
Mohamed G. Gouda — CS 337

Fall 2007 — Midterm 2

Open Book and Notes — 70 Minutes

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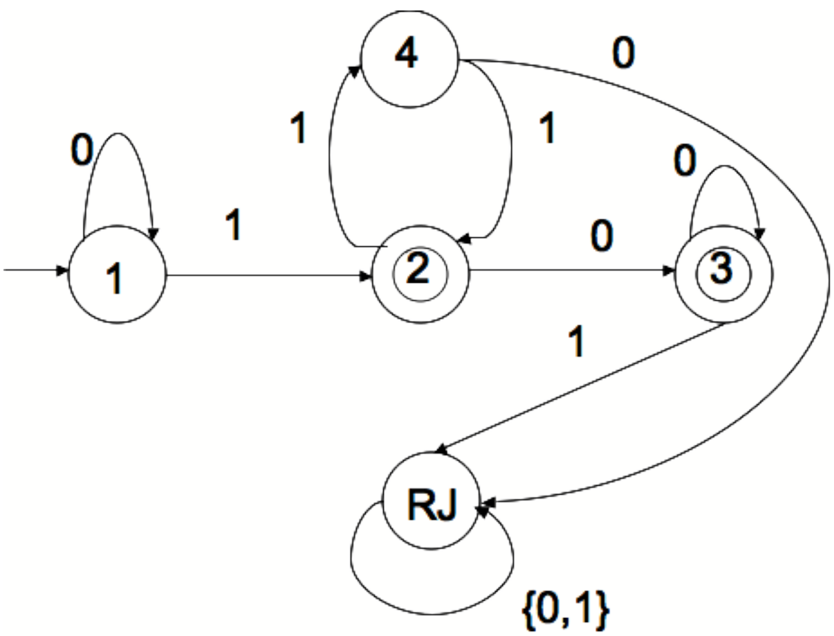
1. Write a regular expression over the alphabet {0, 1} the describes a set S of all strings where each string has an odd number of 1's and the 1's in the string occur consecutively. Design a finite state machine that accepts set S.

Solution:
The regular expression is 0* 1 (11)* 0*
The Finite State Machine that accepts set S is



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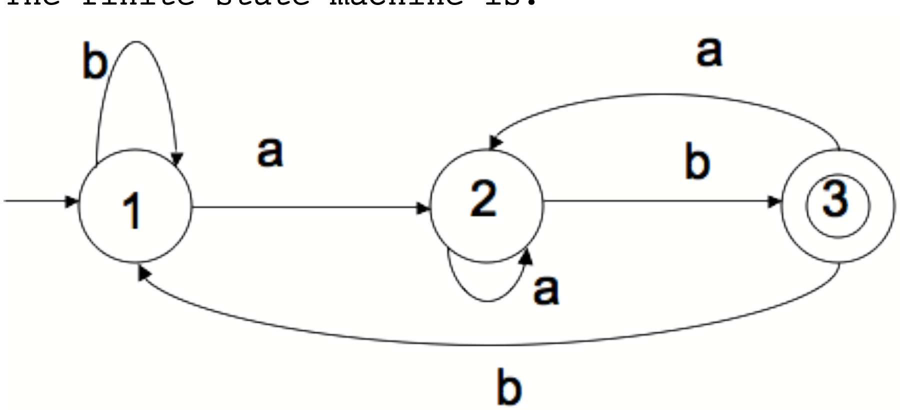
2. Design a finite state machine over the alphabet {a, b} that accepts each string that ends with ab.

Solution:
The finite state machine is:



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3. For each state i in the finite state machine in Problem 2, define a predicate Q.i that defines every string x of symbols "a" and "b" that can move the machine from its initial state to state i.

Solution:
Q.1 = (x is the empty string) V (x is the string b) V (x ends with bb)

Q.2 = (x ends with a)

Q.3 = (x ends with ab)

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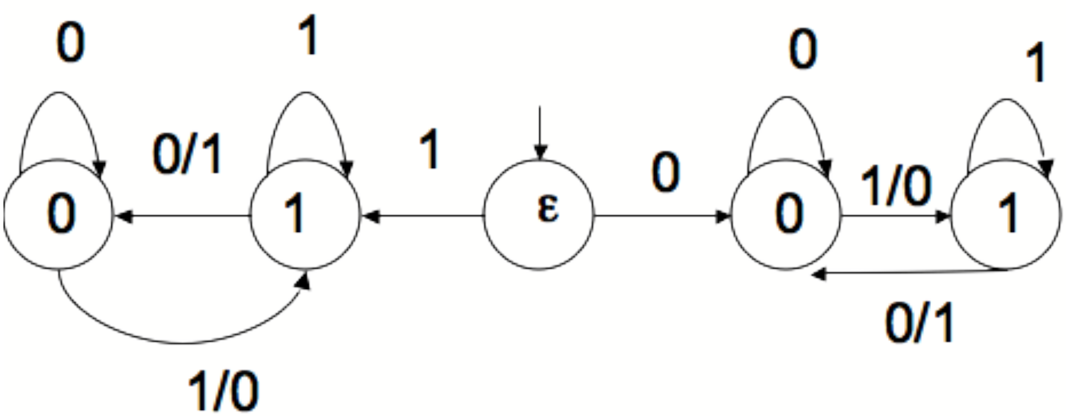
4. Design a transducer T whose input and output alphabets are {0, 1}. If the input string consists of only one symbol, T does not output any string. If the input string <S.1, S.2, .., S.k> has at least 2 symbols, then after receiving the i-th symbol S.i from this string, where i is at least 2, T outputs the symbol S.(i-1) provided S.i is different from S.(i-1). For example, if the input string is <0, 0, 1, 1, 1, 0, 1>, then T outputs the string <0, 1, 0>.

Solution:
The transducer T is



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5. Solve the following two regular expression equations with variables P and Q:

P = (10)* | Q
Q = (10)P

Simplify the regular expressions for P and Q as much as possible.

Solution:
Substitute Q in P = (10)* | Q.

P = (10)* | (10) P
The solution for P is (ε | (10)(10)*) (10)* = (10)* (10)* = (10)*
So, Q = (10) (10)*

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6. Define a Haskell function sumt that takes as parameters integers a, b, and n, and computes, without using the infix operator ^, the sum of n terms T.1, T.2, .., T.n, where the i-th term is a*(b^(i-1)). (Hint: function sumt should call another function term that computes the i-th term T.i in the sequence of terms.)

Solution:

```
term a b 1 = a
term a b i = (term a b (i-1)) * b
sumt a b 1 = term a b 1
sumt a b n = (sumt a b (n-1)) + term a b n
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7. Define a Haskell function min2 that takes as a parameter any list xs of two or more integers and computes a 2-tuple (x, y), where x is the smallest integer in list xs and y is the second smallest integer in xs.

Solution:

```
min2 [x,y] = (min x y, max x y)
min2 (x:xs)
     | x < m = (x,m)
     | x < n = (m,x)
     | True  = (m,n)
        where (m,n) = min2 xs
```
