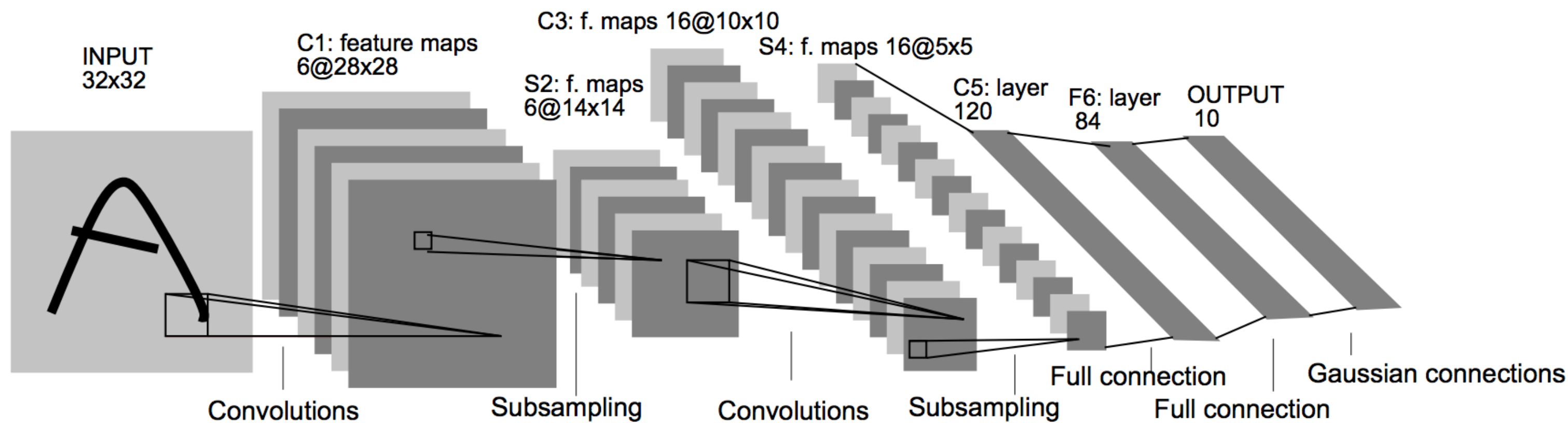
- ▶ Can do this by counting and normalizing distributions!

Neural Net History

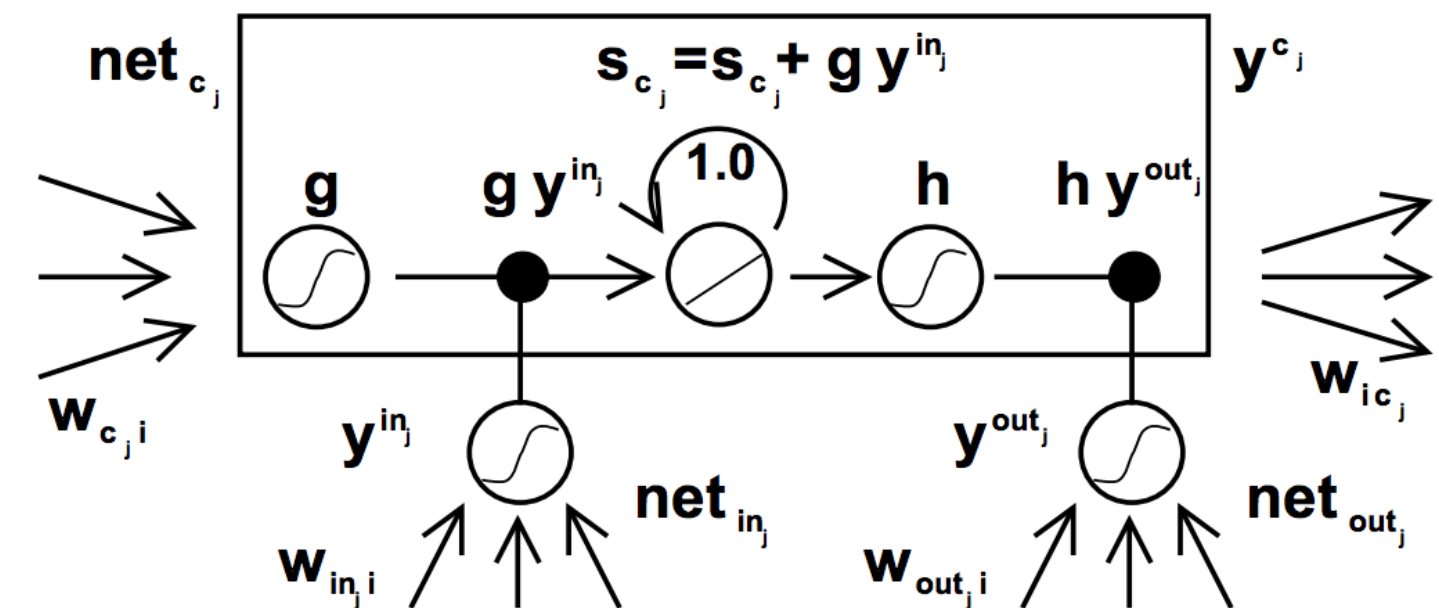


History: NN “dark ages”

- Convnets: applied to MNIST by LeCun in 1998



- LSTMs: Hochreiter and Schmidhuber (1997)

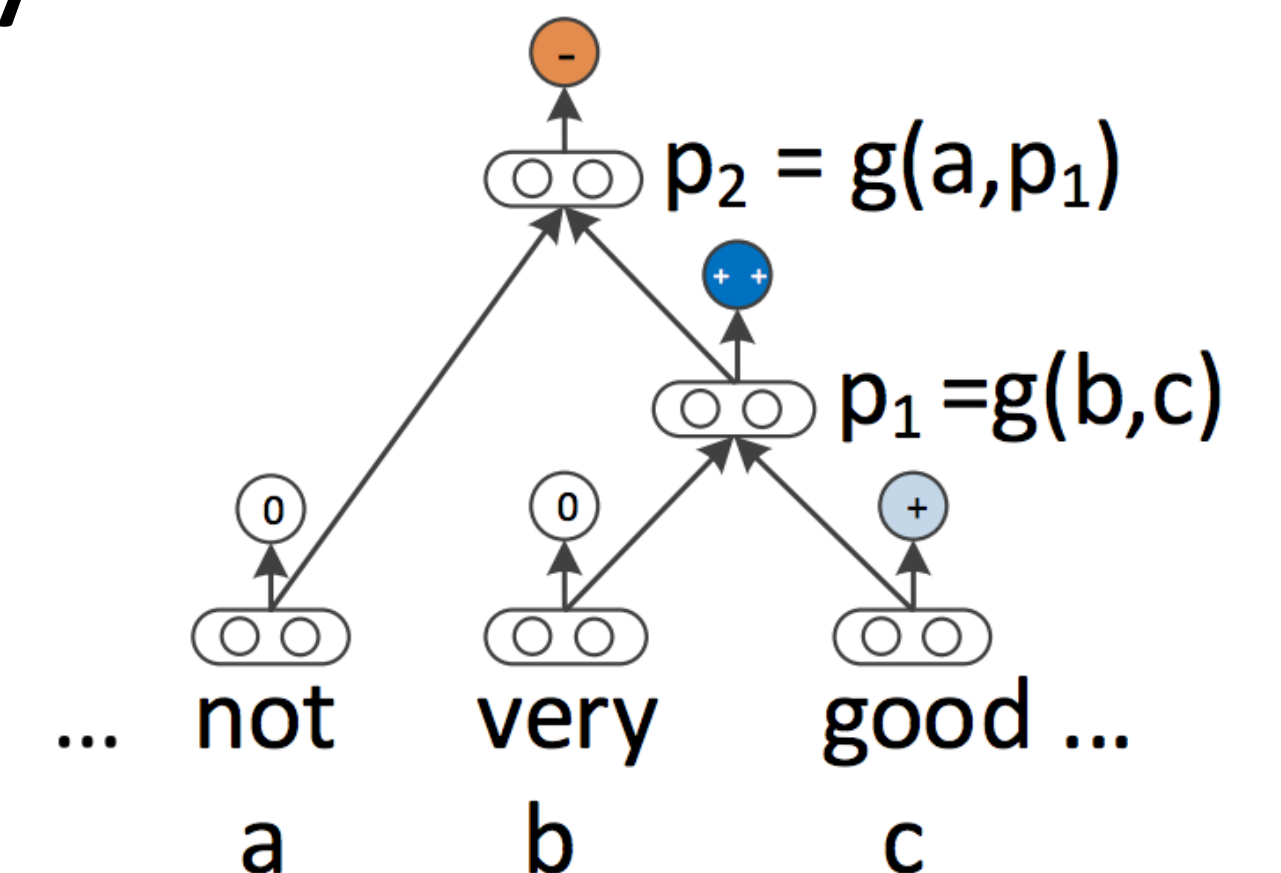
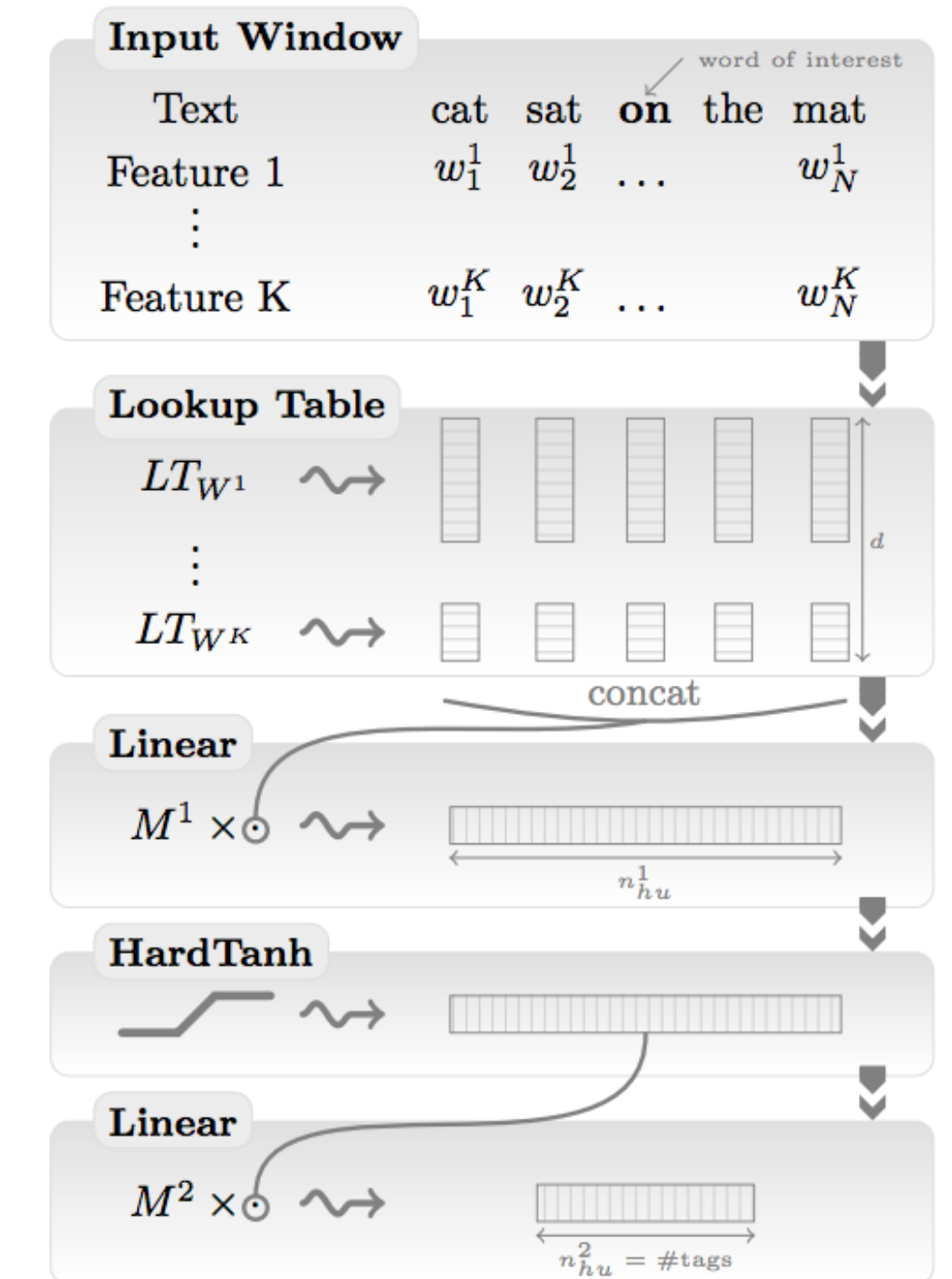


- Henderson (2003): neural shift-reduce parser, not SOTA



2008-2013: A glimmer of light...

- ▶ Collobert and Weston 2011: “NLP (almost) from scratch”
 - ▶ Feedforward neural nets induce features for sequential CRFs (“neural CRF”)
 - ▶ Basically tied SOTA in 2011, but with lots of computation (two weeks of training embeddings)
- ▶ Socher 2011-2014: tree-structured RNNs working okay
- ▶ Krizhevsky et al. (2012): AlexNet for vision





2014: Stuff starts working

- ▶ Kim (2014) + Kalchbrenner et al. (2014): sentence classification / sentiment (convnets)
- ▶ Sutskever et al. + Bahdanau et al.: seq2seq for neural MT (LSTMs)
- ▶ Chen and Manning transition-based dependency parser (based on feedforward networks)
- ▶ What made these work? **Data**, **optimization** (initialization, adaptive optimizers), **representation** (good word embeddings)



Takeaways

- ▶ Two views of multiclass logistic regression:
 - ▶ Different weights: one weight vector per class, fixed features
 - ▶ Different features: single weight vector for all classes, features differ for each class (but in a systematic way)
- ▶ Gradient looks like binary logistic regression gradient: softly move gold weight vector towards the example (also move all other weight vectors away from the example)
- ▶ Next time: neural networks
 - ▶ Extension of multiclass logistic regression with a nonlinearity