Recall: Feedforward NNs

\[ P(y|x) = \text{softmax}(Wg(Vf(x))) \]

- \(d\) hidden units
- \(n\) features
- \(d\times n\) matrix
- nonlinearity (tanh, relu, ...)
- \(num\_classes\times d\) matrix
- \(num\_classes\) probs

Recall: Backpropagation

\[ P(y|x) = \text{softmax}(Wg(Vf(x))) \]

\[
\begin{align*}
\frac{\partial L}{\partial W} & = \text{err}(z) \\
\frac{\partial z}{\partial V} & = f(x) \\
\text{err(root)} & = \text{err}(z)
\end{align*}
\]
Recall: Deep Averaging Networks

- Deep Averaging Networks: feedforward neural network on average of word embeddings from input

\[ h_2 = f(W_2 \cdot h_1 + b_2) \]
\[ h_1 = f(W_1 \cdot \text{av} + b_1) \]
\[ \text{av} = \sum_{i=1}^{4} \frac{s_i}{4} \]

Predator  is  a  masterpiece
\( c_1 \quad c_2 \quad c_3 \quad c_4 \)

This Lecture

- Word representations
- word2vec/GloVe
- Evaluating word embeddings

Word Representations

- Neural networks work very well at continuous data, but words are discrete
- Continuous model <-> expects continuous semantics from input
- “You shall know a word by the company it keeps” Firth (1957)

[Finch and Chater 92, Shuetze 93, many others]
**Discrete Word Representations**

- Brown clusters: hierarchical agglomerative *hard* clustering (each word has one cluster, not some posterior distribution like in mixture models)

- Maximize $P(w_i|w_{i-1}) = P(c_i|c_{i-1})P(w_i|c_i)$

- Useful features for tasks like NER, not suitable for NNs

  Brown et al. (1992)

---

**Word Embeddings**

- Want a vector space where similar words have similar embeddings

  the movie was great

  $\simeq$

  the movie was good

- Goal: come up with a way to produce these embeddings

- For each word, want “medium” dimensional vector (50-300 dims) representing it

---

**Continuous Bag-of-Words**

- Predict word from context

  the *dog* bit *the* man

  $d$-dimensional word embeddings

  $P(w|w_{-1}, w_{+1}) = \text{softmax}(W(c_{w_{-1}} + c_{w_{+1}}))$

- Parameters: $d \times |V|$ (one $d$-length context vector per voc word), $|V| \times d$ output parameters ($W$)

  Mikolov et al. (2013)
**Skip-Gram**

- Predict one word of context from word
  
  \[ \text{the dog bit the man} \]

- The gold word is `dog`
  
  \[ P(w' | w) = \text{softmax}(W e(w)) \]

- Another training example: `bit` -> `the`
  
  Parameters: \( d \times |V| \) vectors, \(|V| \times d\) output parameters (W) (also usable as vectors!)

---

**Using Skip-Gram**

- Context window size: how many words around the “center” word do we look?
  
  \[ \text{the dog bit the man} \]

  \( k=1: \text{two words of context} \)
  \( k=2: \text{four words of context} \)

- Advantages/disadvantages of different sizes of \( k \)?

- Training: maximize log likelihood of the examples derived given \( k \), summed over a corpus (but we'll never use the model as is, only its embeddings)

- Initialization: need to randomly initialize in a reasonable way

- Vector size: controls capacity of model

---

**Hierarchical Softmax**

\[ P(w|w_{-1}, w_{+1}) = \text{softmax}(W(c(w_{-1}) + c(w_{+1}))) \]

\[ P(w' | w) = \text{softmax}(W e(w)) \]

- Matmul + softmax over \(|V|\) is very slow to compute for CBOW and SG

- Huffman encode vocabulary, use binary classifiers to decide which branch to take

- \( \log(|V|) \) binary decisions

- \(|V| \) dot products of size \( d \)

- \(|V| \times d\) parameters

---

**Skip-Gram with Negative Sampling**

- Take (word, context) pairs and classify them as “real” or not. Create random negative examples by sampling from unigram distribution

  \( (bit, the) \Rightarrow +1 \)
  \( (bit, cat) \Rightarrow -1 \)
  \( (bit, a) \Rightarrow -1 \)
  \( (bit, fish) \Rightarrow -1 \)

- Objective: \( \log P(y = 1 | w, c) + \frac{1}{k} \sum_{i=1}^{n} \log P(y = 0 | w_i, c) \)

- Words in similar contexts select for similar \( c \) vectors

- \( d \times |V| \) vectors, \( d \times |V| \) context vectors (same # of params as before)

---

*Mikolov et al. (2013)*
Connections with Matrix Factorization

- Skip-gram model looks at word-word co-occurrences and produces two types of vectors.
- Looks almost like a matrix factorization...

Skip-Gram as Matrix Factorization

- Skip-gram model looks at word-word co-occurrences and produces two types of vectors.
- Looks almost like a matrix factorization..

Skip-gram objective exactly corresponds to factoring this matrix:

- If we sample negative examples from the unigram distribution over words
- ...and it's a weighted factorization problem (weighted by word freq)

GloVe (Global Vectors)

- Also operates on counts matrix, weighted regression on the log co-occurrence matrix
- Objective = \( \sum_{i,j} f(\text{count}(w_i, c_j)) \left( w_i^T c_j + a_i + b_j - \log \text{count}(w_i, c_j) \right)^2 \)
- Constant in the dataset size (just need counts), quadratic in voc size
- By far the most common word vectors used today (30000+ citations)
GloVe (Global Vectors): Example

- Objective = \( \sum_{i,j} f(\text{count}(w_i, c_j)) \left( w_i^T c_j + a_i + b_j - \log \text{count}(w_i, c_j) \right)^2 \)

<table>
<thead>
<tr>
<th>the</th>
<th>dog</th>
<th>cat</th>
<th>ran</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>200</td>
<td>200</td>
<td>0</td>
</tr>
<tr>
<td>200</td>
<td>0</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>200</td>
<td>0</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>0</td>
<td>15</td>
<td>15</td>
<td>0</td>
</tr>
</tbody>
</table>

Linear regression with 16 points: each element is plugged into the above equation

\[ + \text{constant} = \log \text{count of pair} \]

(made up values — matrix will generally be symmetric, though)

Pennington et al. (2014)

Analogies

\((\text{king} - \text{man}) + \text{woman} = \text{queen}\)

\(\text{king} + (\text{woman} - \text{man}) = \text{queen}\)

- Why would this be?
- woman - man captures the difference in the contexts that these occur in
- Dominant change: more “he” with man and “she” with woman — similar to difference between king and queen
- Can evaluate on this as well

GloVe Motivation

Table 1: Co-occurrence probabilities for target words ice and steam with selected context words from a 6 billion token corpus. Only in the ratio does noise from non-discriminative words like water and fashion cancel out, so that large values (much greater than 1) correlate well with properties specific to ice, and small values (much less than 1) correlate well with properties specific of steam.

<table>
<thead>
<tr>
<th>Probability and Ratio</th>
<th>(k = \text{solid})</th>
<th>(k = \text{gas})</th>
<th>(k = \text{water})</th>
<th>(k = \text{fashion})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P(k</td>
<td>\text{ice}))</td>
<td>(1.9 \times 10^{-4})</td>
<td>(6.6 \times 10^{-5})</td>
<td>(3.0 \times 10^{-3})</td>
</tr>
<tr>
<td>(P(k</td>
<td>\text{steam}))</td>
<td>(2.2 \times 10^{-5})</td>
<td>(7.8 \times 10^{-4})</td>
<td>(2.2 \times 10^{-3})</td>
</tr>
<tr>
<td>(P(k</td>
<td>\text{ice})/P(k</td>
<td>\text{steam}))</td>
<td>8.9</td>
<td>8.5 \times 10^{-2}</td>
</tr>
</tbody>
</table>

- GloVe objective is derived to preserve regularities in cooccurrence of words with other words

\[ F \left( (w_i - w_j)^T \tilde{w}_k \right) = \frac{P_{ik}}{P_{jk}} \]

Pennington et al. (2014)

Other Methods
fastText: Sub-word Embeddings

- Same as SGNS, but break words down into n-grams with $n = 3$ to $6$
  where:
  3-grams: <wh, whe, her, ere, re>
  4-grams: <whe, wher, here, ere>,
  5-grams: <wher, where, here>,
  6-grams: <where, where>
- Replace $\mathbf{W} \cdot \mathbf{c}$ in skip-gram computation with $\left( \sum_{g \in \text{ngrams}} w_g \cdot c \right)$
- Advantages?

Bojanowski et al. (2017)

Preview: Context-dependent Embeddings

- How to handle different word senses? One vector for bat

- Train a neural language model to predict the next word given previous words in the sentence, use its internal representations as word vectors
- Context-sensitive word embeddings: depend on rest of the sentence
- Huge improvements across nearly all NLP tasks over GloVe

Peters et al. (2018)

Compositional Semantics

- What if we want embedding representations for whole sentences?
- Skip-thought vectors (Kiros et al., 2015), similar to skip-gram generalized to a sentence level (more later)
- Is there a way we can compose vectors to make sentence representations? Summing?
- Will return to this in a few weeks as we move on to syntax and semantics

Using Word Embeddings

- Approach 1: learn embeddings as parameters from your data
  - Often works pretty well
- Approach 2: initialize using GloVe, keep fixed
  - Faster because no need to update these parameters
- Approach 3: initialize using GloVe, fine-tune
  - Works best for some tasks
Evaluating Word Embeddings

- What properties of language should word embeddings capture?
- Similarity: similar words are close to each other
- Analogy:
  - good is to best as smart is to ???
  - Paris is to France as Tokyo is to ???

**Similarity**

<table>
<thead>
<tr>
<th>Method</th>
<th>WordSim Similarity</th>
<th>WordSim Relatedness</th>
<th>Bruni et al. M. MEN</th>
<th>Radinsky et al. M. Turk</th>
<th>Luong et al. Rare Words</th>
<th>Hill et al. SimLex</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPMI</td>
<td>.755</td>
<td>.697</td>
<td>.745</td>
<td>.686</td>
<td>.462</td>
<td>.393</td>
</tr>
<tr>
<td>SVD</td>
<td><strong>.793</strong></td>
<td>.691</td>
<td><strong>.778</strong></td>
<td>.666</td>
<td><strong>.514</strong></td>
<td>.432</td>
</tr>
<tr>
<td>SGNS</td>
<td>.793</td>
<td>.685</td>
<td>.774</td>
<td><strong>.693</strong></td>
<td>.470</td>
<td><strong>.438</strong></td>
</tr>
<tr>
<td>GloVe</td>
<td>.725</td>
<td>.604</td>
<td>.729</td>
<td>.632</td>
<td>.403</td>
<td>.398</td>
</tr>
</tbody>
</table>

- SVD = singular value decomposition on PMI matrix
- GloVe does not appear to be the best when experiments are carefully controlled, but it depends on hyperparameters + these distinctions don’t matter in practice

Levy et al. (2015)

**Stability**

- To what extent are the relationships captured by word embeddings consistent?
- Stability: percent overlap between nearest neighbors in embedding space if you retrain embeddings from different initialization

Burdick et al. (2018)
**Stability: GloVe**

- Left y-axis: bucketed corpus frequency
- Right y-axis: number of neighbors
- x-axis: percent of neighbors stable across samples
- Being all the way to the right is better (most neighbors are stable)

Burdick et al. (2018)

**GloVe vs. word2vec: w2v much less stable!**

Burdick et al. (2018)

---

**What can go wrong with word embeddings?**

- What’s wrong with learning a word’s “meaning” from its usage?

> Ukraine’s deputy defense minister resigns amid corruption allegations

> From 2015 through 2020, there were at least 2,070 unintentional shootings by children under 18 in the US, according to a report from Everytown. Those shootings resulted in 765 deaths and 1,366 injuries.

> Convicted child sex trafficker Ghislaine Maxwell has said a decades-old photograph of Prince Andrew with his sexual abuse accuser Virginia Giuffre is “fake,” in a series of interviews from prison.

---

**What do we mean by bias?**

- Identify she - he axis in word vector space, project words onto this axis

Bolukbasi et al. (2016)

- Nearest neighbor of (b - a + c)

Manzini et al. (2019)
Debiasing

- Identify gender subspace with gendered words
- Project words onto this subspace
- Subtract those projections from the original word

Bolukbasi et al. (2016)

Hardness of Debiasing

- Not that effective...and the male and female words are still clustered together
- Bias pervades the word embedding space and isn’t just a local property of a few words

Gonen and Goldberg (2019)

Takeaways

- Word vectors: learning word -> context mappings has given way to matrix factorization approaches (constant in dataset size)
- Lots of pretrained embeddings work well in practice, they capture some desirable properties
- Even better: context-sensitive word embeddings (ELMo)
- Next time: language modeling and Transformers