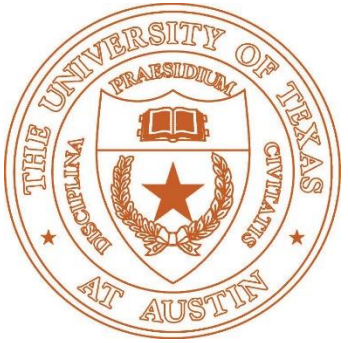
Triangular Mesh/Mesh Simplification

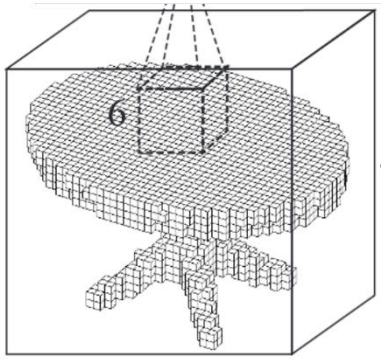
Qixing Huang

Feb. 23th 2017

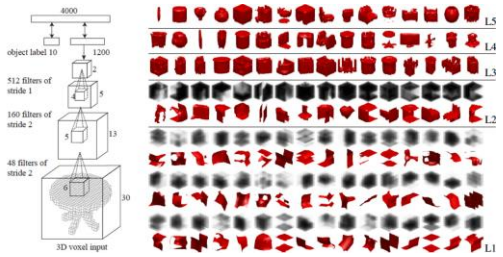


Review: Course Structure

3D Reconstruction



3D Representations

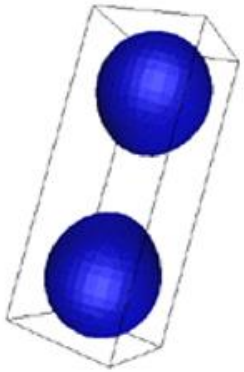


[Wu et al. 15]

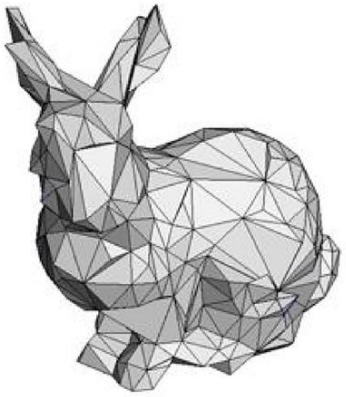
3D Understanding

Review: 3D Representations

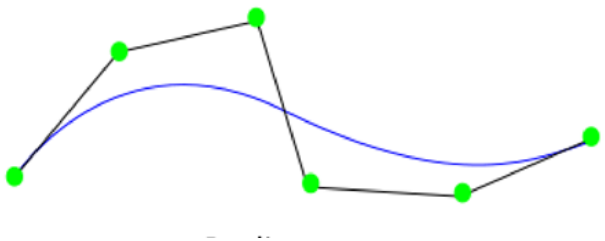
Point Cloud



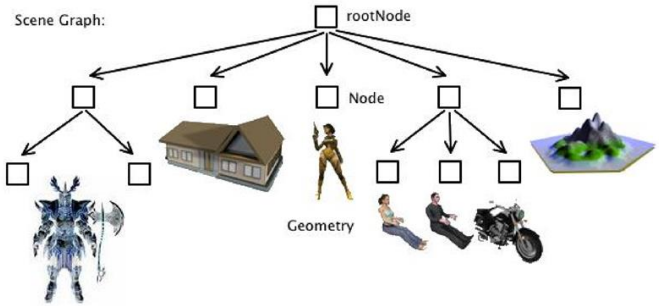
Implicit



Triangular Mesh

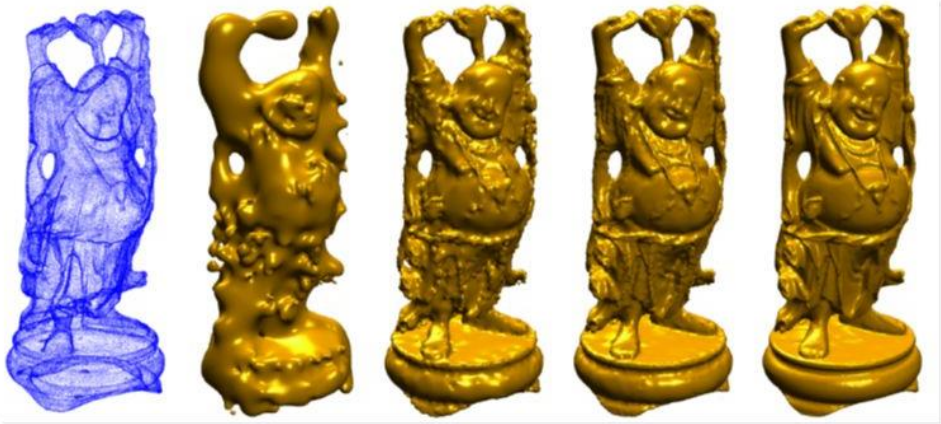


Parametric



Scene Graph

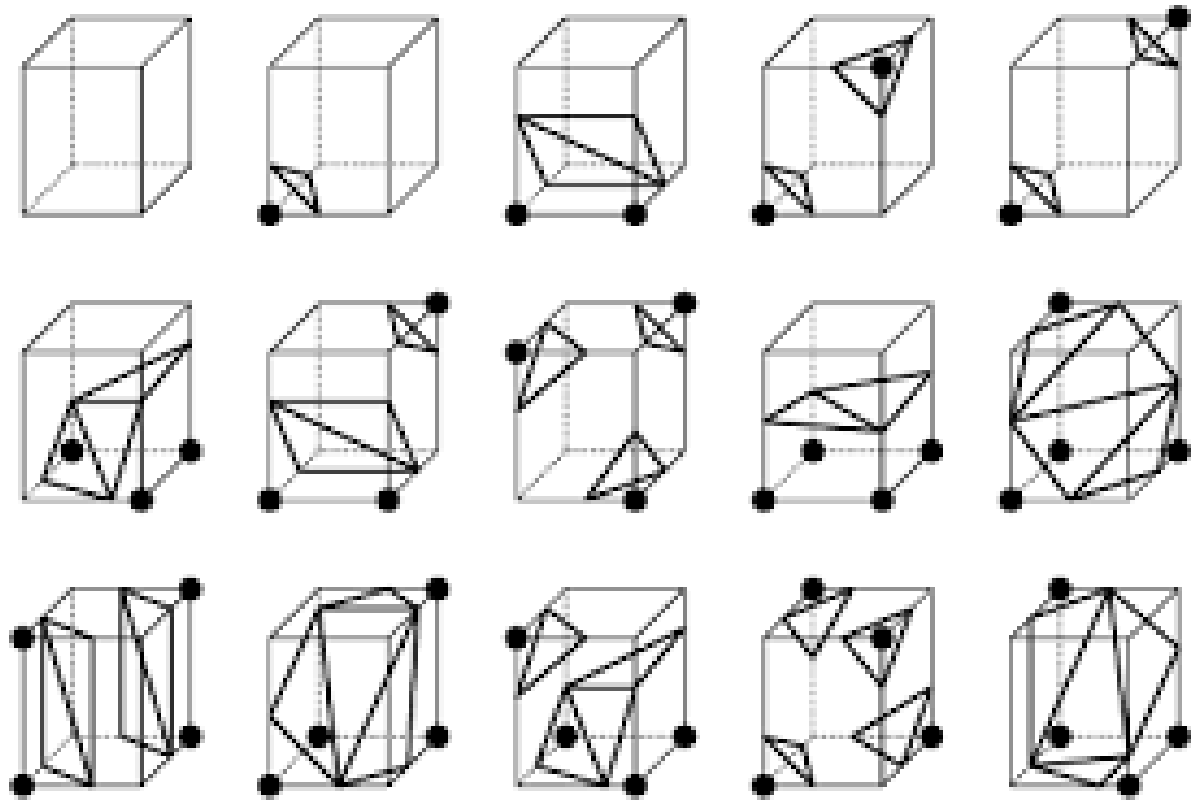
Review: Pointcloud to Implicit Surface



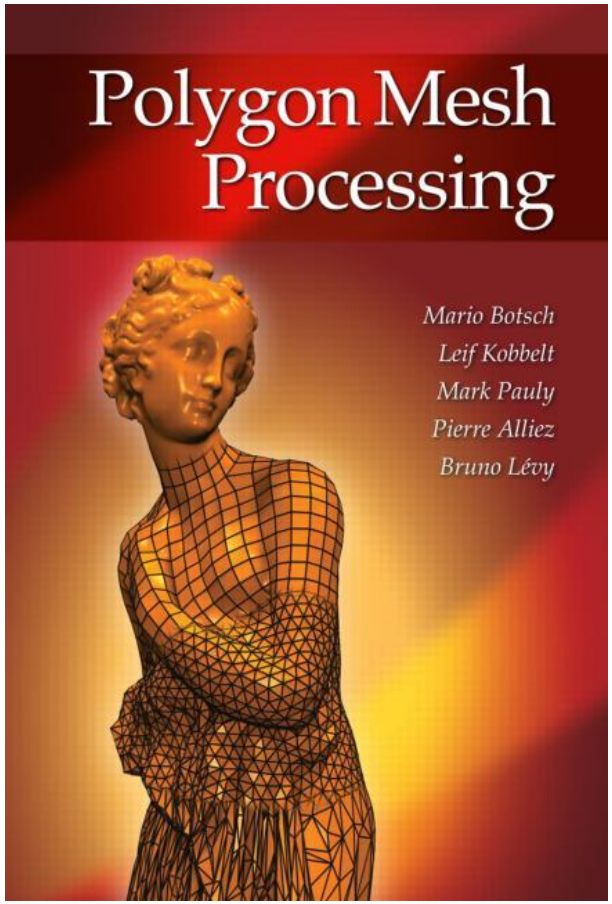
Radial Basis Function
[Carr et al. 01]

Review: Implicit->Mesh

[Lorensen and Cline 87]



Polygon Mesh Processing



Mesh Simplification

Mesh Analysis (3D Understanding)

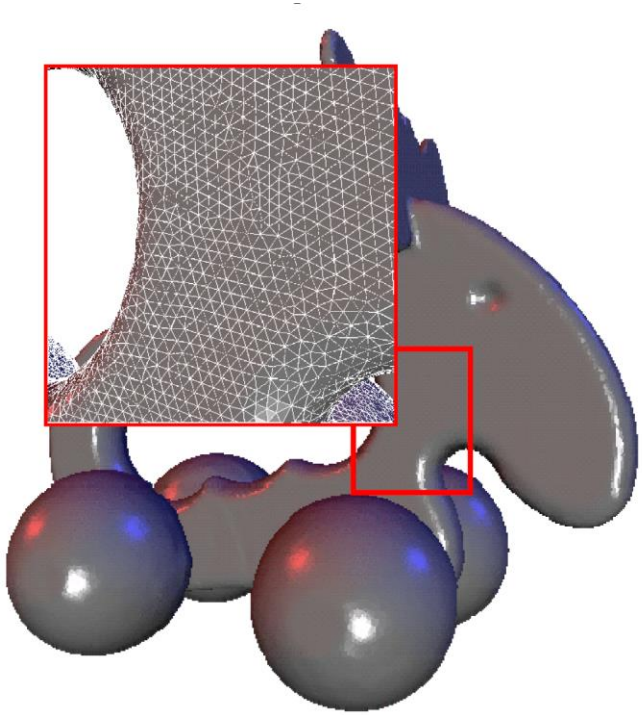
Mesh Parameterization

Mesh Deformation

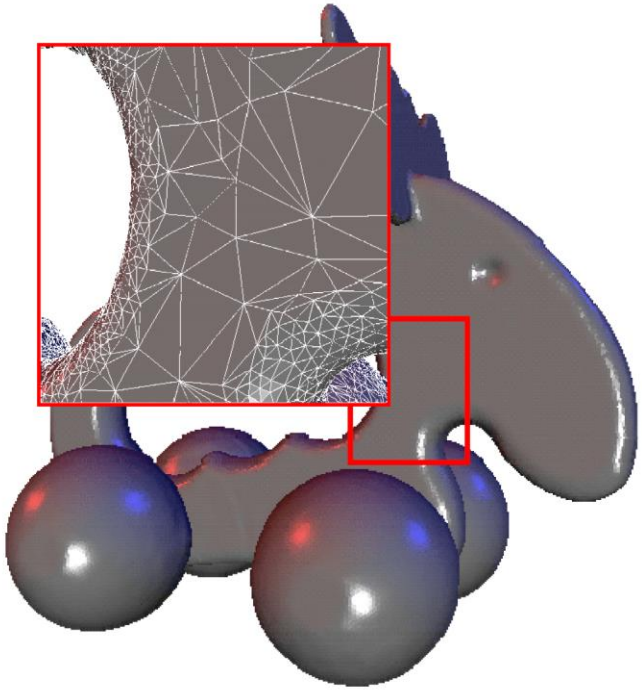
Mesh Simplification

Applications

- Oversampled 3D scan data



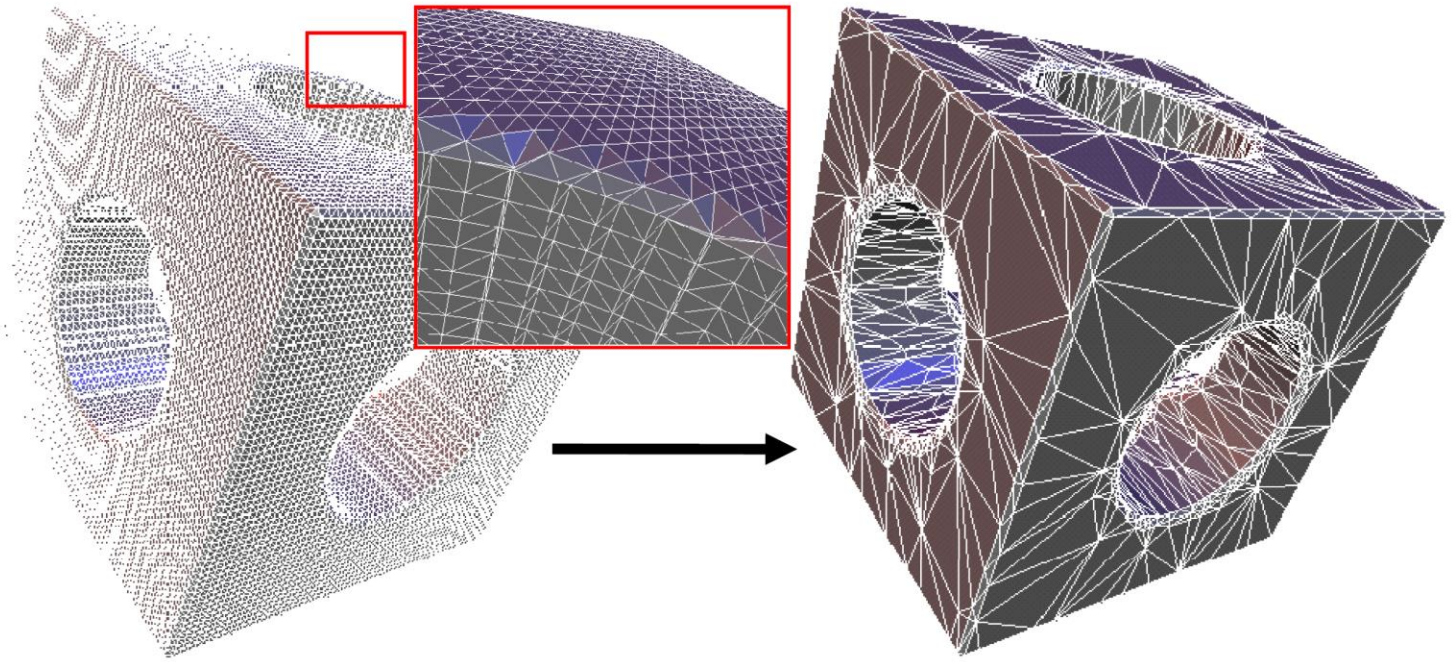
~150k triangles



~80k triangles

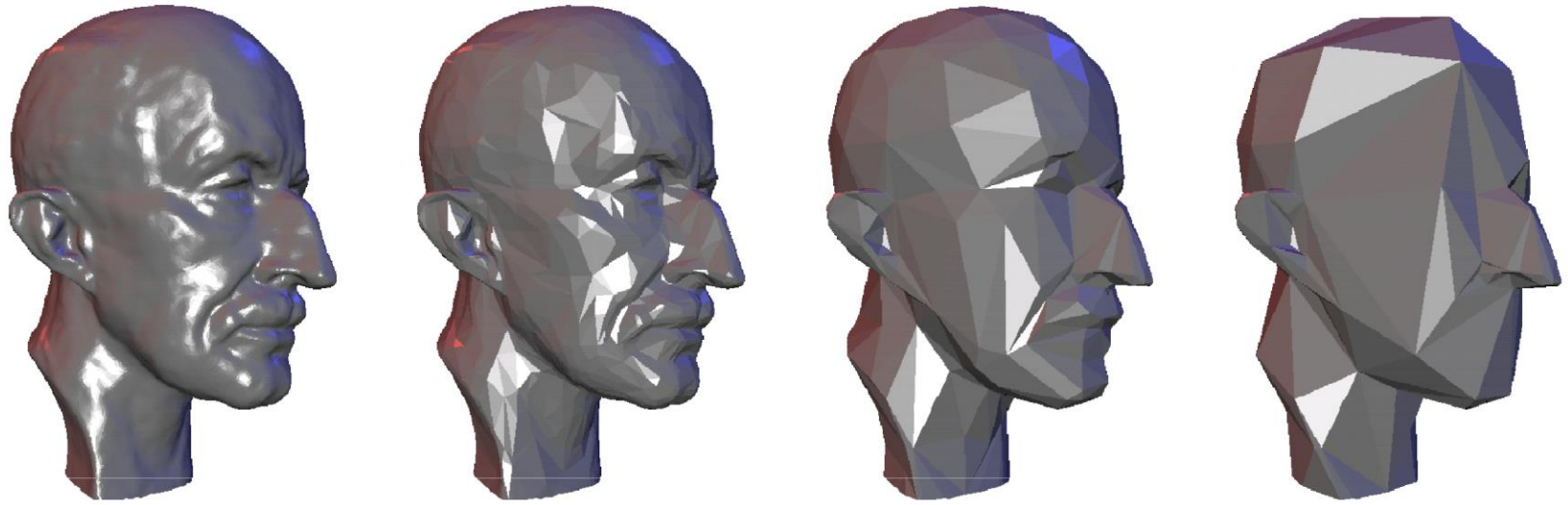
Applications

- Overtessellation: E.g. iso-surface extraction



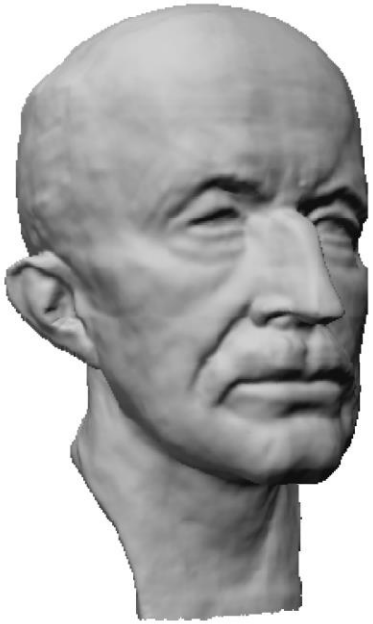
Applications

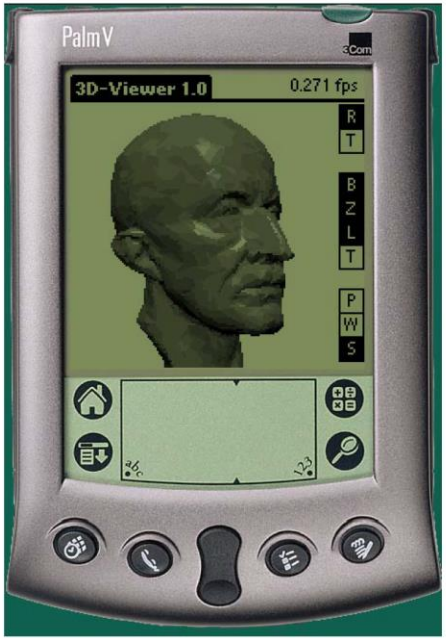
- Multi-resolution hierarchies for
  - efficient geometry processing
  - level-of-detail (LOD) rendering



Applications

- Adaptation to hardware capabilities

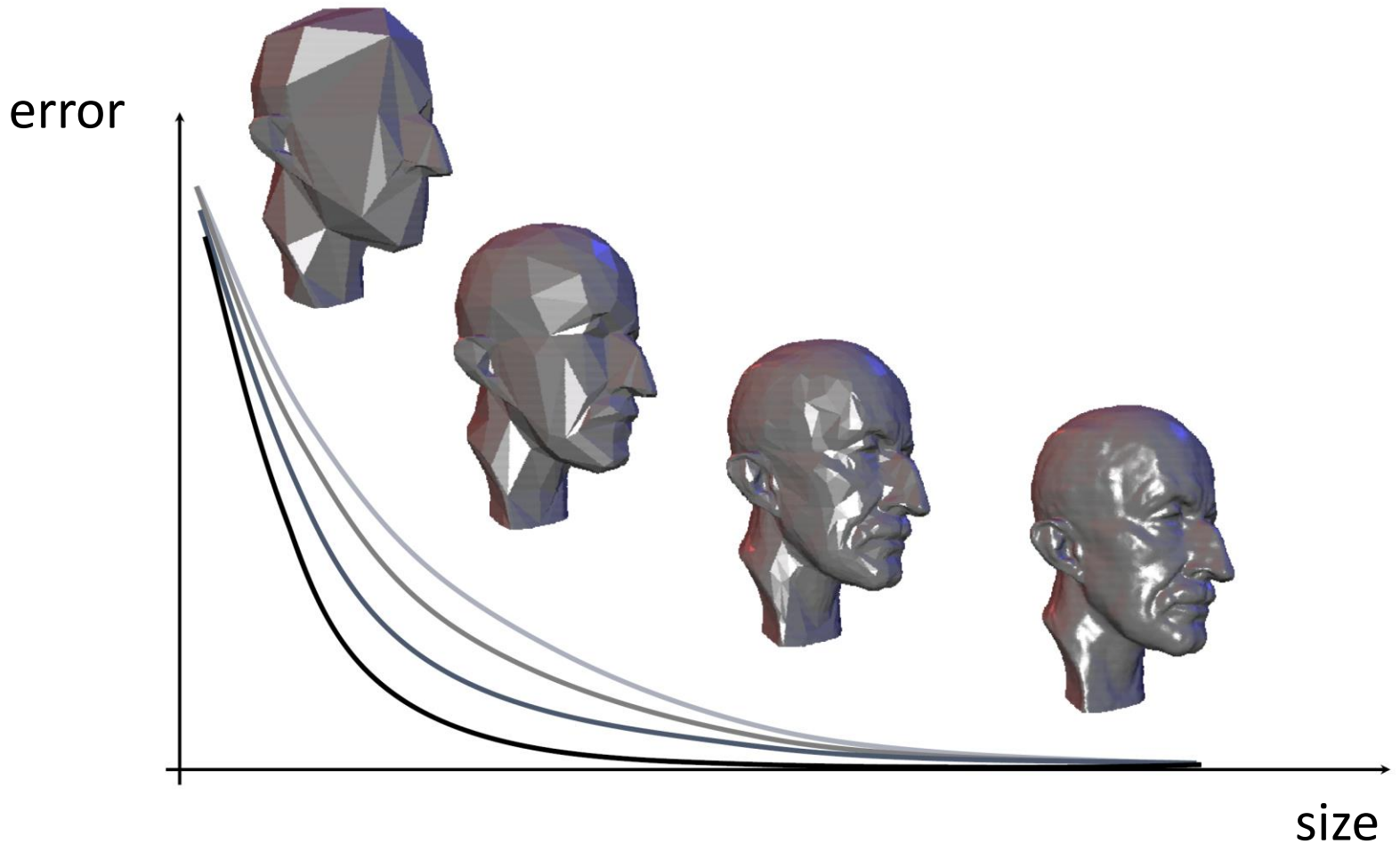




1999

2012

Size-Quality Tradeoff



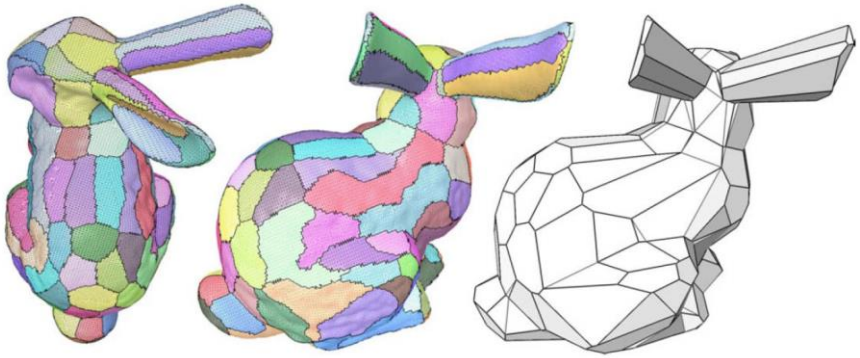
Problem Statement

- Given: $M = (V,F)$
- Find: $M' = (V',F')$ such that
  - $|V'| = n < |V|$ and $d(M,M')$ is minimal, or
  - $d(M,M') < eps$ and $|V'|$ is minimal

- Respect additional fairness criteria
  - Normal deviation, triangle shape, scalar attributes, etc.

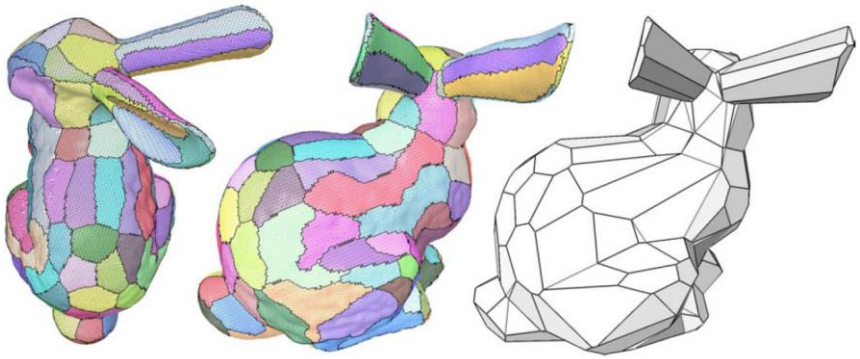
Mesh Decimation Methods

- Vertex clustering
- Incremental decimation
- Resampling
- Mesh approximation



Mesh Decimation Methods

- **Vertex clustering**
- **Incremental decimation**
- Resampling
- Mesh approximation

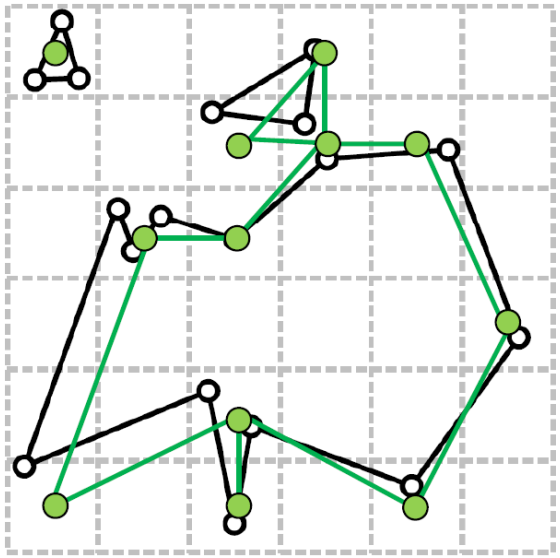


Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation
- Topology changes

Vertex Clustering

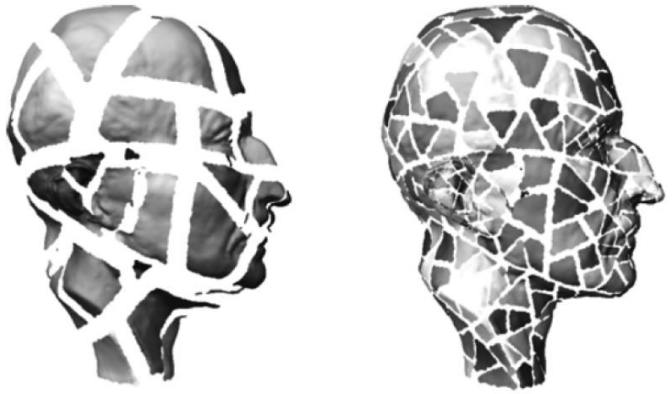
- Cluster Generation
  - Uniform 3D grid
  - Map vertices to cluster cells



- Computing a representative
- Mesh generation
- Topology changes

Vertex Clustering

- Cluster Generation
  - Hierarchical approach
  - Top-down or bottom-up



- Computing a representative
- Mesh generation
- Topology changes

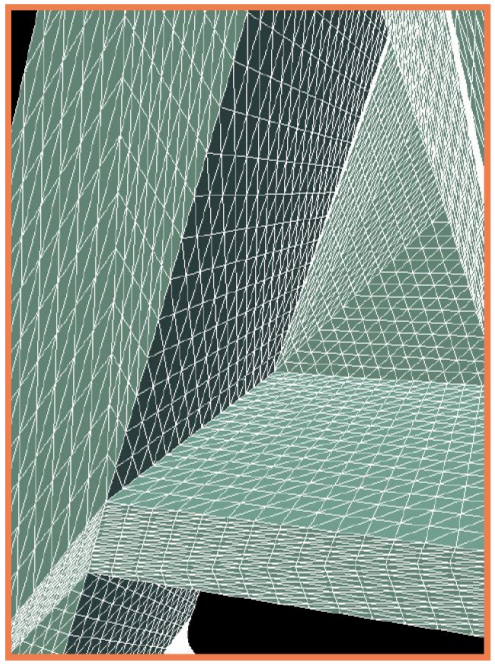
Vertex Clustering

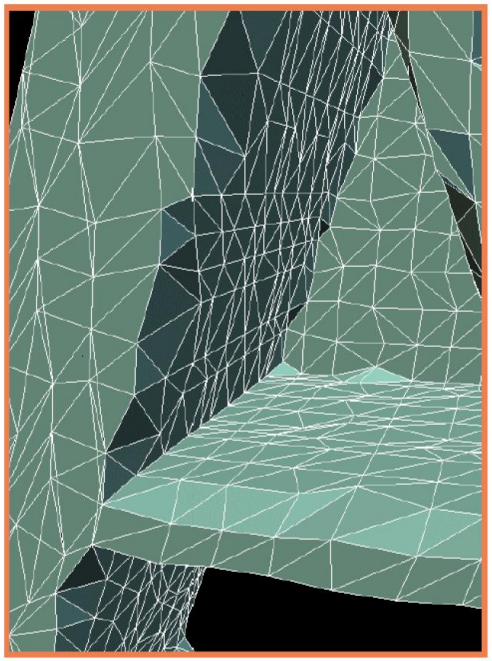
- Cluster Generation

- Computing a representative
  - Average/median vertex position
  - Error quadrics

- Mesh generation
- Topology changes

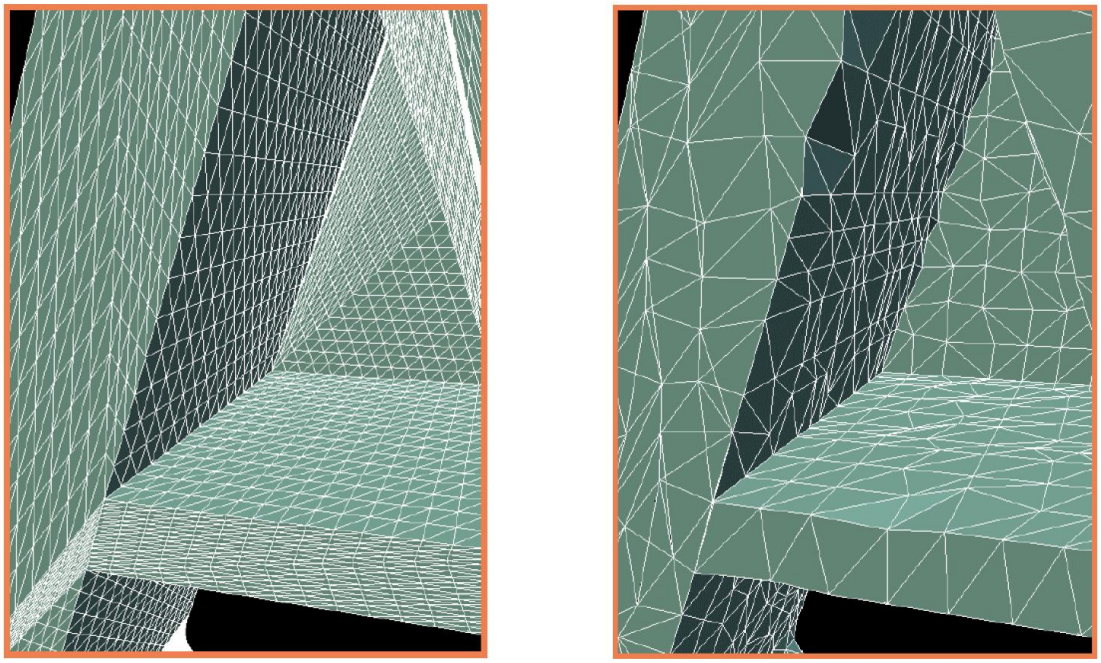
Computing a Representative





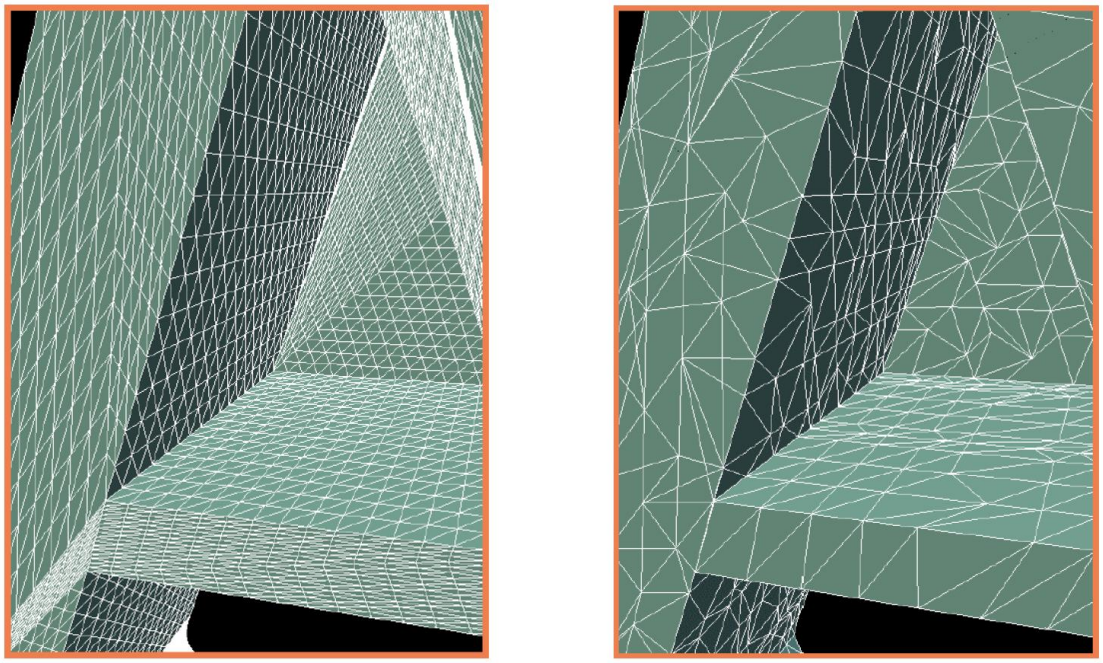
Average vertex position

Computing a Representative



Median vertex position

Computing a Representative

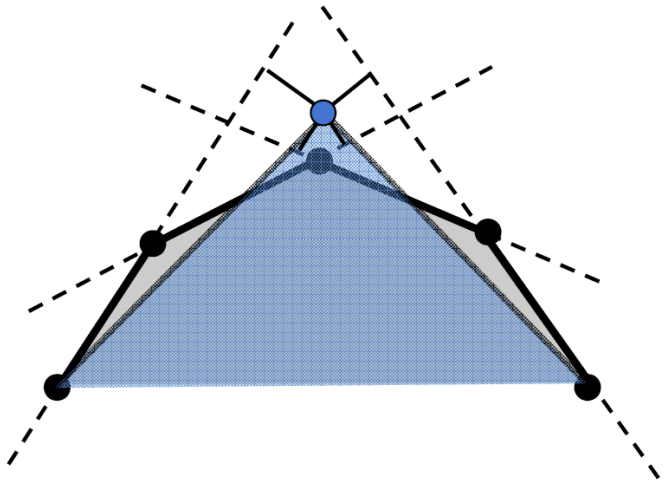


Error quadrics

Error Quadrics

- Patch is expected to be piecewise flat

- Minimize distance to neighboring triangles' planes



Error Quadrics

- Squared distance of point *p* to plane *q*

$$p = (x, y, z, 1)^T, \quad q = (a, b, c, d)^T$$

$$dist(q, p)^2 = (q^T p)^2 = p^T (q q^T) p =: p^T Q_q p$$

$$Q_q = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix}$$

Error Quadrics

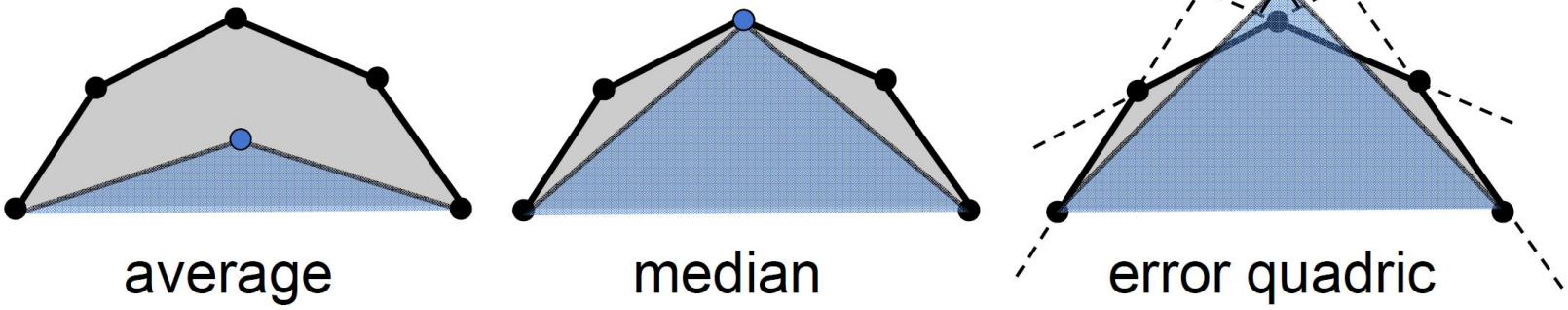
- Sum distances to planes q_i of vertex' neighboring triangles

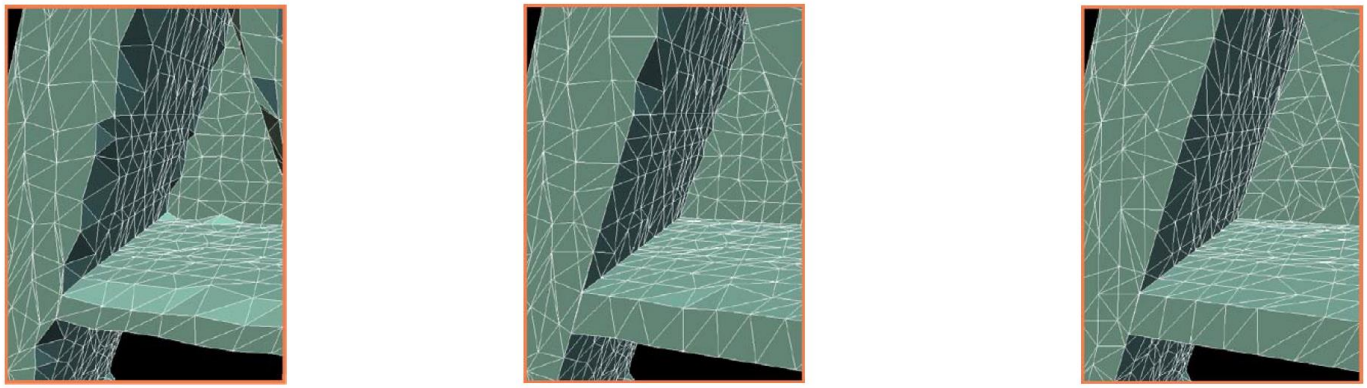
$$\sum_i dist(q_i, p)^2 = \sum_i p^T Q_{q_i} p = p^T \left(\sum_i Q_{q_i} \right) p =: p^T Q_p p$$

- Point p* that minimizes the error satisfies:

$$\begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} p^* = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Comparison





Vertex Clustering

- Cluster Generation
- Computing a representative

- Mesh generation
  - Clusters p<-> $\{p_0,\ldots, p_n\}$, q<-> $\{q_0,\ldots, q_m\}$

- Topology changes

Vertex Clustering

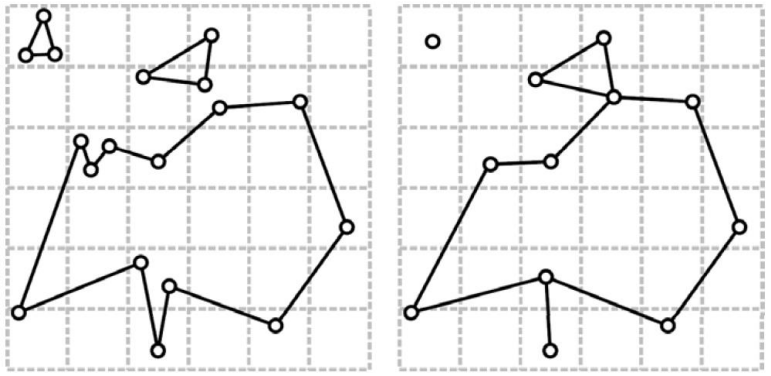
- Cluster Generation
- Computing a representative

- Mesh generation
  - Clusters p<-> $\{p_0,\ldots, p_n\}$, q<-> $\{q_0,\ldots, q_m\}$
  - Connect (p,q) if there was an edge (p_i, q_i)
- Topology changes

Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation

- Topology changes
  - If different sheets pass through one cell
  - Can be non-manifold



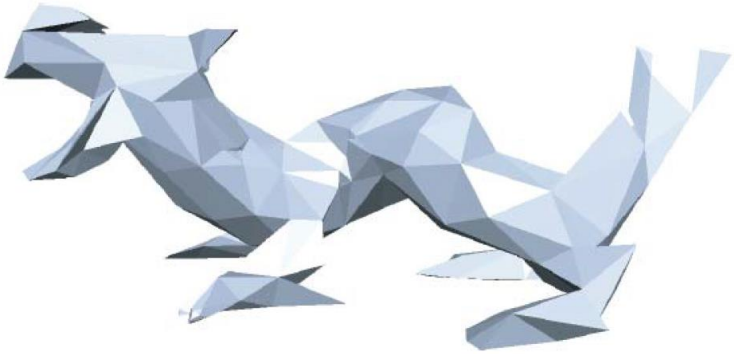
Incremental Decimation

Incremental Decimation

500K

50K

5K



0.5K

Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria

General Setup

- Repeat:
  - Pick mesh region
  - Apply decimation operator
- Until no further reduction possible

Variant I --- Greedy Optimization

- For each region
  - evaluate quality after decimation
  - enqeue(quality, region)

- Repeat:
  - get best mesh region from queue
  - apply decimation operator
  - update queue
- Until no further reduction possible

Variant II --- Global Error Control

- For each region
  - evaluate quality after decimation
  - enqeue(quality, region)

- Repeat:
  - get best mesh region from queue
  - If error < eps
    - Apply decimation operator
    - Update queue
- Until no further reduction possible

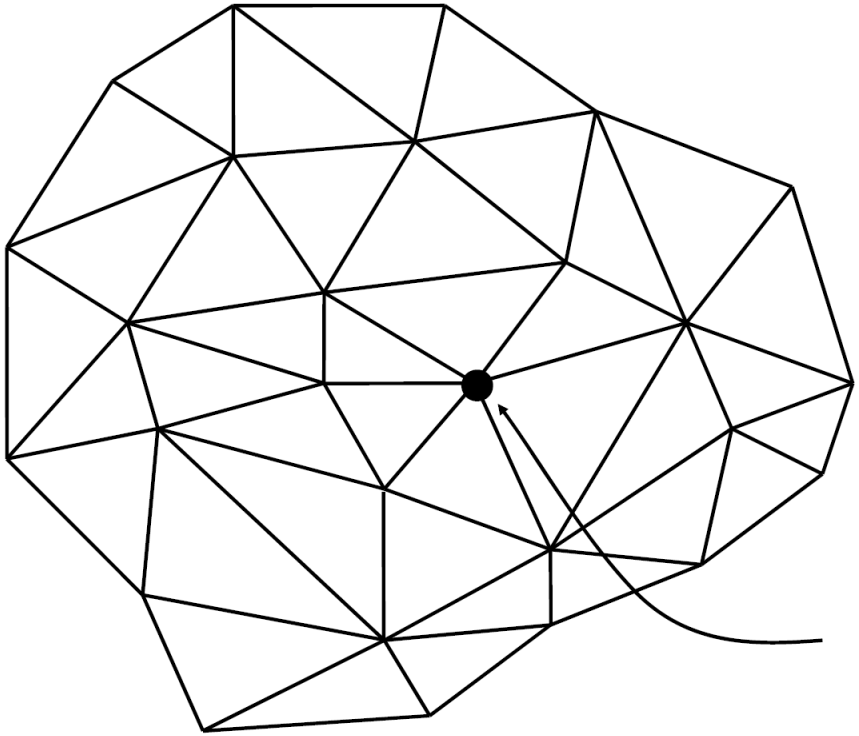
Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria

Decimation Operators

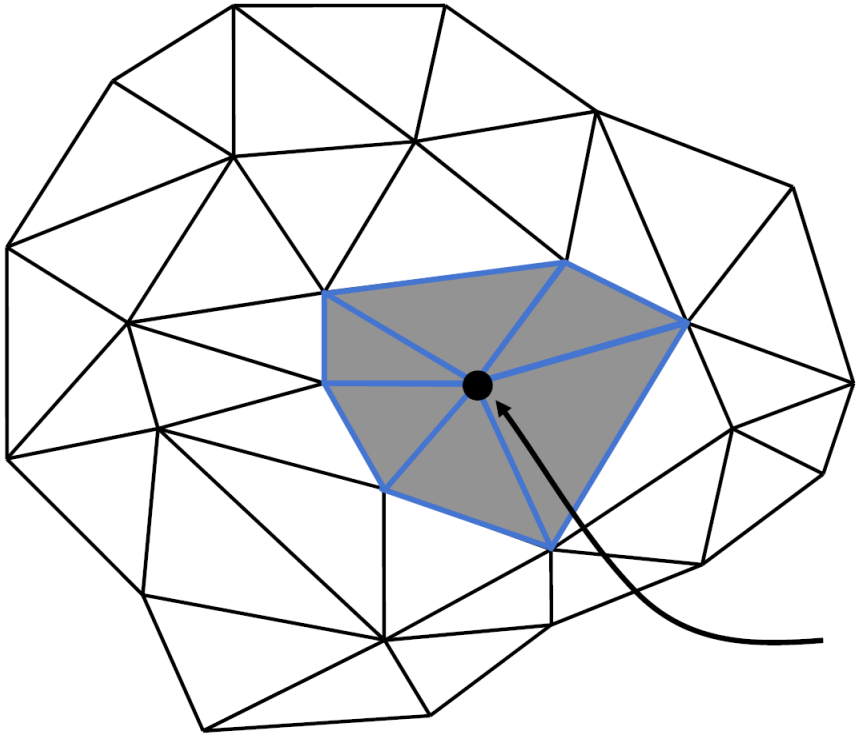
- What is a “region”?
- What are the DOF for re-triangulation?
- Classification
  - Topology-changing vs. topology-preserving
  - Inverse operation -> progressive meshes [Hoppe et al….]

Vertex Removal



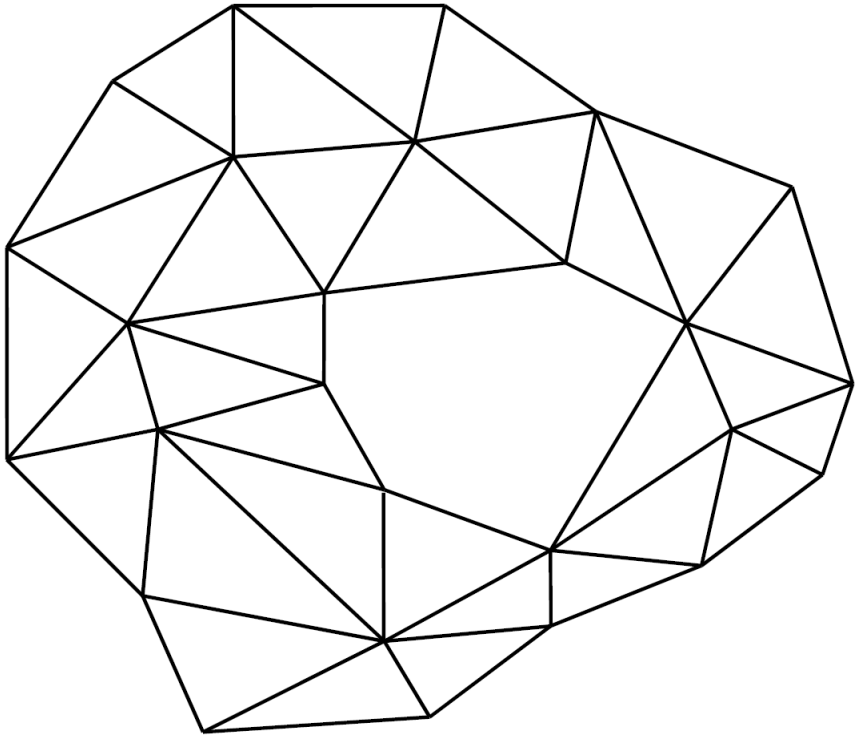
Select a vertex to be eliminated

Vertex Removal



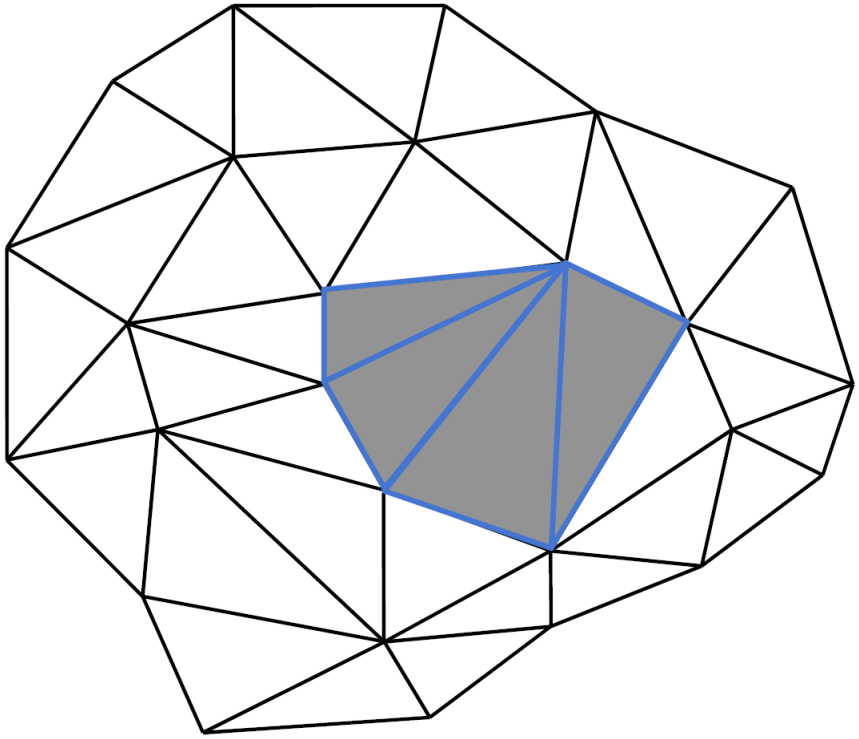
Select all triangles
sharing this vertex

Vertex Removal



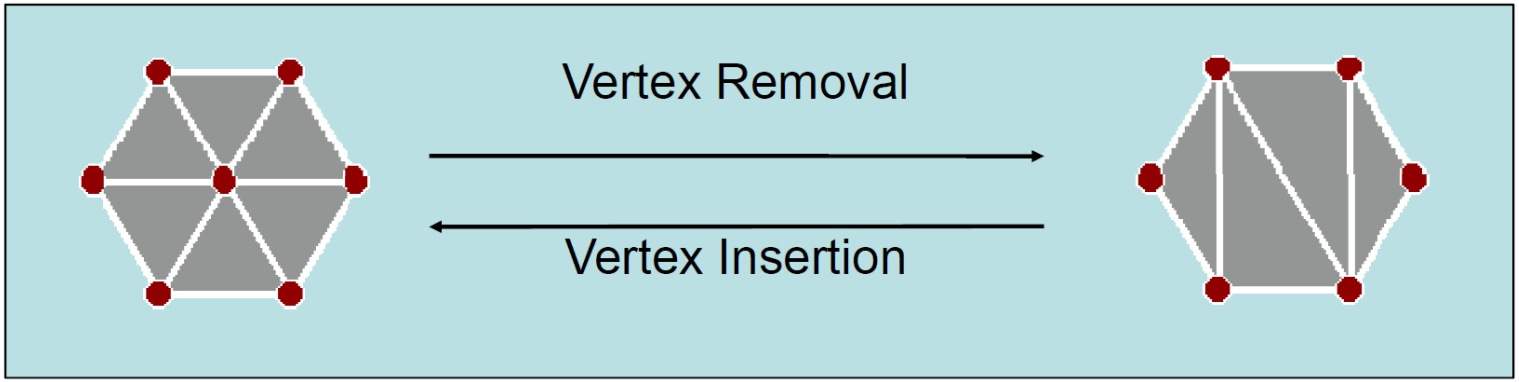
Remove the
selected triangles,
creating the hole

Vertex Removal



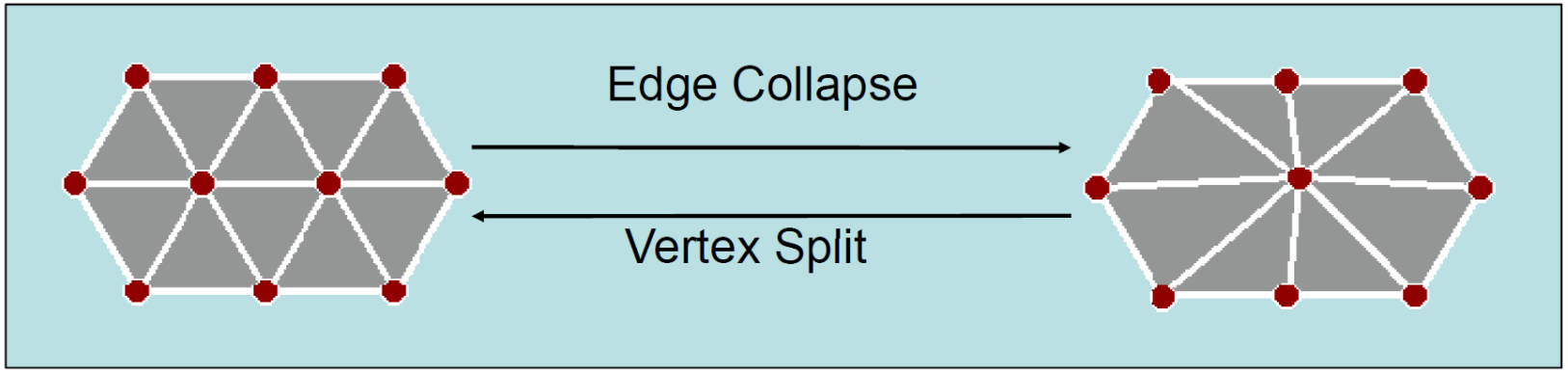
Fill the hole with
new triangles

Decimation Operators



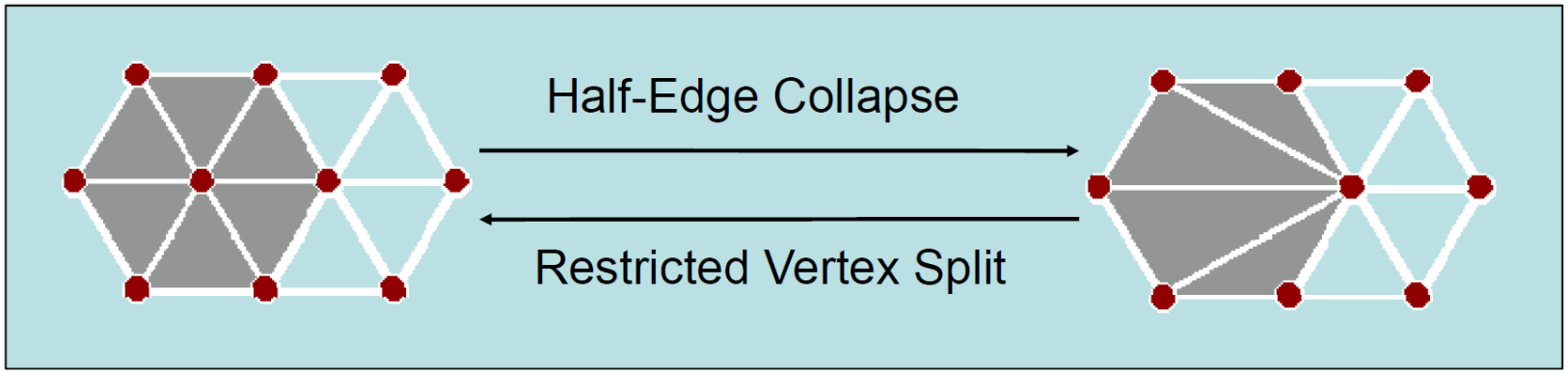
- Remove vertex
- Re-triangulate hole
  - Combinatorial degrees of freedom

Decimation Operators



- Merge two adjacent vertices
- Define new vertex position
  - Continuous degrees of freedom

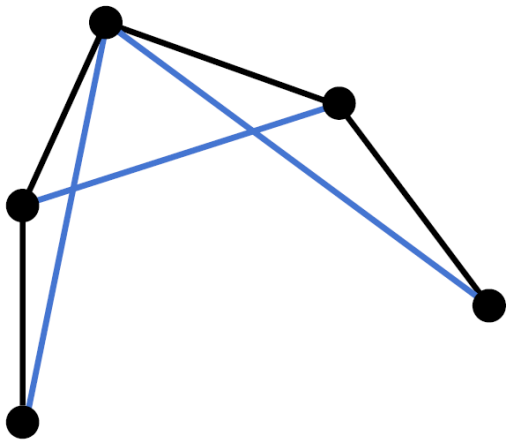
Decimation Operators



- Collapse edge into one end point
  - Special case of vertex removal
  - Special case of edge collapse
- No degrees of freedom

Fairness Criteria

- Rate quality of decimation operation
  - Approximation error
  - Triangle shape
  - Dihedral angles
  - Valence balance
  - …



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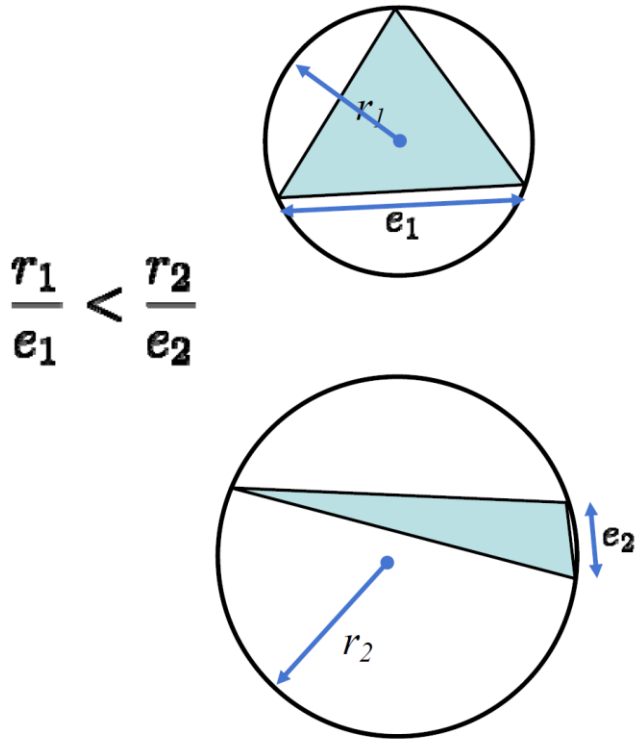
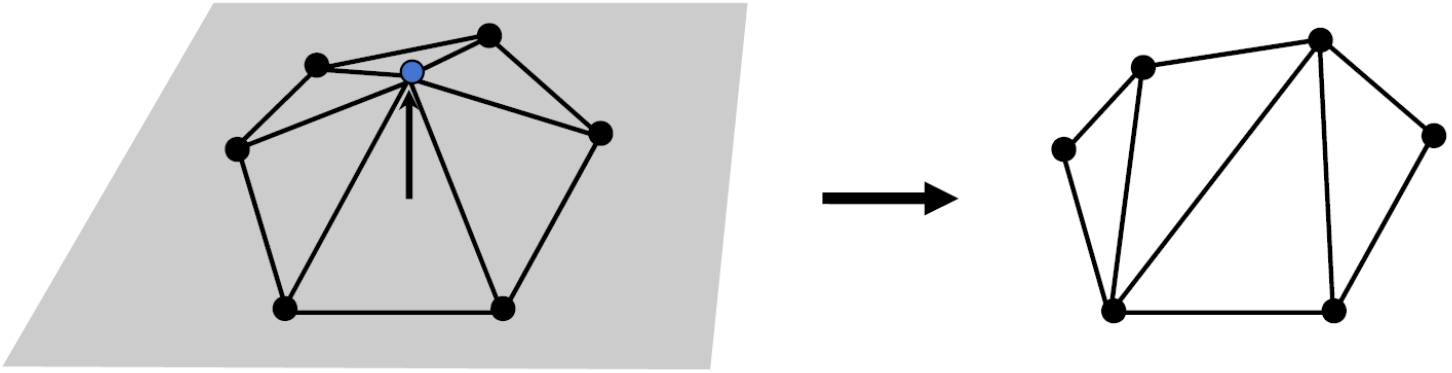


Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria

Local Error Metrics

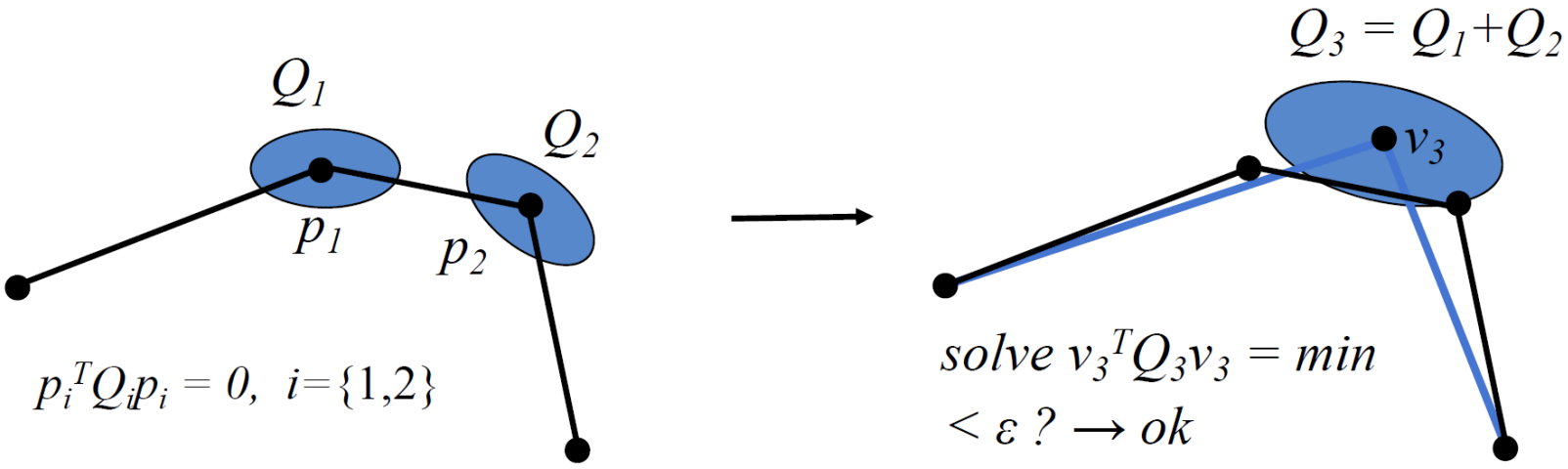
- Local distance to mesh
  - Compute average plane
  - No comparison to *original* geometry



Global Error Metrics

[Garland and Heckbert 97]
4k+ citations on Google Scholar

- Error quadrics
  - Squared distance to planes at vertex
  - No bound on true error



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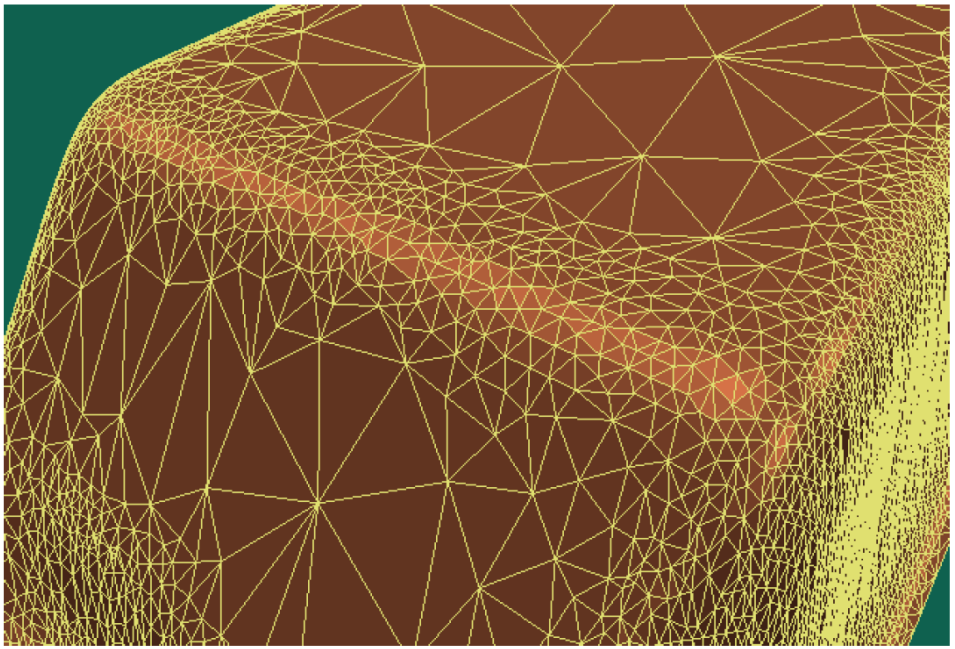

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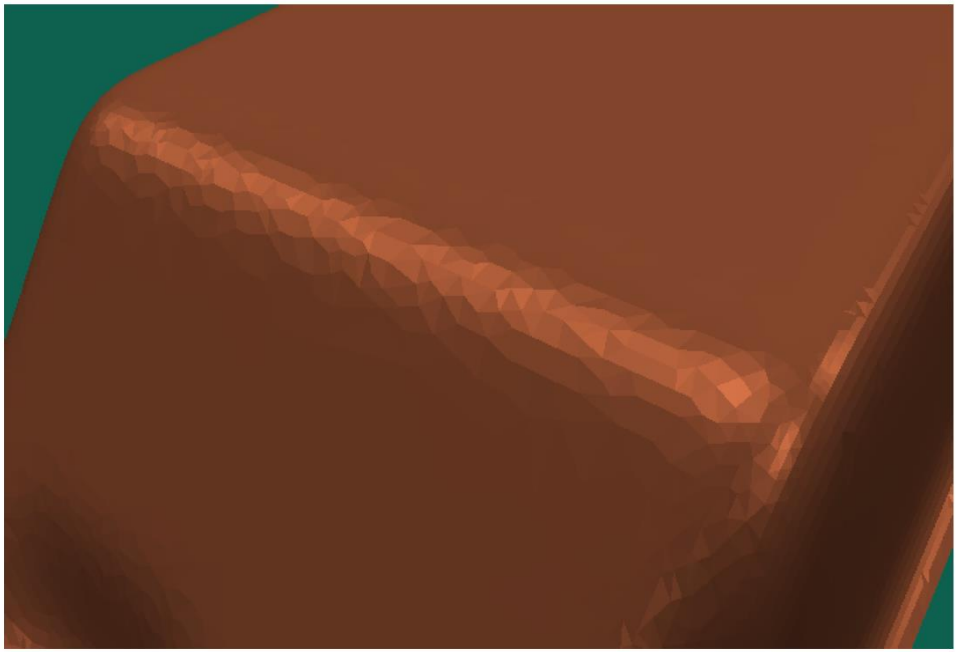

Fairness Criteria

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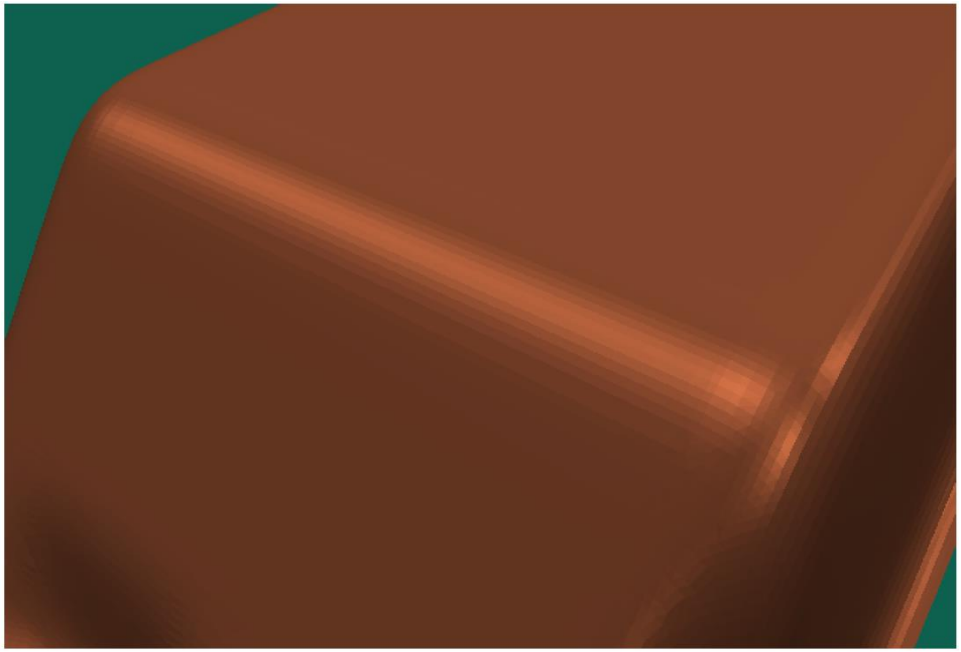

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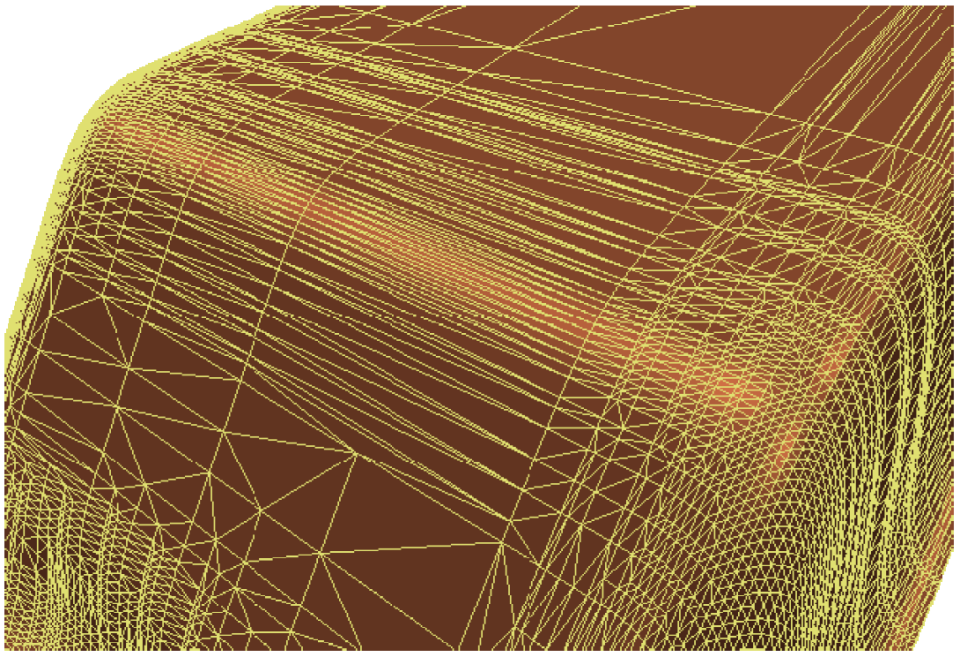

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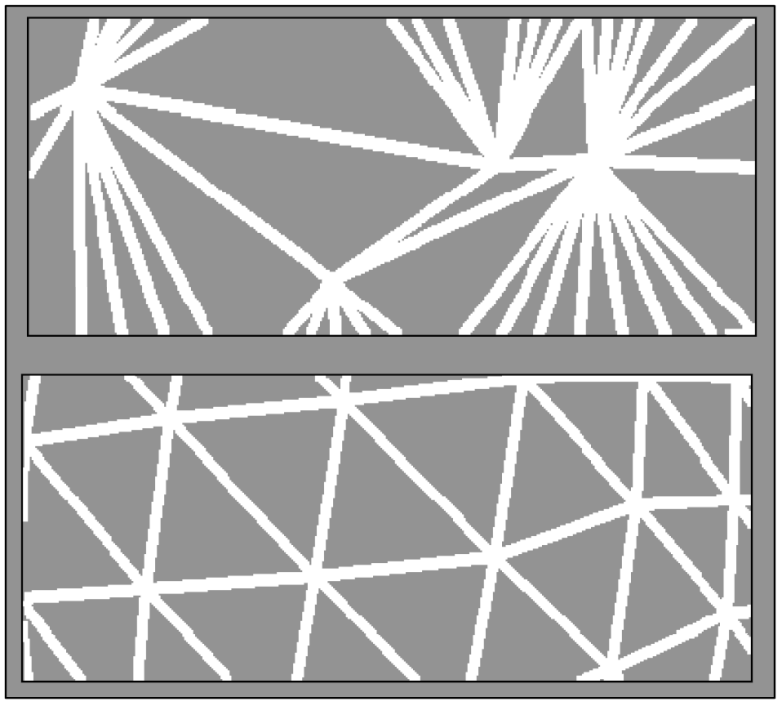
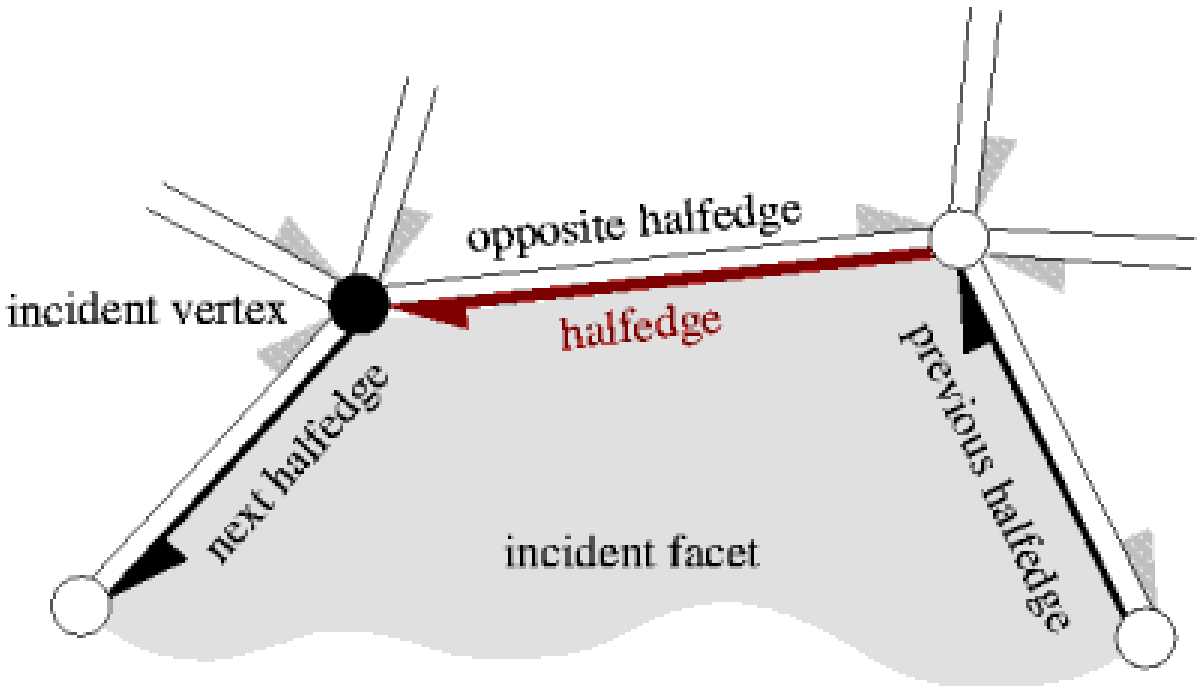


Implementation – Half Edge Data Structure



Reading: https://doc.cgal.org/latest/HalfedgeDS/index.html

Comparison

- Vertex clustering
  - fast, but difficult to control simplified mesh
  - Topology changes, non-manifold meshes
  - Global error bound, but often not close to optimum

- Incremental decimation with quadratic error metrics
  - good trade-off between mesh quality and speed
  - explicit control over mesh topology
  - restricting normal deviation improves mesh quality