

CS395T: Numerical Optimization for Graphics and AI: Homework IV

1 Guideline

- Please complete **1** problems out of **2** problems.
- You are welcome to complete more problems.

2 Programming

Problem 1. The goal of this optional assignment is to implement an ICP algorithm to align depth scans: Following what we have discussed in class, you are required to implement a basic ICP algorithm, which

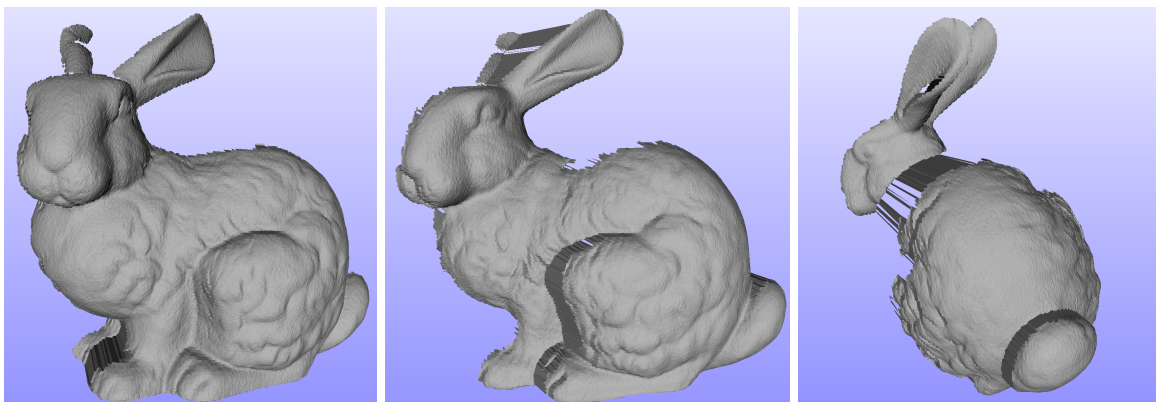


Figure 1: Snapshots of the depth scans of the Stanford Bunny Model.

alternates between finding closest point correspondences and fitting a rigid transformation to these correspondences. Since the scans are partially overlapping, you may find using a robust-norm beneficial, which can be solved using reweighted least squares. Please align the first two scans bun000 and bun045. The dataset can be assessed from <http://graphics.stanford.edu/data/3Dscanrep/>. You may sample points randomly and use 'knnsearch' to find closest points.

Bonus point: Implement a multi-way matching algorithm to align multiple scans together.

Please email (1) your MATLAB code in a zip format with file name and (2) a PDF report file with name to the TA.

3 Theory

Problem 2. Show that ICP converges linearly. Specifically, suppose you have a point cloud $P = \{p_i, 1 \leq i \leq n\}$ and a target surface Q . ICP alternates between two steps. At iteration k , the first step computes the

closest point $\mathbf{q}_i^{(k)}$ of each point $\mathbf{p}_i$ in Q :

$$\mathbf{q}_i^{(k)} = \operatorname{argmin}_{\mathbf{q} \in Q} \|\mathbf{p}_i - \mathbf{q}\|^2, \quad 1 \leq i \leq n.$$

The second step fits a rigid transformation between P and $Q^{(k)} := \{\mathbf{q}_i^{(k)}, 1 \leq i \leq n\}$:

$$(R^{(k)}, \mathbf{t}^{(k)}) := \operatorname{argmin}_{\mathbf{q} \in Q} \sum_{i=1}^n \|R\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i^{(k)}\|^2.$$

- Provides condition on P and Q , so that $T^{(k)}$ converge to a stationary point T^* .
- Provides conditions on P and Q , so that $T^{(k)}$ converge to T^* linearly, i.e.,

$$\|T^{(k+1)} - T^*\| \leq c \|T^{(k)} - T^*\|, \tag{1}$$

for some constant $c < 1$.