GAMES
Registration

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Motivation

- Align two shapes/scans given initial guess for relative transform
Task classification
Three axis

- Pairwise vs multiple
- Global vs local
- Fully overlap vs partial overlap
Outline

• Pairwise registration
  – Full overlap
  – Partial overlap
  – Global methods
  – Learning-based

• Multiple registration
  – Joint pairwise registration
  – Simultaneous registration and reconstruction
ICP for pairwise registration

• If correct correspondences are known, can find correct relative rotation/translation

Construct error function:

\[ E := \sum_{i} (Rp_i + t - q_i)^2 \]

Minimize (closed form solution in [Horn 87])

Slide credit: Niloy Mitra and Szymon Rusinkiewicz
ICP for pairwise registration

• Assume: Closest points as corresponding

\[ p_i \rightarrow C(p_i) \]
ICP for pairwise registration

• ... and iterate to find alignment
• Iterative Closest Points (ICP) [Besl and McKay 92]
• Converges if starting poses are close enough

Slide credit: Niloy Mitra and Szymon Rusinkiewicz
From the optimization perspective

• The registration problem shall be formulated in a least squares sense as follows. Compute the rigid body transformation $\alpha^*$, which minimizes

$$F(\alpha) = \sum_i d^2(\alpha(x_i^0), \Phi)$$

Here, $d^2(\alpha(x_i^0), \Phi)$ denotes the squared distance of $\alpha(x_i^0)$ to $\Phi$.

• ICP is alternating minimization
  – Always reduces the objective function
  – Linear convergence
Reducing objective value does not guarantee convergence

- $f(x) = x^2$

- $x_i = 3 + \frac{1}{i}$
Gauss-Newton optimization

• Review of Gauss-Newton method

Given $m$ functions $r = (r_1, \ldots, r_m)$ (often called residuals) of $n$ variables $\beta = (\beta_1, \ldots, \beta_n)$, with $m \geq n$, the Gauss–Newton algorithm iteratively finds the value of the variables that minimizes the sum of squares\(^3\)

$$S(\beta) = \sum_{i=1}^{m} r_i(\beta)^2.$$  

Starting with an initial guess $\beta^{(0)}$ for the minimum, the method proceeds by the iterations

$$\beta^{(s+1)} = \beta^{(s)} - (J_r^T J_r)^{-1} J_r^T r \left( \beta^{(s)} \right),$$

where, if $r$ and $\beta$ are column vectors, the entries of the Jacobian matrix are

$$(J_r)_{ij} = \frac{\partial r_i \left( \beta^{(s)} \right)}{\partial \beta_j},$$

and the symbol $^T$ denotes the matrix transpose.

Gauss-Newton optimization

• Review of Gauss-Newton method

The Gauss–Newton algorithm can be derived by linearly approximating the vector of functions \( r_j \). Using Taylor’s theorem, we can write at every iteration:

\[
r(\beta) \approx r(\beta^{(s)}) + J_r(\beta^{(s)}) \Delta
\]

with \( \Delta = \beta - \beta^{(s)} \). The task of finding \( \Delta \) minimizing the sum of squares of the right-hand side; i.e.,

\[
\min \left\| r(\beta^{(s)}) + J_r(\beta^{(s)}) \Delta \right\|^2_2,
\]

is a linear least-squares problem, which can be solved explicitly, yielding the normal equations in the algorithm.

Convergence rate of Gauss-Newton method

- Quasi-quadratic convergence

\[ \|x_{k+1} - x^*\| = O\left(F(x^*)\|x_k - x^*\| + \|x_k - x^*\|^2\right) \]

Error of at the next iteration
Residual of the optimal solution
Error of the current iteration
Point-2-plane distance

• Gauss-Newton leads to the following optimization problem

$$\min \sum_{i} \left[ \mathbf{n}_i \cdot (\bar{c} + c \times x_i) + d_i \right]^2$$

where $c$ gives a linear parameterization of $\text{SO}(3)$

Using point-to-plane distance instead of point-to-point allows flat regions slide along each other

[Chen and Medioni 91]
Squared distance function

$$F_d(x_1, x_2, x_3) = \frac{d}{d - \varrho_1} x_1^2 + \frac{d}{d - \varrho_2} x_2^2 + x_3^2.$$
Practical considerations

• Nearest neighbor computation (Kd-tree)
Squared distance field

[Pottmann et al. 06]
Stable Sampling [Gelfand et al. 2003]

- Select samples that constrain all degrees of freedom of the rigid-body transformation
Stable Sampling [Gelfand et al. 2003]
Partial Overlaps
Registration under robust functions

- Use a robust norm under the point-2-plane distance metric

\[
\min_{R, t} \sum_{i=1}^{N} \rho(||(Rp_i + t - f_i)^T n_i||)
\]

\[
\rho_{GM}(t) = \frac{t^2}{\sigma^2 + t^2} \quad \rho_1(t) = |t| \quad \rho_2(t) = t^2
\]
Optimization

• Still alternate between optimizing the correspondences and optimizing the transformation

• Optimization strategy I: Gauss-Newton optimization

• Optimization strategy II: reweighted least squares

\[
\min_{R,t} \sum_{i=1}^{N} w_i |(Rp_i + t - f_i)^T n_i|^2
\]

where

\[
w_i = \frac{\rho(|(Rp_i + t - f_i)^T n_i|)}{|(Rp_i + t - f_i)^T n_i|^2}
\]
Mean versus Median

- **Mean**

\[
\min_x \sum_{i=1}^{N} (x - x_i)^2
\]

- **Median**

\[
\min_x \sum_{i=1}^{N} |x - x_i|
\]
Median computation

• Weighted average

\[ \min_x \sum_{i=1}^{N} w_i |x - x_i|^2 \quad x^* = \frac{\sum_{i=1}^{N} w_i x_i}{\sum_{i=1}^{N} w_i} \]

• Weighting

\[ w_i = \frac{1}{|x^* - x_i|} \]
Bi-directional pruning

[Mitra et al. 05]
Efficient variants of ICP Registration

- Selection of points
- Matching points
- Weighting of pairs
- Rejecting pairs
- Error Metric and Minimization
Global Matching
Global matching

Feature extraction + Feature matching

Reassembling fractured surfaces
[Huang et al. 06]
Global matching

Feature extraction + Feature matching

Reassembling fractured surfaces
[Huang et al. 06]

Relative pose extraction
Relative poses/pair-wise matching in the neural network era

FlowNet: Learning Optical Flow with Convolutional Networks
[Fischer et al. 15]
Feature descriptors – Spin images

https://www.ri.cmu.edu/publications/spin-images-a-representation-for-3-d-surface-matching/
Feature descriptors – integral invariants

Related to mean curvature and robust

Integral Invariants for Robust Geometry Processing. Pottmann et al., 09.
Other features

3D SIFT

Patch features

A 3-Dimensional Sift Descriptor and Its Application to Action Recognition. Scovanner et al., 07. ACM MM
Salient Geometric Features for Partial Shape Matching and Similarity. Gal and Cohen-Or’ 06. ACM TOG
Global matching --- RANSAC

• How many point-pairs specify a rigid transform?
  – In $\mathbb{R}^2$?
  – In $\mathbb{R}^3$?

• Additional constraints?
  – Distance preserving
  – Stability?
Software

Geomagic
RANSAC

- Preprocessing: sample each object
- Recursion:
  - Step I: Sample three (two) pairs, check distance constraints
  - Step II: Fit a rigid transform
  - Step III: Check how many point pairs agree. If above threshold, terminates; otherwise goes to Step I
RANSAC --- facts

• Sampling
  – Feature point detection

• Correspondences
  – Use feature descriptors
  – The candidate correspondences $m \ll O(n^2)$
  – Denote the success rate $p \frac{1}{4} \frac{n}{m}$

• Basic analysis
  – The probability of having a valid triplet $p^3$
  – The probability of having a valid triplet in $N$ trials is $1-(1-p^3)^N$
RANSAC+

- How many surfel (position + normal) correspondences specify a rigid transform?

\[
[\mathbf{n}_1, \mathbf{n}_2, d] \xrightarrow{R} [\mathbf{n}'_1, \mathbf{n}'_2, d']
\]

Constraints:
1. \[\|\mathbf{p}_1 - \mathbf{p}_2\| \approx \|\mathbf{p}'_1 - \mathbf{p}'_2\|\]
2. \[\angle(\mathbf{n}_1, \mathbf{d}) = \angle(\mathbf{n}'_1, \mathbf{d}')\]
3. \[\angle(\mathbf{n}_2, \mathbf{d}) = \angle(\mathbf{n}'_2, \mathbf{d}')\]
4. \[\angle(\mathbf{n}_1, \mathbf{n}_2) = \angle(\mathbf{n}'_1, \mathbf{n}'_2)\]

Reduce the number of trials from \(O(m^3)\) to \(O(m^2)\)

Success rate: \[1 - (1 - p^2)^N\]
Hough transform for line fitting

• Line detection in an image
  – what is the line?
  – How many lines?
  – Point-line associations?

• **Hough Transform** is a voting technique that can be used to answer all of these questions
  – Record vote for each possible line on which each edge point lies
  – Look for lines that get many votes.
Voting

image space

Hough (parameter) space
Clustering
Rigid matching

• Rigid transform detection from feature correspondences
Symmetry detection

Partial and Approximate Symmetry Detection for 3D Geometry, N. Mitra, L. Guibas, and M. Pauly, SIGGRAPH’ 06
Spectral Approach
Distance preservation = Rigidity?

\[ P = \{p_i\} \quad Q = \{q_i\} \]

\[ \|p_i - p_j\| = \|\phi(p_i) - \phi(p_j)\| \quad \phi(p_i) = R \cdot p_i + t \]

\[ \det(R) = -1 \]
Spectral approach

Correspondences

Consistency matrix

A Spectral Technique for Correspondence Problems using Pairwise Constraints, M. Leordeanu and M. Hebert, ICCV 2005
## Clique extraction

### Consistency matrix

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Algorithm

• Step 1: Compute the maximum eigenvector $\mathbf{v}$ of $\mathbf{C}$

• Step 2: Sort the vertices based on magnitude of $\mathbf{v}$ and initialize the cluster

• Step 3: Incrementally insert vertices while checking the clique constraint

• Step 4: Stop if the size of the cluster is small, otherwise accept the cluster and go to Step 1
Geometric consistency

$$\Delta_1^2 (c, c') := \| f(q_1) - f(q_2) \|^2 + \| f(q_1') - d(q_2) \|^2$$

Also used for initializing correspondences

$$\Delta_2 (c, c') := \| p(q_1) - p(q_1') \| - \| p(q_2) - p(q_2') \|$$

$$\Delta_3 (c, c') := \angle (n(q_1), n(q_1')) - \angle (n(q_2), n(q_2'))$$

$$\Delta_4 (c, c') := \angle (n(q_1), p(q_1)p(q_1')) - \angle (n(q_2), p(q_2)p(q_2'))$$

$$\Delta_5 (c, c') := \angle (n(q_1'), p(q_1)p(q_1')) - \angle (n(q_2'), p(q_2)p(q_2'))$$

$$w_\gamma (c, c') = \exp \left( -\frac{1}{2} \sum_{i=1}^{5} \left( \frac{\Delta_i (c, c')}{\gamma_i} \right)^2 \right)$$
Hybrid Method
Robust geometric matching

Input:

\[ S_1 \quad C \quad S_2 \]

Reweighted non-linear least squares

Loss term:

\[ r_{(R,t)}(c) = (\|R p(q_1) + t - p(q_2)\|^2 + \|R n(q_1) - n(q_2)\|^2)^\frac{1}{2} \]

Total objective term:

\[ \min_{R,t} \sum_{c \in C} r_{(R,t)}(c) \]

Can only tolerate 50% of incorrect correspondences

Spectral matching

\[ \max_{\{x_c\}} \sum_{c,c'} w(c,c') x_c x_{c'} \]

subject to \[ \sum_c x_c = 1 \]

Indicators associated with initial corres.

Can tolerate more incorrect correspondences

Not a clean separation between inliers/outliers

[Yang et al. 19]
Spectral matching + reweighted least squares

\[
\begin{align*}
\text{maximize} & \quad \sum_{c, c' \in \mathcal{C}} w_{\gamma}(c, c') x_c x_{c'} (\delta - r_{R,t}(c) - r_{R,t}(c')) \\
\text{subject to} & \quad \sum_{c \in \mathcal{C}} x_c^2 = 1
\end{align*}
\]

Optimization:

1. When \( R, t \) are fixed:

\[
\max_{x_c} \sum_{c, c'} a_{cc'} x_c x_{c'} \quad \text{subject to} \quad \sum_c x_c^2 = 1 \rightarrow \text{Leading eigenvector computation}
\]

\[
a_{cc'} := w_{\gamma}(c, c') (\delta - r_{R,t}(c) - r_{R,t}(c'))
\]

2. When \( \{x_c\} \) are fixed:

\[
\min_{R, t} \sum_{c \in \mathcal{C}} a_c r_{R,t}(c), \quad a_c := x_c \sum_{c' \in \mathcal{C}} w_{\gamma}(c, c') x_{c'}
\]

Reduces to the standard setting of reweighted non-linear least squares

Training details in the paper
Side-by-side comparison

source

target

Input

Ground truth
Side-by-side comparison

source

target

Ground truth

Input

Spectral matching
Side-by-side comparison

- Source
- Target
- Input
- Ground truth
- Spectral matching
- Reweighted non-linear LS
Side-by-side comparison

source

target

Input

Ground truth

Spectral matching

Reweighted non-linear LS

Spectral matching + Reweighted non-linear LS
Learning-based methods
Learning registration

Use transformers to build correspondences
Solve for the rigid transformation

[Wang and Solomon 19]
From overlapping scans to non-overlapping scans

Complete scene

Overlapping scans

Complete scene

Small/no overlapping scans
Diverse applications

Early detection of loop closure

Reconstruction from a few snapshots [Furukawa and Hernandez 15]

Solving jigsaw puzzle [Cho et al. 10]
Challenges

- No or few features to match
- Black-box deep networks do not work
- Overlapping ratios vary

Small or No-overlaps
Human perception

Human body reconstruction from a pair of front and back scans
Key Idea: Completion + Relative pose estimation
Scene Completion
Combine depth/normal/color/learned semantic class descriptors
Scene completion from two inputs
A generic constraint

- The completed scene from each scan should contain the other scan
Update the completed scenes using both input scans

– Allow the geometry of the second scan to move when performing completion
Qualitative Results --- Small overlap
Qualitative Results --- No overlap

G.T. Color

G.T. Scene

Ours

4PCS

DL

GReg

CGReg
Qualitative Results --- SUNCG
Qualitative Results --- Matterport
Qualitative Results --- ScanNet
Understand the quality of the completions

Relative pose estimation in the presence of large outlier ratios
Representations for Completion

Top-view and plane features generalize better in far-away invisible regions
Representations for local registration
Qualitative results
Multiple methods
Joint pairwise registration

Tam, Gary KL et al. 2012
Joint pairwise registration

\[ S_i \]

\[ T_i = (R_i, t_i) \]

\[ T_i' = (R_i', t_i') \]

\[ S_i' \]

\[ p_{ij} \]

\[ p_{i'j'} \]

\[ n_{i'j'} \]

\[ \minimize_{\{T_i\}} \sum_{(i, i') \in \mathcal{E}} d^2(S_i, T_i, S_{i'}, T_{i'}) \]

subject to \( R_1 = I_3, t_1 = 0. \)

\[ d^2(S_i, T_i, S_{i'}, T_{i'}) := \]

\[ \sum_{(p_{ij}, p_{i'j'}) \in C_{ii'}} \left( (R_i p_{ij} + t_i - R_{i'} p_{i'j'} - t_{i'})^T (R_{i'} n_{i'j'}) \right)^2 \]

Gauss-Newton method

Newton’s method
Joint pairwise registration - applications

- Surface reconstruction
- Graph/Depth-based SLAM
- Organizing shape collections
Joint pairwise registration - limitations

- Need to determine overlapping scans and overlapping regions
  - Potentially a quadratic number of pairs

- Slow convergence when there are a lot of scans
Simultaneous registration and reconstruction
Simultaneous registration and reconstruction

Huang et al. 2007, Huang and Anguelov 2010
Step I – Latent surface creation

- Fit planes to points associated with each cell
Step II – Scan-surface alignment

\[ T_1 = (R_1, t_1) \]

Latent Surface \((d_k, n_k)\)
Simultaneous registration and reconstruction

\[
\begin{align*}
\arg\min_{\{R_i, t_i\}, \{(d_k, n_k)\}} & \sum_{i=1}^{N} \sum_{j=1}^{N_i} ((R_i p_{ij} + t_i)^T n_{kij} - d_{kij})^2 \\
\text{subject to} & \quad R_1 = I_3, t_1 = 0.
\end{align*}
\]

• Fix the scans to optimize the latent surface

• Fix the latent surface to optimize the scan poses
Simultaneous registration and reconstruction

Works for un-organized point sets

Efficient – no range query
Large-scale registration
Large-scale registration
Topics that are not covered

• Non-rigid registration
  – Will have a guest lecture on this

• Methods that are based on probabilistic modeling

• Other learning-based methods
  – Will talk about this later