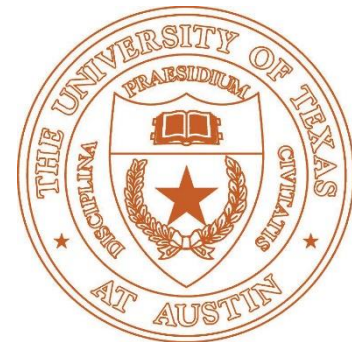


# GAMES

## Point-Based Representation

Qixing Huang  
August 19<sup>th</sup> 2021



# Acknowledgement

- The first part of the slides adopted from

## ***Point-Based Computer Graphics SIGGRAPH 2004 Course Notes***

Marc Alexa, Darmstadt University of Technology

Markus Gross, ETH Zurich

Mark Pauly, Stanford University

Hanspeter Pfister, Mitsubishi Electric Research Laboratories

Marc Stamminger, University of Erlangen-Nuremberg

Matthias Zwicker, Massachusetts Institute of Technology

# Goal

- Learn traditional stuff which will be useful for developing point-based neural networks

**Tzu-Mao Li** @tzumaoli · Aug 13

...

Read the books those papers cited. Learn the basics. Learn Monte Carlo methods. Learn finite element methods. Learn differential geometry. Learn differential equations. Learn how compilers work. Learn linear algebra. Don't say "it's irrelevant". It's usually relevant.

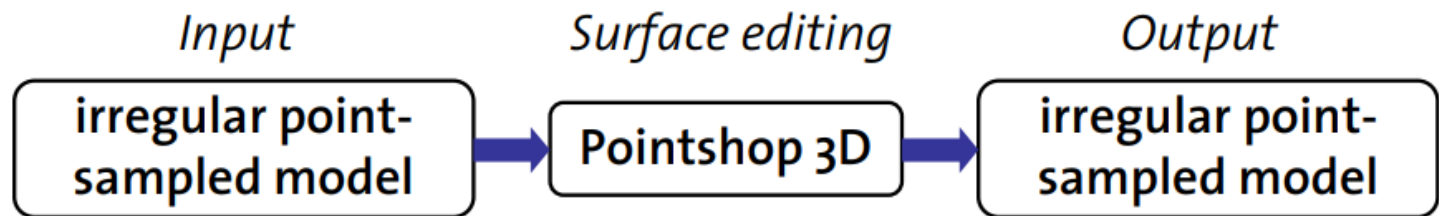
# Pointshop 3D

# Overview

- Introduction
- Pointshop3D System Components
  - Point Cloud Parameterization
  - Resampling Scheme
  - Editing Operations

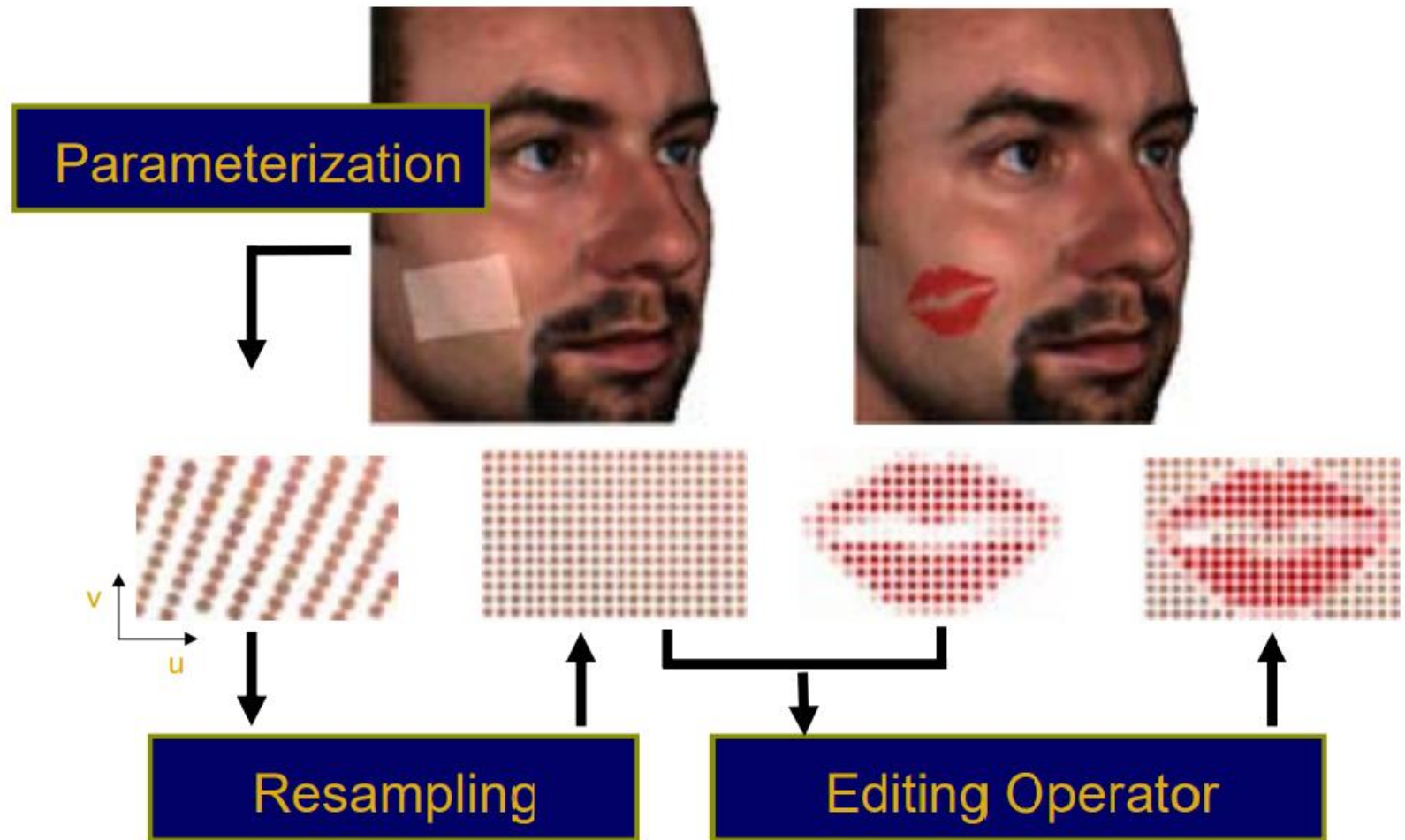
# Pointshop 3D

- Interactive system for point-based surface editing
- Generalize 2D photo editing concepts and functionality to 3D point-sampled surfaces
- Use 3D surface pixels (*surfels*) as versatile display and modeling primitive



- Does not require intermediate triangulation

# Concept



# Key Components

- Point cloud parameterization  $\Phi$ 
  - brings surface and brush into common reference frame
- Dynamic resampling  $\Psi$ 
  - creates one-to-one correspondence of surface and brush samples
- Editing operator  $\Omega$ 
  - combines surface and brush samples

$$S' = \Omega(\Psi(\Phi(S)), \Psi(B))$$

↑                      ↑                      ↑

modified surface      original surface      brush



# Parameterization

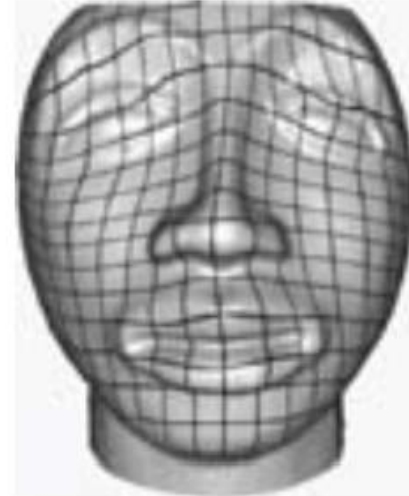
- Constrained minimum distortion parameterization of point clouds

$$\mathbf{u} \in [0,1]^2 \Rightarrow X(\mathbf{u}) = \begin{bmatrix} x(\mathbf{u}) \\ y(\mathbf{u}) \\ z(\mathbf{u}) \end{bmatrix} = \mathbf{x} \in P \subset R^3$$

# Parameterization



constraints = matching  
of feature points



minimum distortion =  
maximum smoothness

# Parameterization

- Find mapping  $X$  that minimizes objective function:

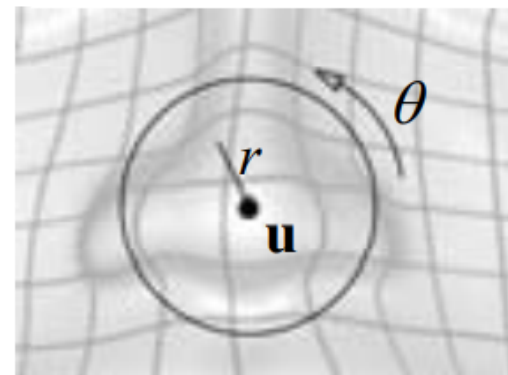
$$C(X) = \sum_{j \in M} \underbrace{(X(\mathbf{p}_j) - \mathbf{x}_j)^2}_{\text{fitting constraints}} + \underbrace{\varepsilon \int_P \gamma(\mathbf{u}) d\mathbf{u}}_{\text{distortion}}$$

brush points      surface points

# Parameterization

- Measuring distortion

$$\gamma(\mathbf{u}) = \int_{\theta} \left( \frac{\partial^2}{\partial r^2} X_{\mathbf{u}}(\theta, r) \right)^2 d\theta$$



- Integrates squared curvature using local polar reparameterization

$$X_{\mathbf{u}}(\theta, r) = X\left(\mathbf{u} + r \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}\right)$$

# Parameterization

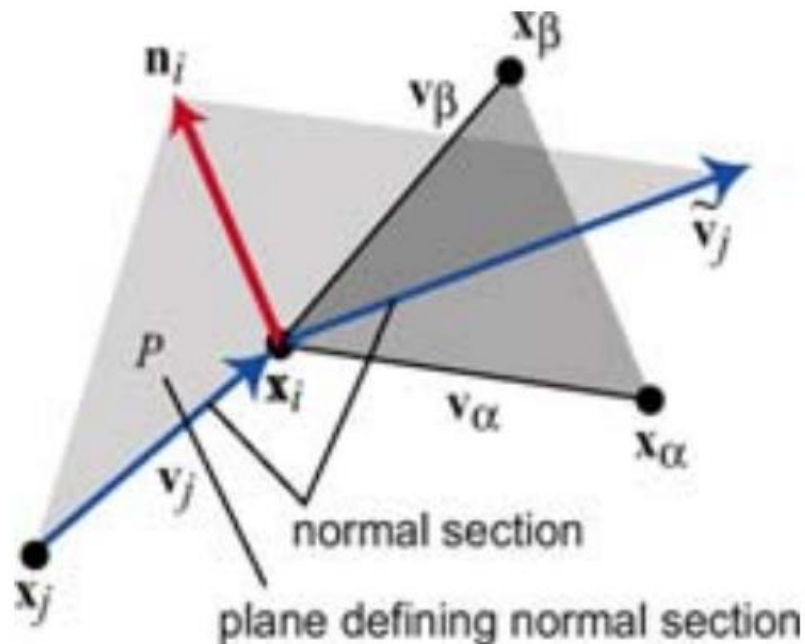
- Discrete formulation:

$$\tilde{C}(U) = \sum_{j \in M} (\mathbf{p}_j - \mathbf{u}_j)^2 + \varepsilon \sum_{i=1}^n \sum_{j \in N_i} \left( \frac{\partial U(\mathbf{x}_i)}{\partial \mathbf{v}_j} - \frac{\partial U(\mathbf{x}_i)}{\partial \tilde{\mathbf{v}}_j} \right)^2$$

- Approximation: mapping is piecewise linear

# Parameterization

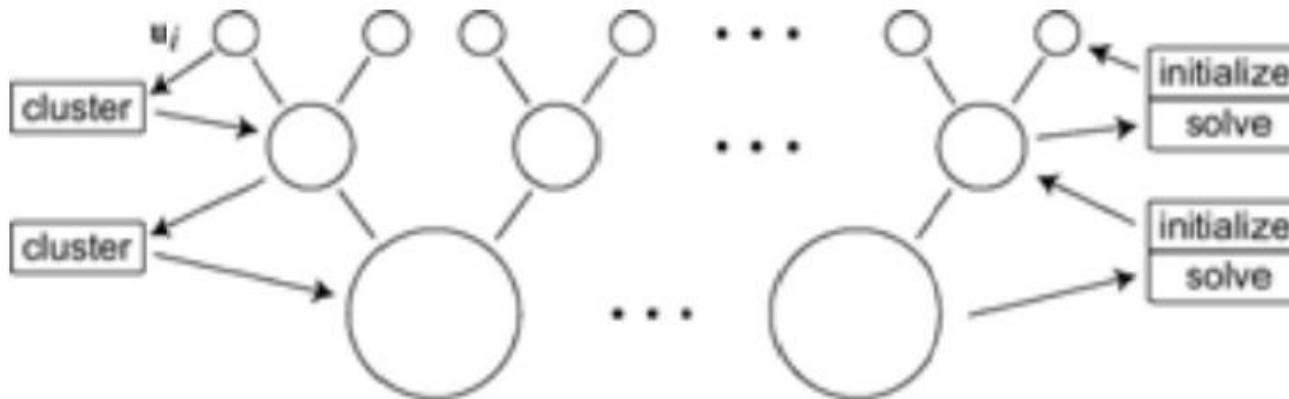
- Directional derivatives as extension of divided differences based on k-nearest neighbors



# Parameterization

- Hierarchical solver for efficient computation of resulting sparse linear least squares problem

$$\tilde{\mathcal{C}}(U) = \sum_j \left( \mathbf{b}_j - \sum_{i=1}^n a_{j,i} \mathbf{u}_i \right)^2 = \|\mathbf{b} - A\mathbf{u}\|^2$$



# Reconstruction

- Parameterized scattered data approximation

$$X(\mathbf{u}) = \frac{\sum_i \Phi_i(\mathbf{u}) r_i(\mathbf{u})}{\sum_i r_i(\mathbf{u})}$$

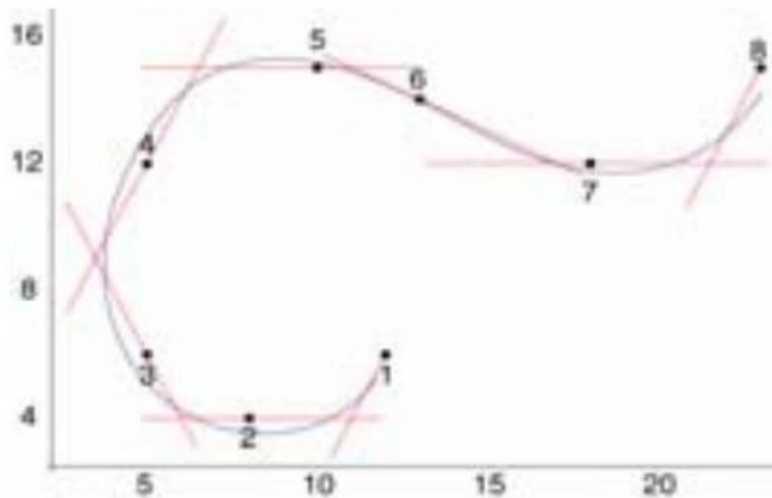
Diagram illustrating the reconstruction formula with annotations:

- fitting functions** (points to  $\Phi_i(\mathbf{u})$ )
- weight functions** (points to  $r_i(\mathbf{u})$  in the numerator)
- normalization** (points to  $\sum_i r_i(\mathbf{u})$  in the denominator)

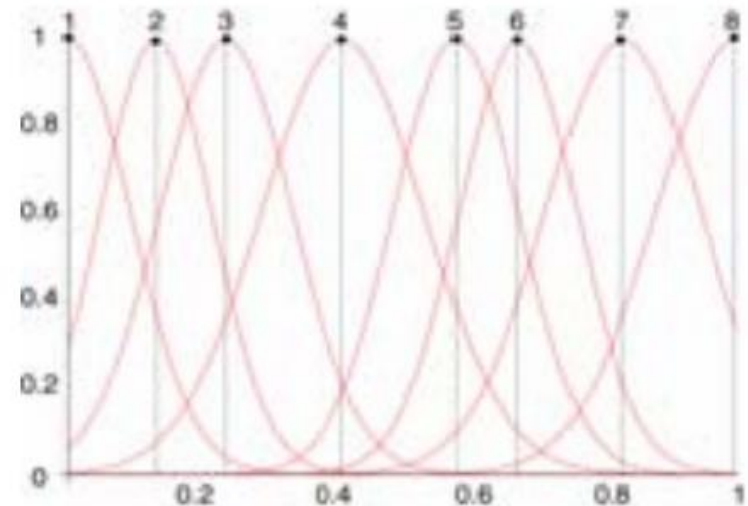
- Fitting functions
  - Compute local fitting functions using local parameterizations
  - Map to global parameterization using global parameter coordinates of neighboring points



# Reconstruction



reconstruction with  
linear fitting functions



weight functions in  
parameter space

# Reconstruction

- Reconstruction with linear fitting functions is equivalent to surface splatting!
  - Use the surface splatting renderer to reconstruct our surface function (see chapter on rendering)
- Provides:
  - Fast evaluation
  - Anti-aliasing (Band-limit the weight functions before sampling using Gaussian low-pass filter)
  - Distortions of splats due to parameterization can be computed efficiently using local affine mappings

# Editing Operators

- Painting
  - Texture, material properties, transparency



# Editing Operators

- Sculpting
  - Carving, normal displacement



texture map



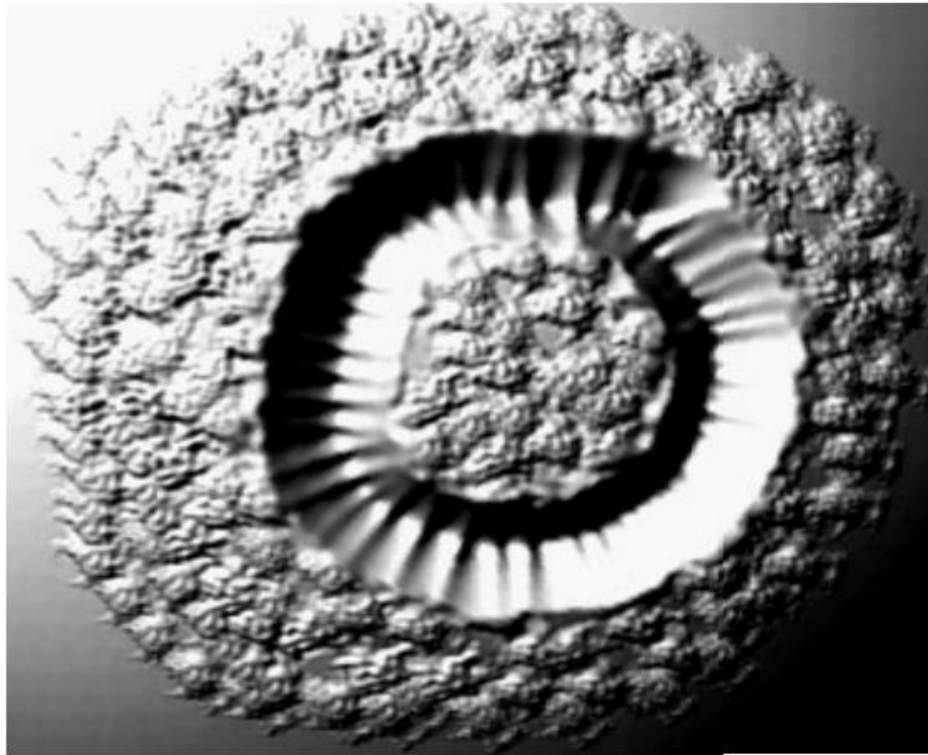
displacement maps



carved and texture mapped  
point-sampled surface

# Editing Operators

- Engraving surface detail



# Editing Operators

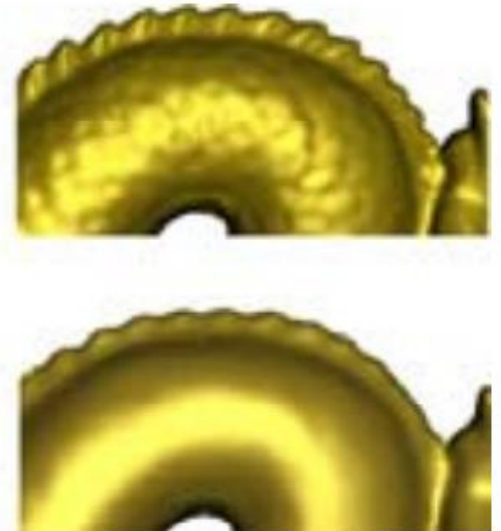
- Filtering appearance and geometry





# Editing Operators

- Filtering appearance and geometry
  - Scalar attributes, geometry



# Advanced Processing

- Multiscale feature extraction





# Point-Cloud Simplification

# Overview

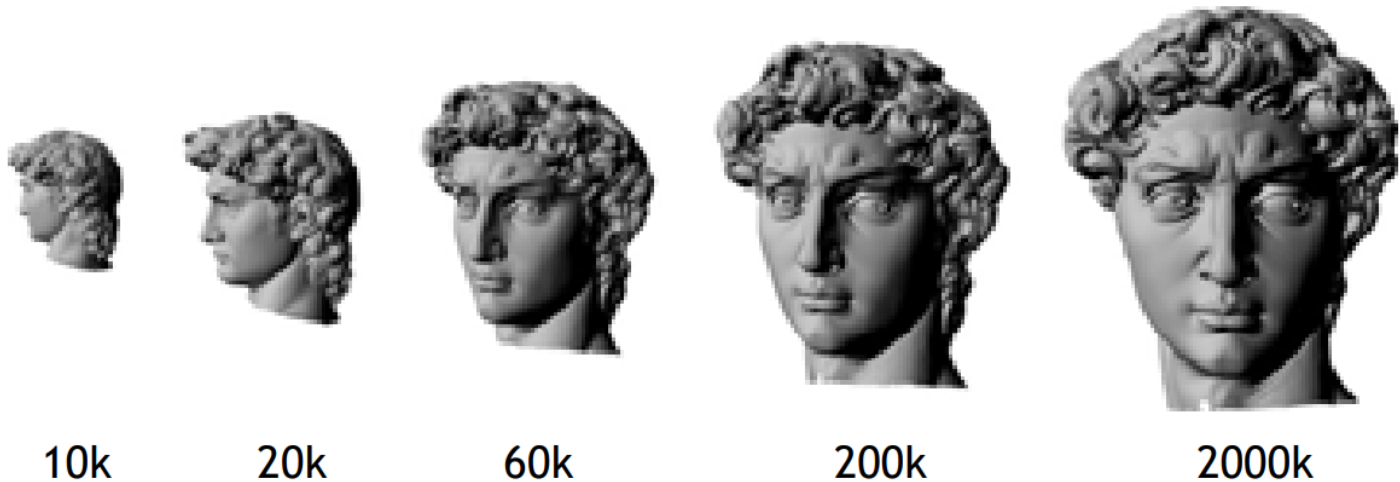
- Introduction
- Local surface analysis
- Simplification methods
- Error measurement
- Comparison

# Introduction

- Point-based models are often sampled very densely
- Many applications require coarser approximations, e.g. for efficient
  - Storage
  - Transmission
  - Processing
  - Rendering
- We need simplification methods for reducing the complexity of point-based surfaces

# Introduction

- Example: Level-of-detail (LOD) rendering



# Introduction

- Different simplification methods from triangle meshes to point clouds:
  - Hierarchical clustering
  - Iterative simplification
  - Particle simulation
- Each method has its pros and cons
- We will talk about mesh simplifications later

# Local Surface Analysis

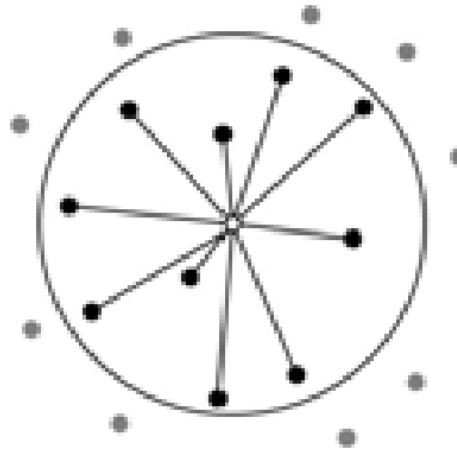
- Cloud of point samples describes underlying (manifold) surface
- We need:
  - Mechanisms for locally approximating the surface -> MLS approach
  - Fast estimation of tangent plane and curvature -  
> principal component analysis of local neighborhood

# Neighborhood

- Replace geodesic proximity with spatial proximity (requires sufficiently high sampling density!)
- Compute neighborhood according to Euclidean distance

# Neighborhood

- K-nearest neighbors

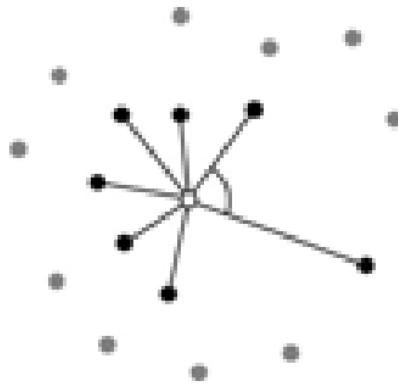


- Can be quickly computed using spatial data-structures (e.g. kd-tree, octree, bsp-tree)
- Requires isotropic point distribution



# Neighborhood

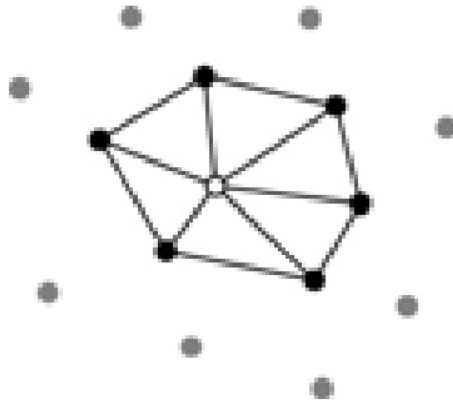
- Improvement: Angle criterion (Linsen)



- Project points onto tangent plane
- Sort neighbors according to angle
- Include more points if angle between subsequent points is above some threshold

# Neighborhood

- Local Delaunay triangulation (Floater)



- Project points into tangent plane
- Compute local Voronoi diagram

# Covariance Analysis

- Covariance matrix of local neighborhood  $N$ :

$$\mathbf{C} = \begin{bmatrix} \mathbf{p}_{i_1} - \bar{\mathbf{p}} \\ \dots \\ \mathbf{p}_{i_n} - \bar{\mathbf{p}} \end{bmatrix}^T \cdot \begin{bmatrix} \mathbf{p}_{i_1} - \bar{\mathbf{p}} \\ \dots \\ \mathbf{p}_{i_n} - \bar{\mathbf{p}} \end{bmatrix}, \quad i_j \in N$$

- with centroid  $\bar{\mathbf{p}} = \frac{1}{|N|} \sum_{i \in N} \mathbf{p}_i$

# Covariance Analysis

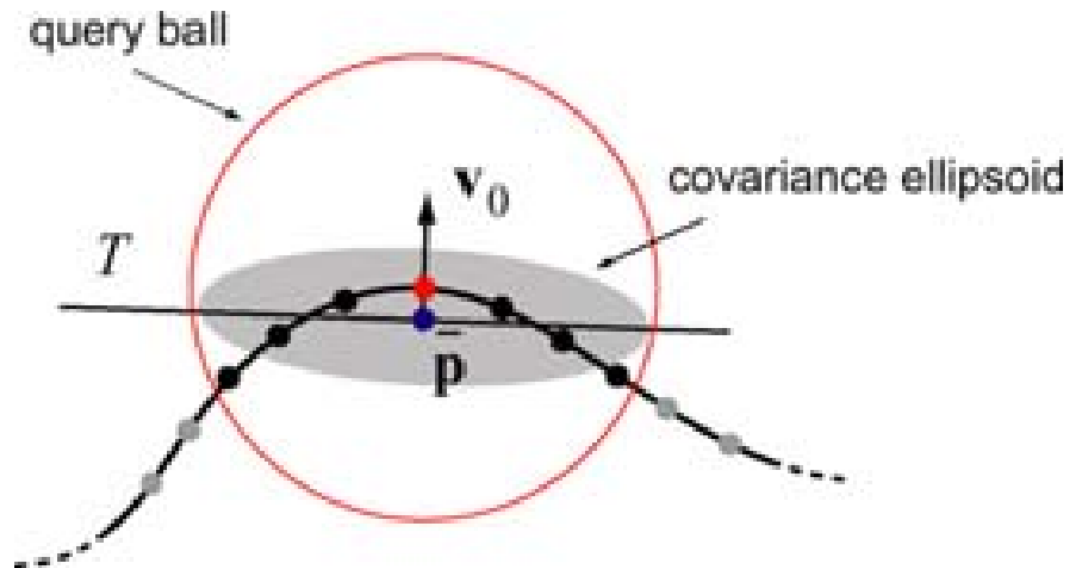
- Consider the eigenproblem:

$$\mathbf{C} \cdot \mathbf{v}_l = \lambda_l \cdot \mathbf{v}_l, \quad l \in \{0,1,2\}$$

- C is a 3x3, positive semi-definite matrix
  - ⇒ All eigenvalues are real-valued
  - ⇒ The eigenvector with smallest eigenvalue defines the least-squares plane through the points in the neighborhood, i.e. approximates the surface normal

# Covariance Analysis

- Covariance ellipsoid spanned by the eigenvectors scaled with corresponding eigenvalue



# Covariance Analysis

- The total variation is given as:

$$\sum_{i \in N} |\mathbf{p}_i - \bar{\mathbf{p}}|^2 = \lambda_0 + \lambda_1 + \lambda_2$$

- We define surface variation as:

$$\sigma_n(\mathbf{p}) = \frac{\lambda_0}{\lambda_0 + \lambda_1 + \lambda_2}, \quad \lambda_0 \leq \lambda_1 \leq \lambda_2$$

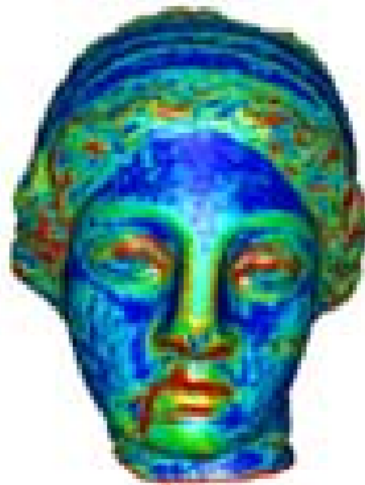
- Measures the fraction of variation along the surface normal, i.e. quantifies how strong the surface deviates from the tangent plane  $\Rightarrow$  estimate for curvature

# Covariance Analysis

- Comparison with curvature:



original



mean curvature



variation n=20



variation n=50

# Surface Simplification

- Hierarchical clustering
- Iterative simplification
- Particle simulation

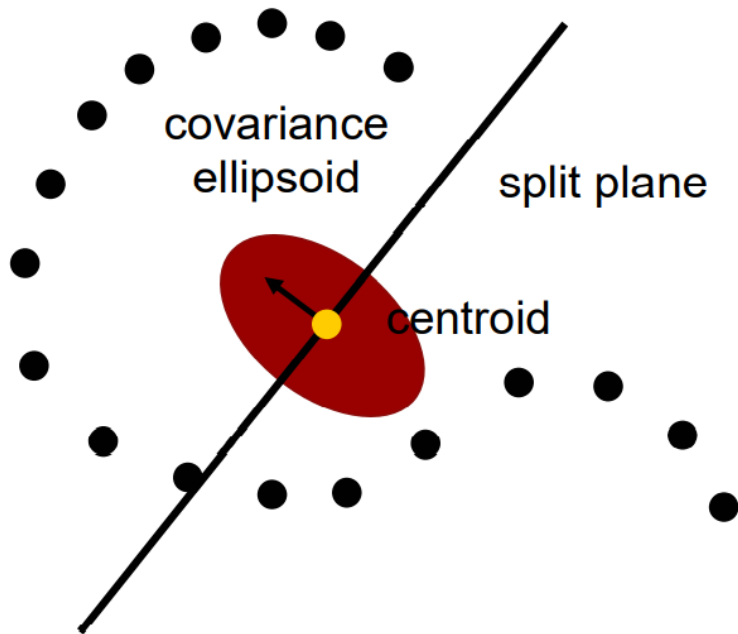


# Hierarchical Clustering

- Top-down approach using binary space partition
- Split the point cloud if:
  - Size is larger than user-specified maximum or
  - Surface variation is above maximum threshold
- Split plane defined by centroid and axis of greatest variation (= eigenvector of covariance matrix with largest associated eigenvector)
- Leaf nodes of the tree correspond to clusters
- Replace clusters by centroid

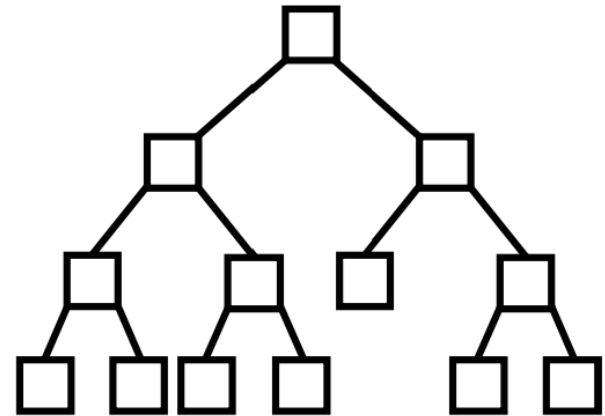
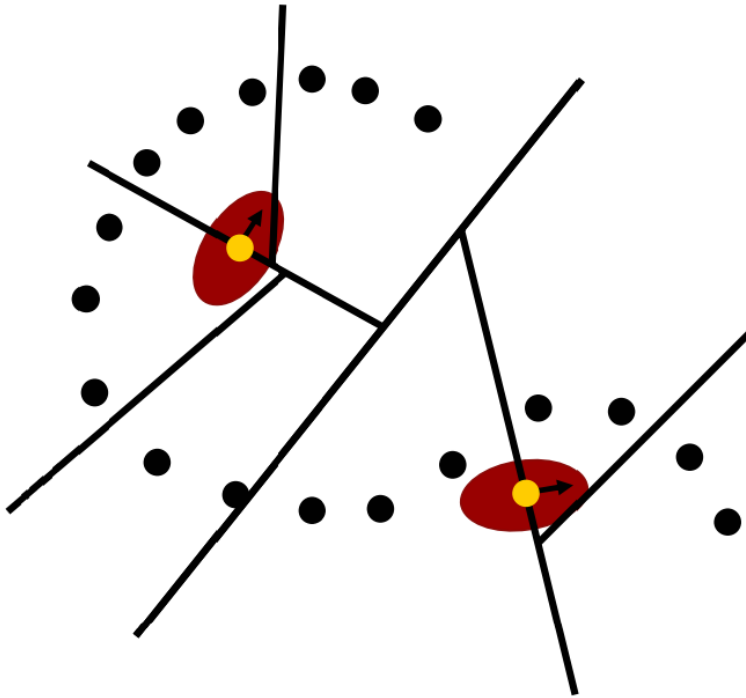
# Hierarchical Clustering

□ root



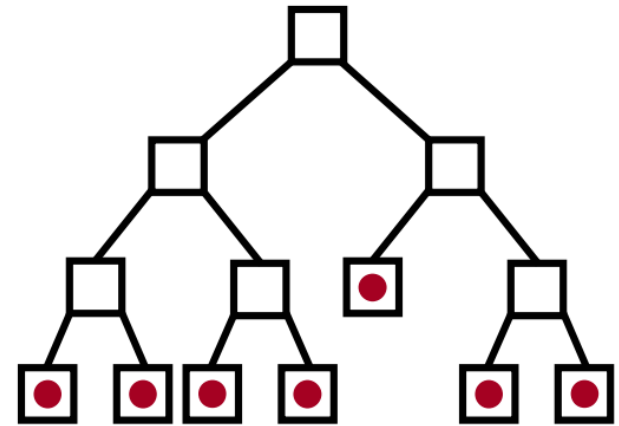
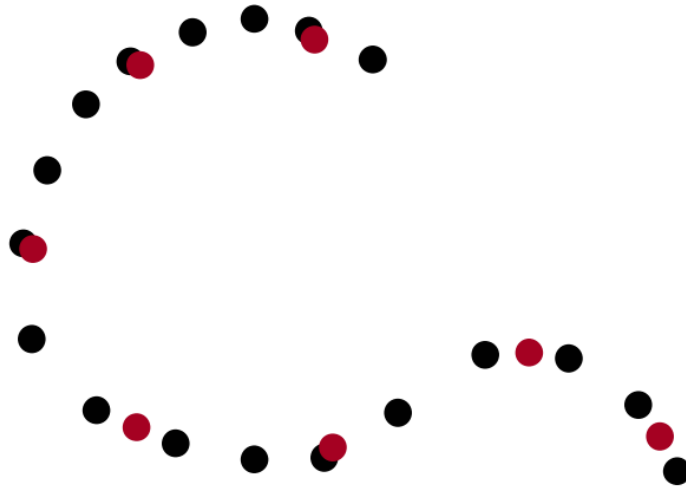
# Hierarchical Clustering

- 2D example



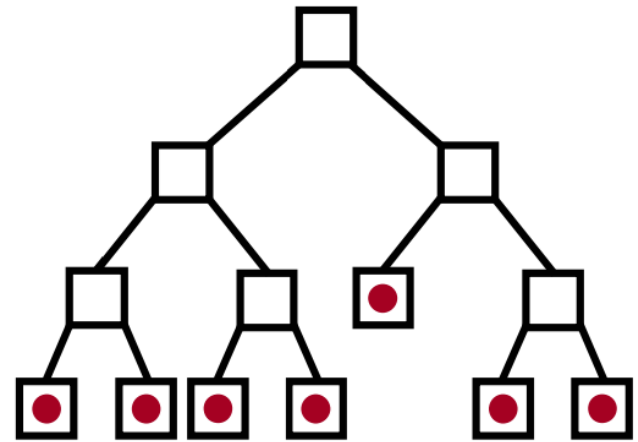
# Hierarchical Clustering

- 2D example



# Hierarchical Clustering

- 2D example



# Hierarchical Clustering



43 Clusters



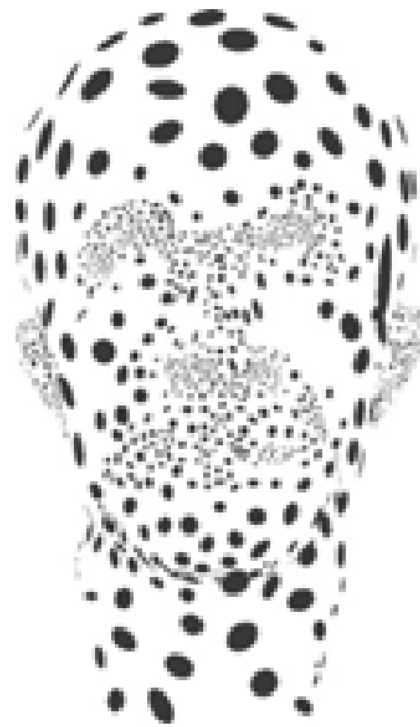
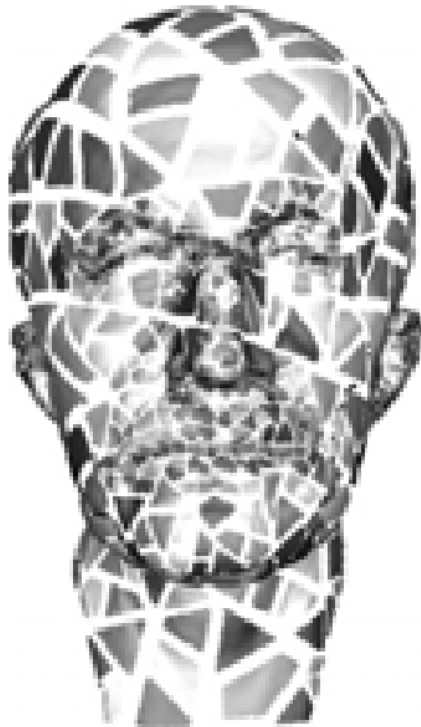
436 Clusters



4,280 Clusters

# Hierarchical Clustering

- Adaptive Clustering



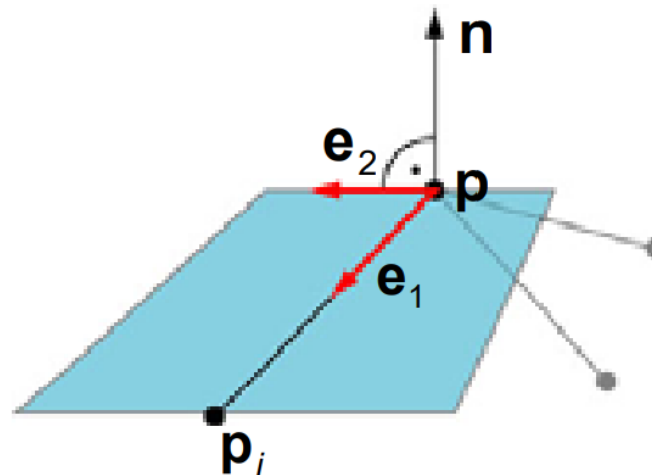
# Iterative Simplification

- Iteratively contracts point pairs
  - Each contraction reduces the number of points by one
- Contractions are arranged in priority queue according to quadric error metric (Garland and Heckbert)
- Quadric measures cost of contraction and determines optimal position for contracted sample
- Equivalent to QSlim except for definition of approximating planes



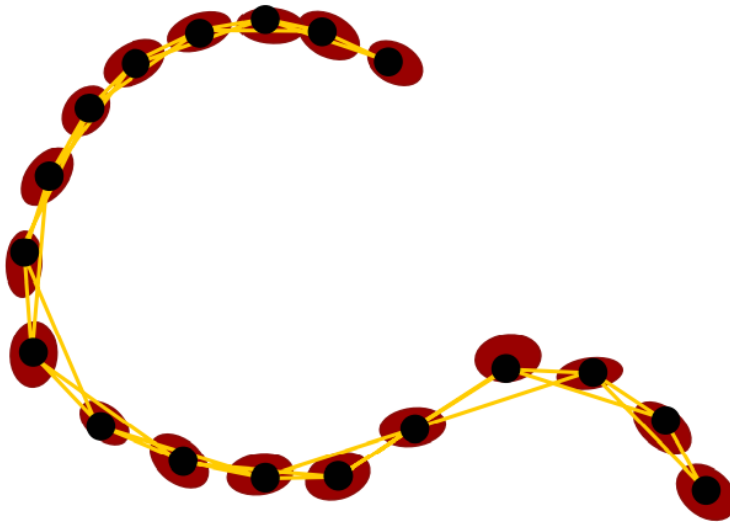
# Iterative Simplification

- Quadric measures the squared distance to a set of planes defined over *edges* of neighborhood
  - plane spanned by vectors  $\mathbf{e}_1 = \mathbf{p}_i - \mathbf{p}$  and  $\mathbf{e}_2 = \mathbf{e}_1 \times \mathbf{n}$



# Iterative Simplification

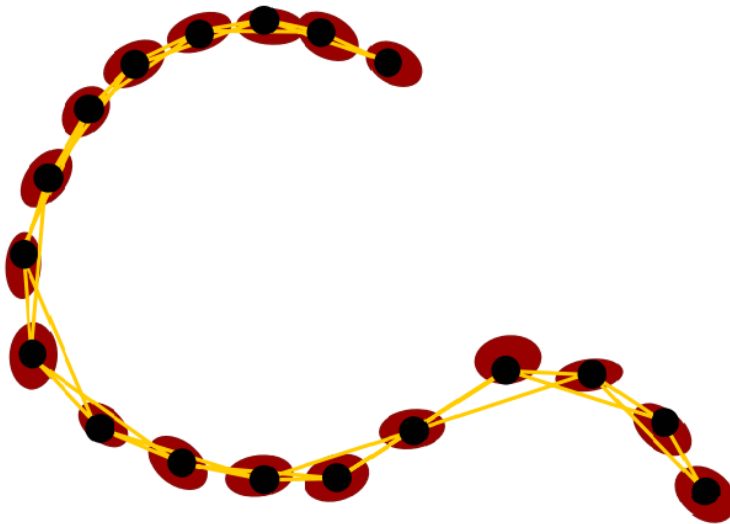
- 2D example



- Compute initial point-pair contraction candidates
- Compute fundamental quadrics
- Compute edge costs

# Iterative Simplification

- 2D example

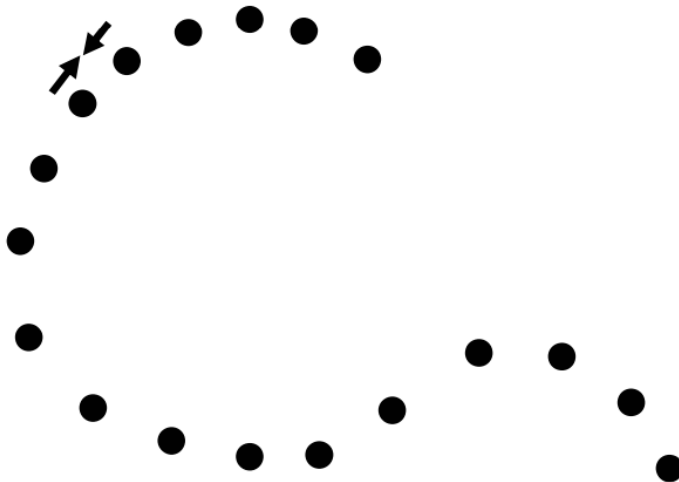


priority queue

| edge | cost |
|------|------|
| 6    | 0.02 |
| 2    | 0.03 |
| 14   | 0.04 |
| 5    | 0.04 |
| 9    | 0.09 |
| 1    | 0.11 |
| 13   | 0.13 |
| 3    | 0.22 |
| 11   | 0.27 |
| 10   | 0.36 |
| 7    | 0.44 |
| 4    | 0.56 |

# Iterative Simplification

- 2D example

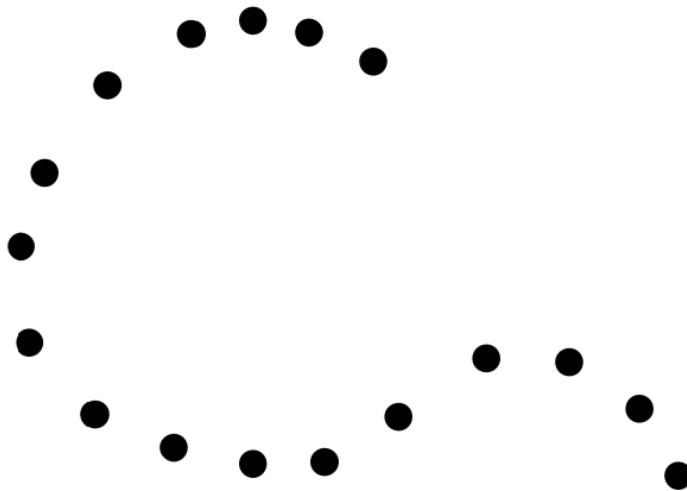


→

| priority queue |      |
|----------------|------|
| edge           | cost |
| 6              | 0.02 |
| 2              | 0.03 |
| 14             | 0.04 |
| 5              | 0.04 |
| 9              | 0.09 |
| 1              | 0.11 |
| 13             | 0.13 |
| 3              | 0.22 |
| 11             | 0.27 |
| 10             | 0.36 |
| 7              | 0.44 |
| 4              | 0.56 |

# Iterative Simplification

- 2D example



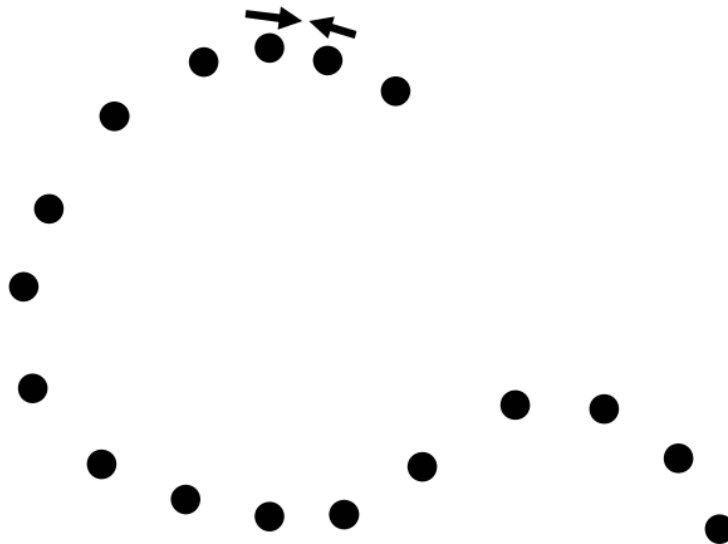
priority queue

edge      cost

|     |      |
|-----|------|
| → 6 | 0.02 |
| 2   | 0.03 |
| 14  | 0.04 |
| 5   | 0.06 |
| 9   | 0.09 |
| 1   | 0.11 |
| 13  | 0.13 |
| 3   | 0.23 |
| 11  | 0.27 |
| 10  | 0.36 |
| 7   | 0.49 |
| 4   | 0.56 |

# Iterative Simplification

- 2D example

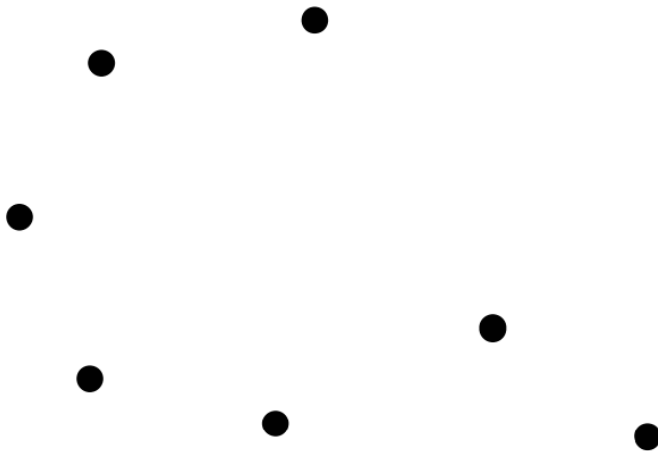


priority queue

| edge | cost |
|------|------|
| 2    | 0.03 |
| 14   | 0.04 |
| 5    | 0.06 |
| 9    | 0.09 |
| 1    | 0.11 |
| 13   | 0.13 |
| 3    | 0.23 |
| 11   | 0.27 |
| 10   | 0.36 |
| 7    | 0.49 |
| 4    | 0.56 |

# Iterative Simplification

- 2D example

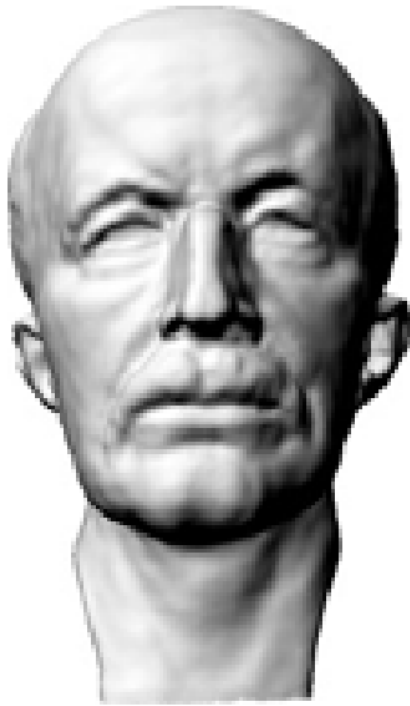


priority queue

edge      cost

|    |      |
|----|------|
| 11 | 0.27 |
| 10 | 0.36 |
| 7  | 0.49 |
| 4  | 0.56 |

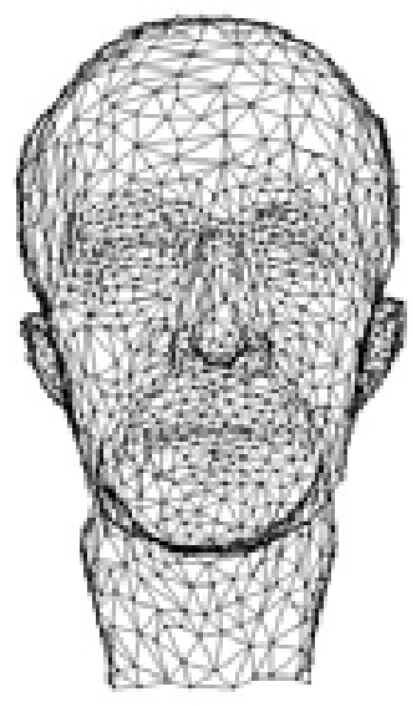
# Iterative Simplification



original model  
(296,850 points)



simplified model  
(2,000 points)



remaining point pair  
contraction candidates



# Particle Simulation

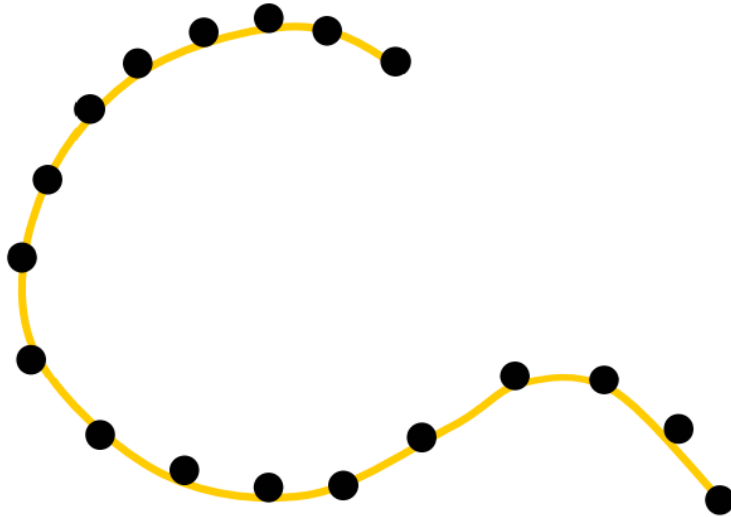
- Resample surface by distributing particles on the surface
- Particles move on surface according to interparticle repelling forces
- Particle relaxation terminates when equilibrium is reached (requires damping)
- Can also be used for up-sampling!

# Particle Simulation

- Initialization
  - Randomly spread particles
- Repulsion
  - Linear repulsion force  $F_i(\mathbf{p}) = k(r - \|\mathbf{p} - \mathbf{p}_i\|) \cdot (\mathbf{p} - \mathbf{p}_i)$   
 $\Rightarrow$  only need to consider neighborhood of radius  $r$
- Projection
  - Keep particles on surface by projecting onto tangent plane of closest point
  - Apply full MLS projection at end of simulation

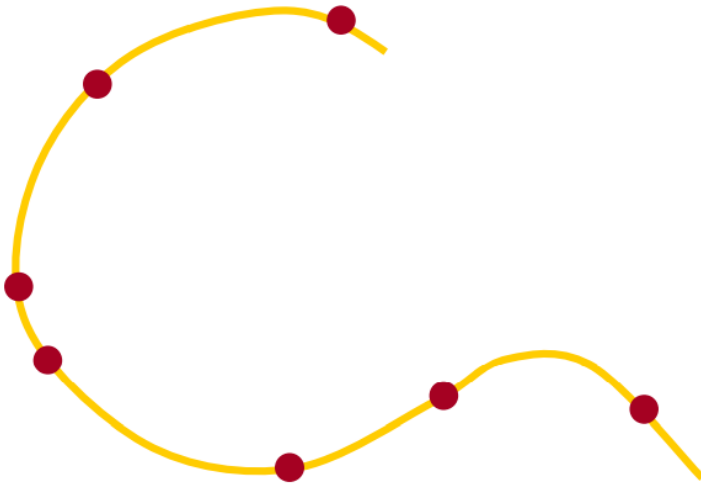
# Particle Simulation

- 2D example



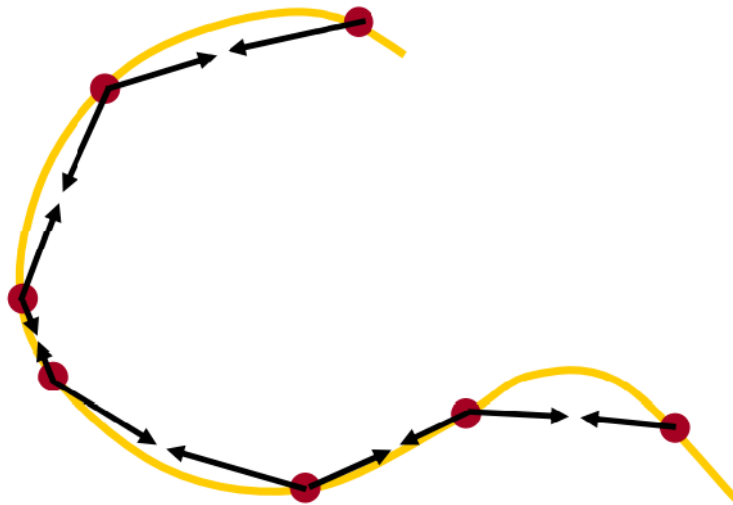
# Particle Simulation

- 2D example
- Initialization
  - randomly spread particles



# Particle Simulation

- 2D example



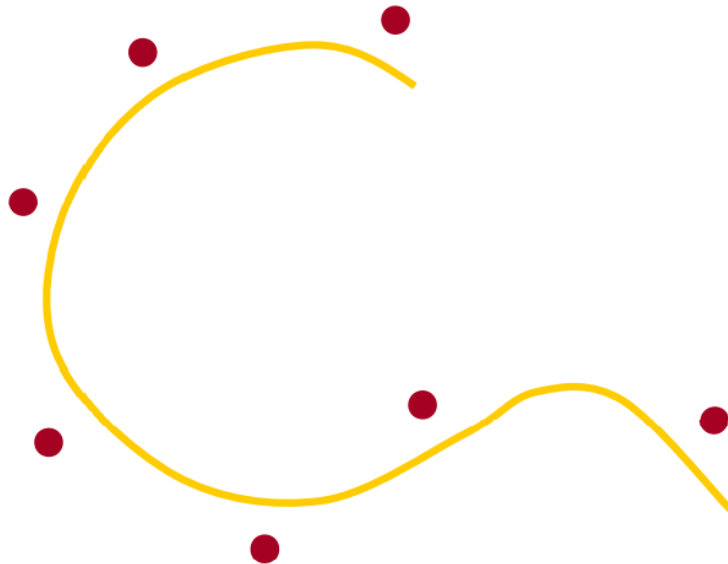
- Initialization
  - randomly spread particles

- Repulsion
  - linear repulsion force

$$F_i(\mathbf{p}) = k(r - \|\mathbf{p} - \mathbf{p}_i\|) \cdot (\mathbf{p} - \mathbf{p}_i)$$

# Particle Simulation

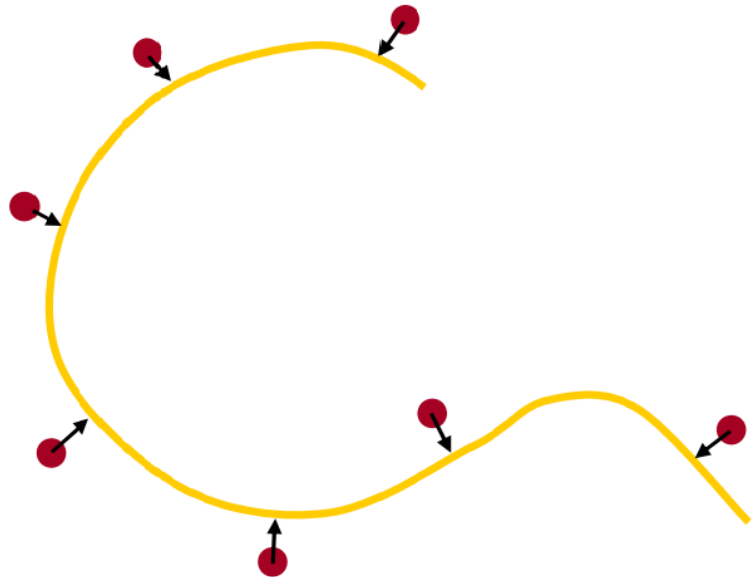
- 2D example



- Initialization
  - randomly spread particles
- Repulsion
  - linear repulsion force
$$F_i(\mathbf{p}) = k(r - \|\mathbf{p} - \mathbf{p}_i\|) \cdot (\mathbf{p} - \mathbf{p}_i)$$

# Particle Simulation

- 2D example



- Initialization
  - randomly spread particles

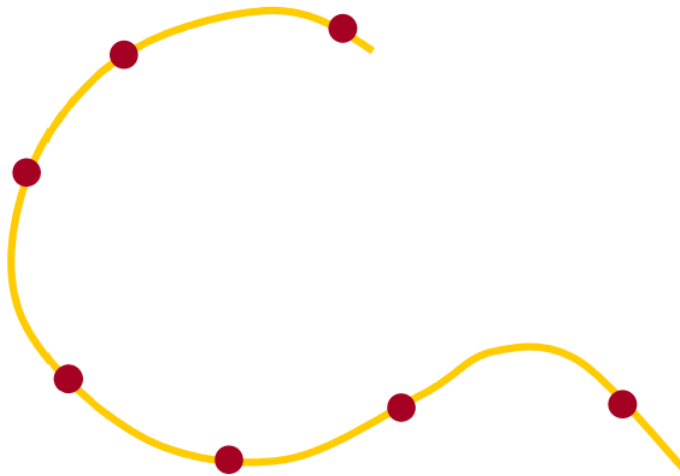
- Repulsion
  - linear repulsion force

$$F_i(\mathbf{p}) = k(r - \|\mathbf{p} - \mathbf{p}_i\|) \cdot (\mathbf{p} - \mathbf{p}_i)$$

- Projection
  - project particles onto surface

# Particle Simulation

- 2D example



- Initialization
  - randomly spread particles
- Repulsion
  - linear repulsion force
$$F_i(\mathbf{p}) = k(r - \|\mathbf{p} - \mathbf{p}_i\|) \cdot (\mathbf{p} - \mathbf{p}_i)$$
- Projection
  - project particles onto surface

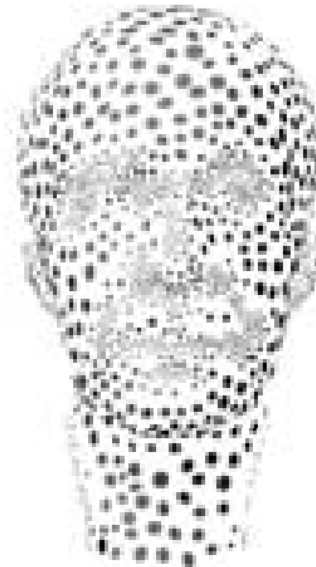


# Particle Simulation

- Adjust repulsion radius according to surface variation  $\Rightarrow$  more samples in regions of high variation



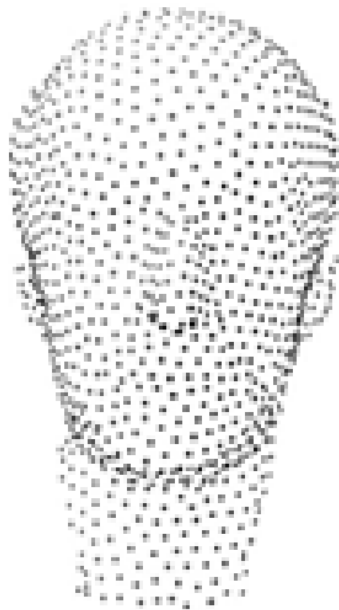
variation  
estimation



simplified model  
(3,000 points)

# Particle Simulation

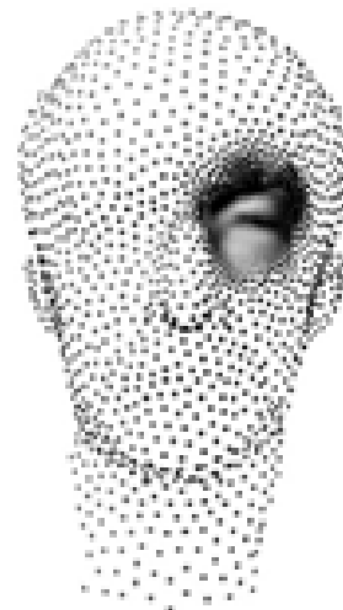
- User-controlled simulation
  - Adjust repulsion radius according to user input



uniform



original



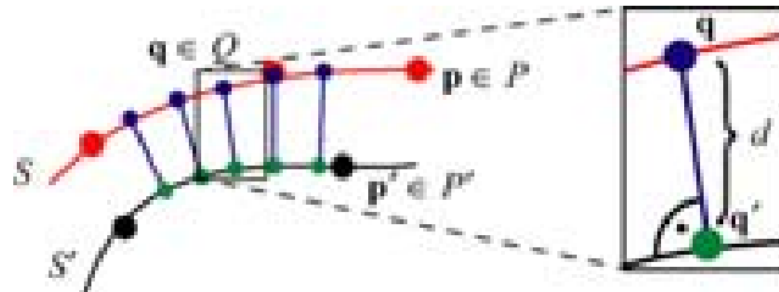
selective

# Measuring Error

- Measure the distance between two point-sampled surfaces using a sampling approach
- Maximum error:  $\Delta_{\max}(S, S') = \max_{\mathbf{q} \in Q} d(\mathbf{q}, S')$ 
  - $\Rightarrow$  Two-sided Hausdorff distance
- Mean error:  $\Delta_{\text{avg}}(S, S') = \frac{1}{|Q|} \sum_{\mathbf{q} \in Q} d(\mathbf{q}, S')$ 
  - $\Rightarrow$  Area-weighted integral of point-to-surface distances
- $Q$  is an up-sampled version of the point cloud that describes the surface  $S$

# Measuring Error

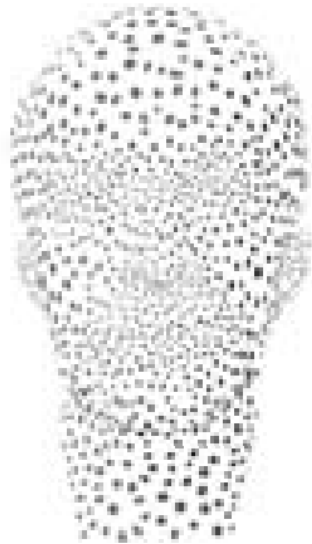
- $d(\mathbf{q}, S)$  approximates the distance of point  $\mathbf{q}$  to surface  $S$  using the MLS projection operator



# Measuring Error



original



simplified



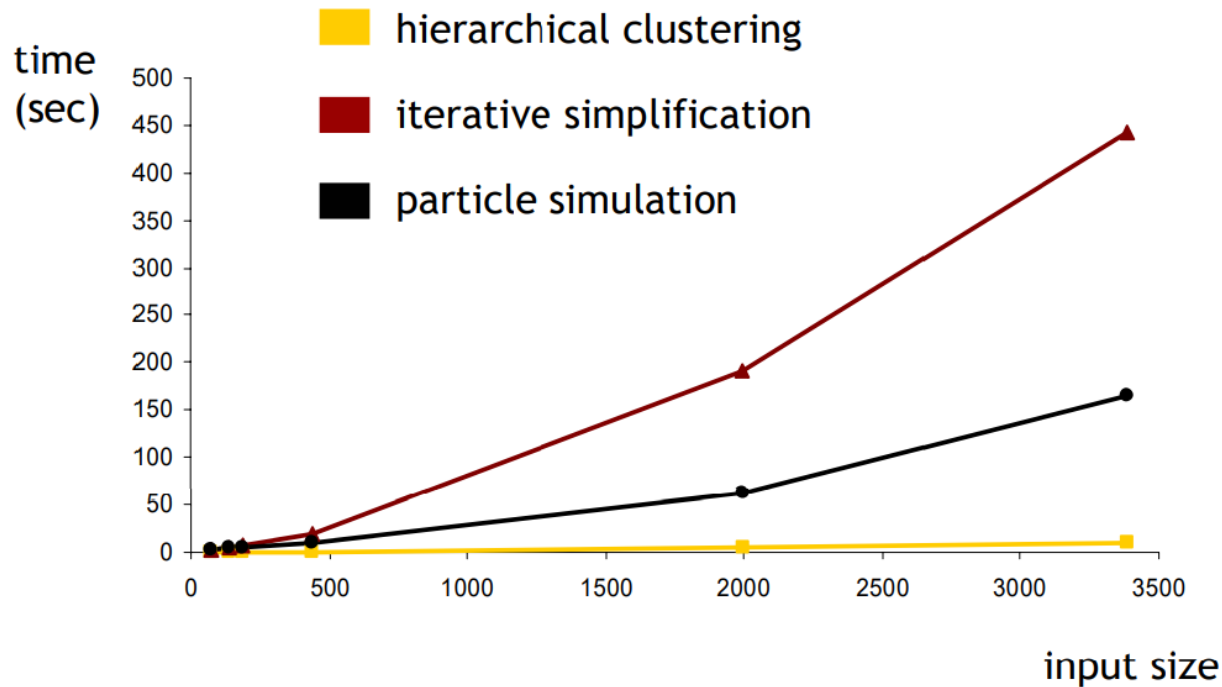
upsampled



error

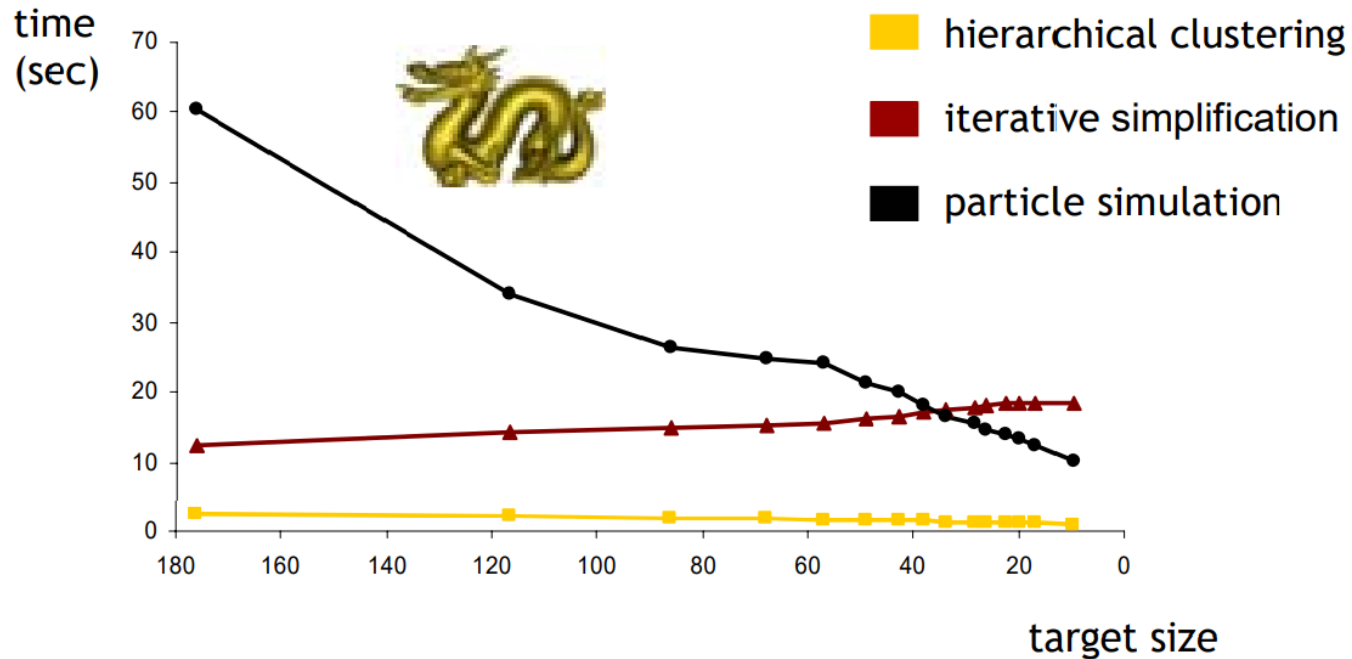
# Comparison

- Execution time as a function of input model size (reduction to 1%)



# Comparison

- Execution time as a function of target model size (input: dragon, 535,545 points)



# Comparison

- Summary

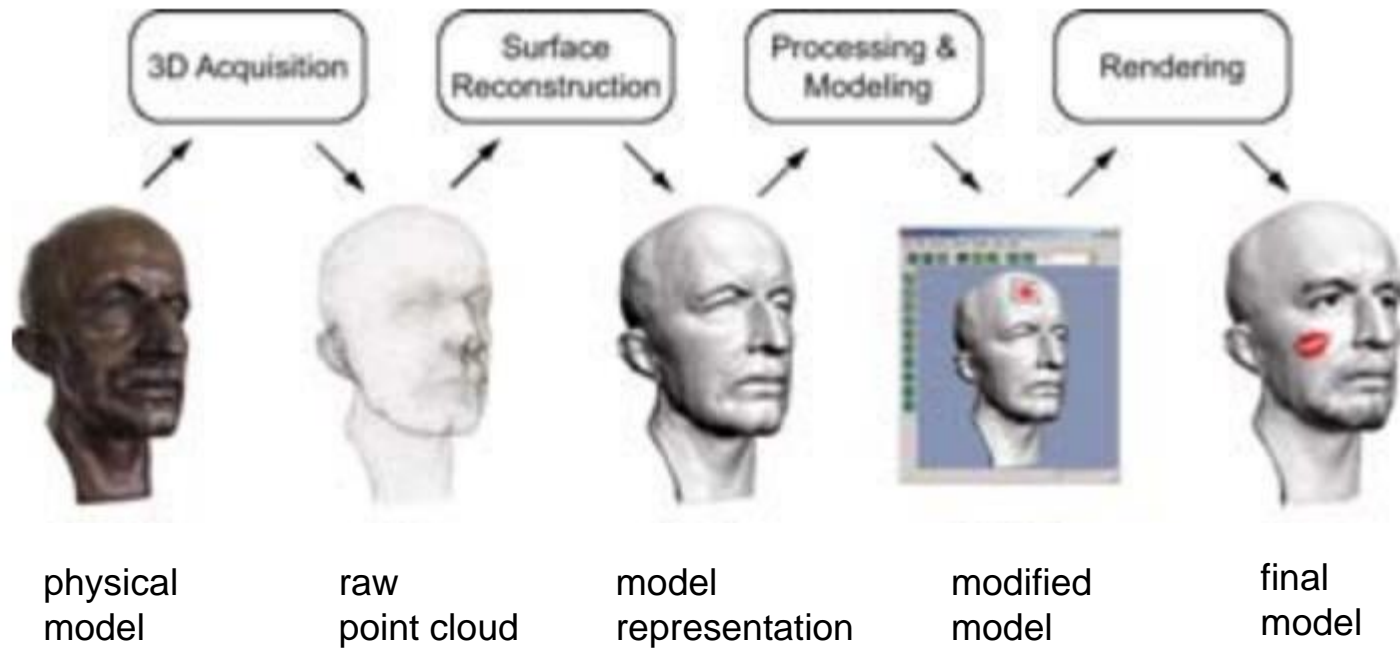
|                          | Efficiency | Surface Error | Control | Implementation |
|--------------------------|------------|---------------|---------|----------------|
| Hierarchical Clustering  | +          | -             | -       | +              |
| Iterative Simplification | -          | +             | o       | o              |
| Particle Simulation      | o          | +             | +       | -              |



# Shape Modeling

# Motivation

- 3D content creation pipeline



# Surface Representations

- Explicit surfaces (B-reps)
  - Polygonal meshes
  - Subdivision surfaces
  - NURBS
- Limitations
  - Efficient rendering
  - Sharp edges
  - Intuitive editing

# Surface Representations

- Implicit surfaces
  - Level sets
  - Radial basis functions
  - Algebraic surfaces
- Limitations
  - Boolean operations
  - Changes of topology
  - Extreme deformations

# Hybrid Representation

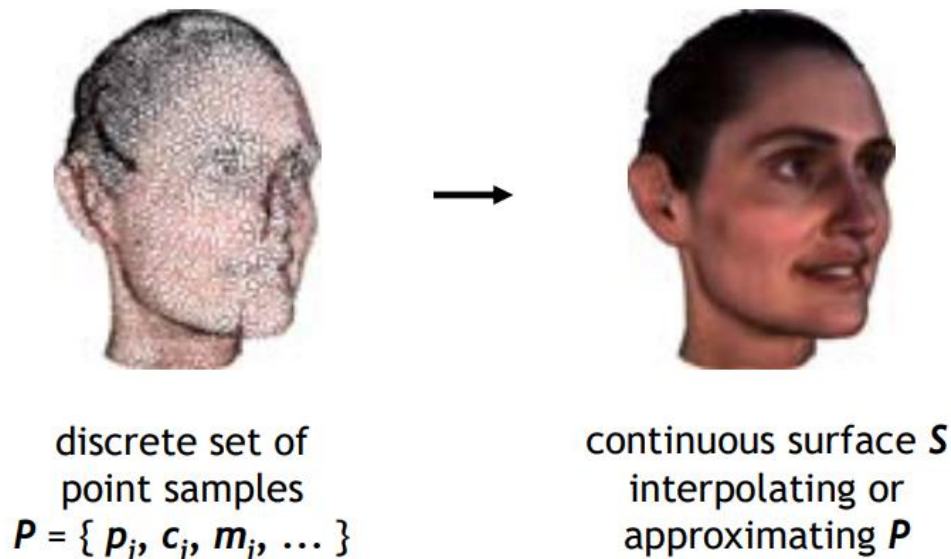
- Goals
  - Explicit cloud of point samples
  - Implicit dynamic surface model
- Point cloud representation
  - Minimal consistency requirements for extreme deformations (dynamic re-sampling)
  - Fast inside/outside classification for boolean operations and collision detection
  - Explicit modeling and rendering of sharp feature curves
  - Integrated, intuitive editing of shape and appearance

# Interactive Modeling

- Interactive design and editing of point-sampled models
  - Shape modeling
    - Boolean operations
    - Free-form deformation
  - Appearance modeling
    - Painting & texturing
    - Embossing & engraving

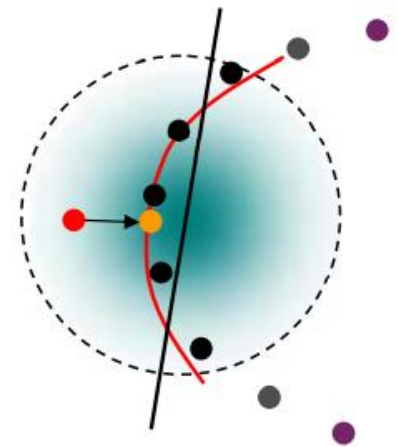
# Surface Model

- Goal: Define continuous surface from a set of discrete point samples



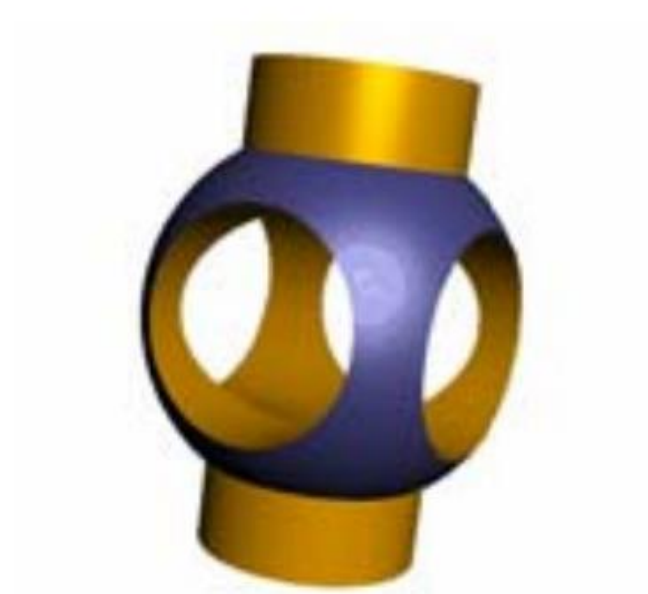
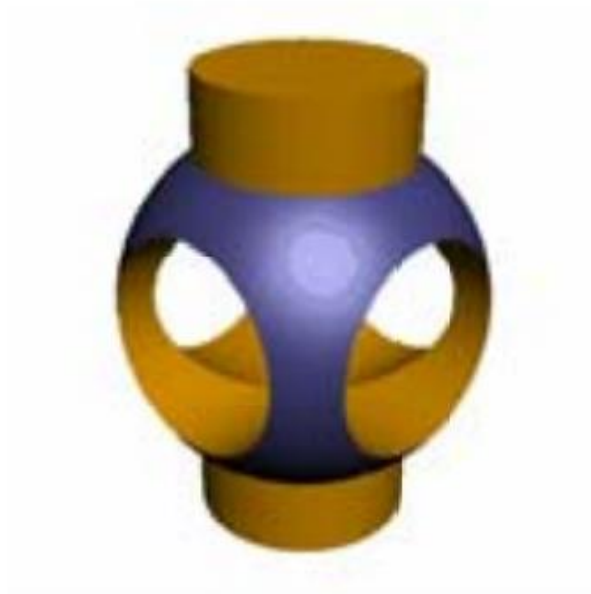
# Surface Model

- Moving least squares (MLS) approximation (Levin, Alexa et al.)
  - Surface defined as stationary set of projection operator
$$S_P = \{x \in \mathbf{R}^3 \mid \Psi_P(x) = x\}$$
  - Weighted least squares optimization
    - Gaussian kernel function
      - local, smooth
      - mesh-less, adaptive





# Boolean Operations



# Boolean Operations

- Create new shapes by combining existing models using union, intersection, or difference operations
- Powerful and flexible editing paradigm mostly used in industrial design applications (CAD/CAM)

# Boolean Operations

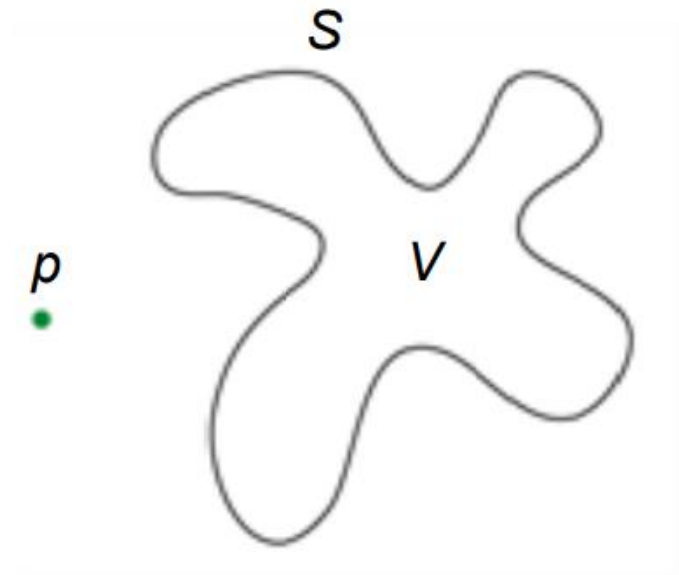
- Easily performed on implicit representations
  - Requires simple computations on the distance function
- Difficult for parametric surfaces
  - Requires surface-surface intersection
- Topological complexity of resulting surface depends on geometric complexity of input models

# Boolean Operations

- Point-Sampled Geometry
  - Classification
    - Inside-outside test using signed distance function induced by MLS projection
  - Sampling
    - Compute exact intersection of two MLS surfaces to sample the intersection curve
  - Rendering
    - Accurate depiction of sharp corners and creases using point-based rendering

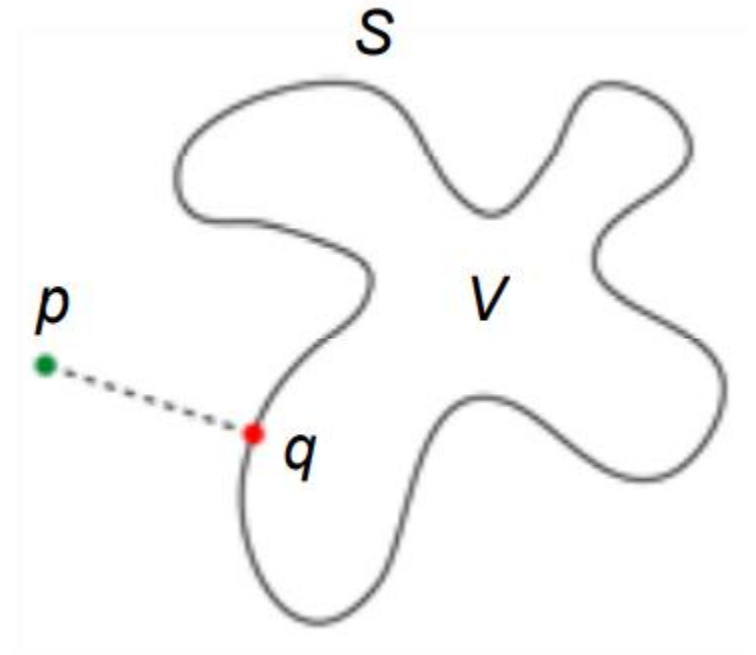
# Boolean Operations

- Classification:
  - Given a smooth, closed surface  $S$  and point  $p$ . Is  $p$  inside or outside of the volume  $V$  bounded by  $S$ ?



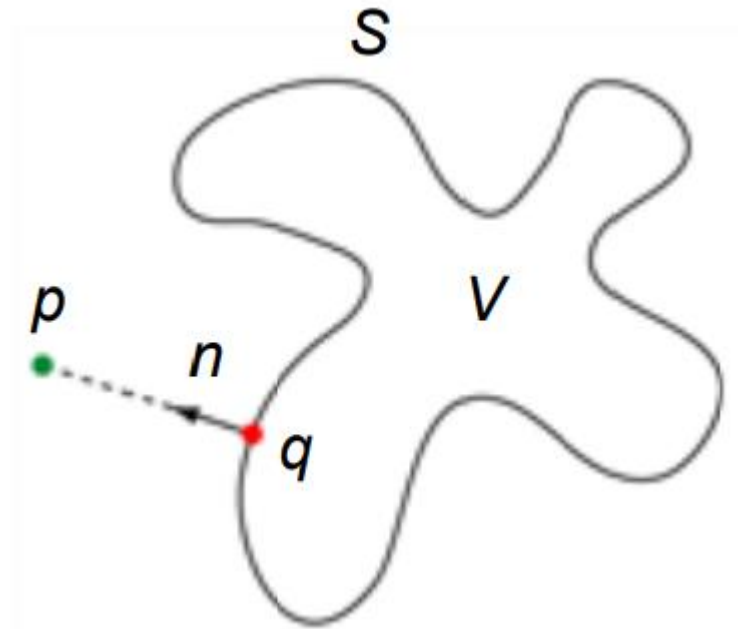
# Boolean Operations

- Classification:
  - Given a smooth, closed surface  $S$  and point  $p$ . Is  $p$  inside or outside of the volume  $V$  bounded by  $S$ ?
  - 1. find closest point  $q$  on  $S$



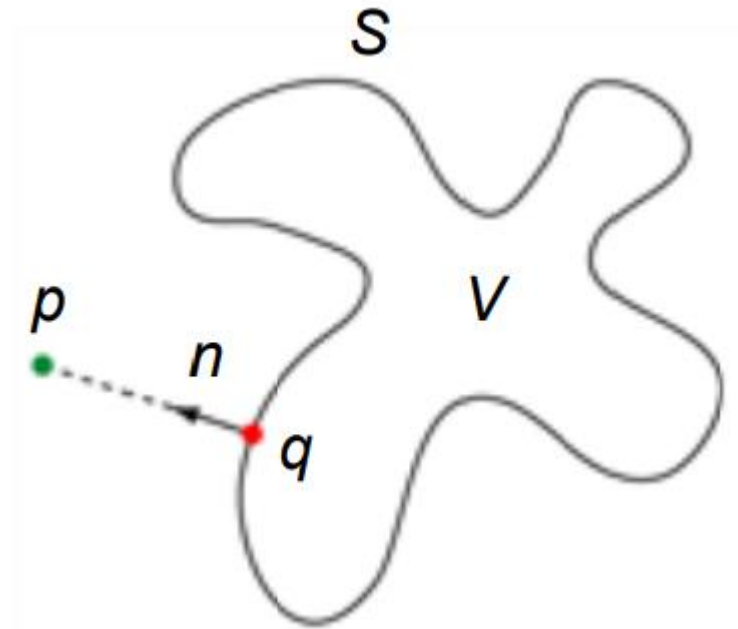
# Boolean Operations

- Classification:
  - Given a smooth, closed surface  $S$  and point  $p$ . Is  $p$  inside or outside of the volume  $V$  bounded by  $S$ ?
  - 1. find closest point  $q$  on  $S$
  - 2.  $d = (p-q)' \cdot n$  defines signed distance of  $p$  to  $S$



# Boolean Operations

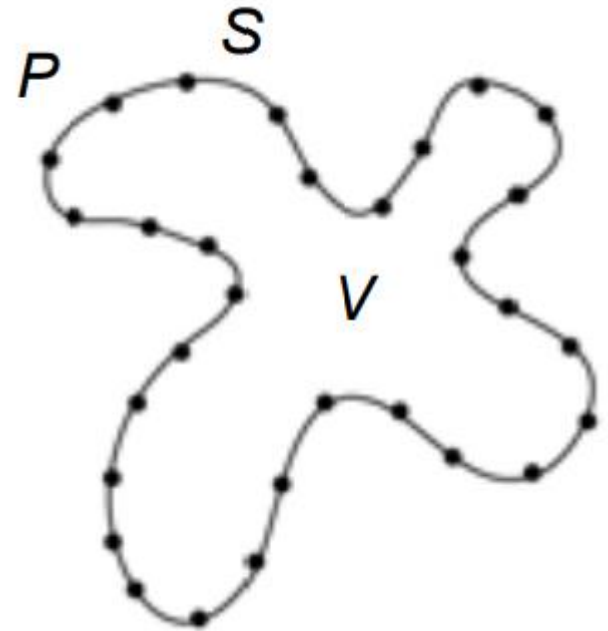
- Classification:
  - Given a smooth, closed surface  $S$  and point  $p$ . Is  $p$  inside or outside of the volume  $V$  bounded by  $S$ ?
  - 1. find closest point  $q$  on  $S$
  - 2.  $d = (p-q)' \cdot n$  defines signed distance of  $p$  to  $S$
  - 3. classify  $p$  as
    - inside  $V$ , if  $d < 0$
    - Outside  $V$ , if  $d > 0$





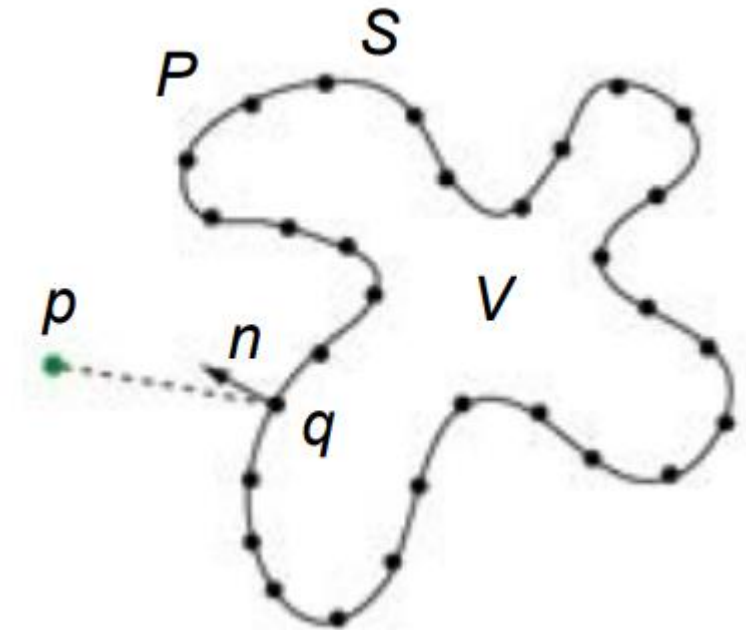
# Boolean Operations

- Classification:
  - Represent smooth surface  $S$  by point cloud  $P$



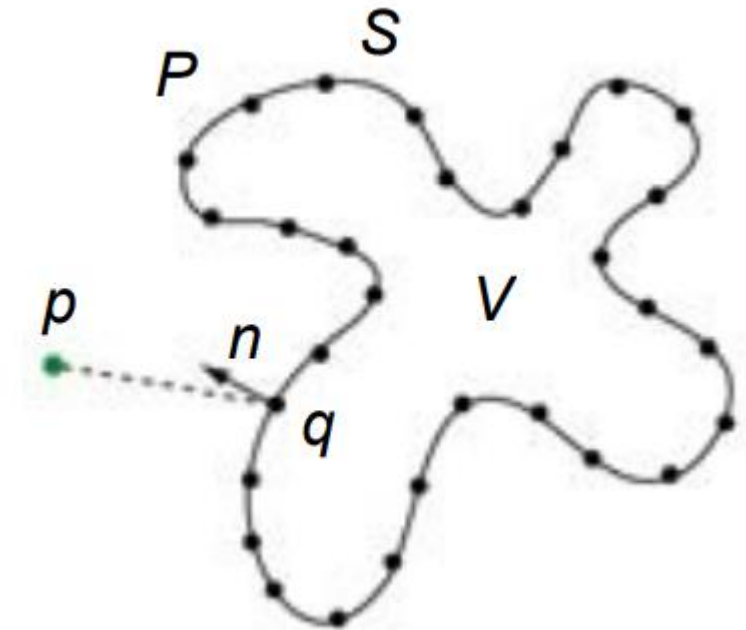
# Boolean Operations

- Classification:
  - Represent smooth surface  $S$  by point cloud  $P$
  - 1. find closest point  $q$  in  $P$



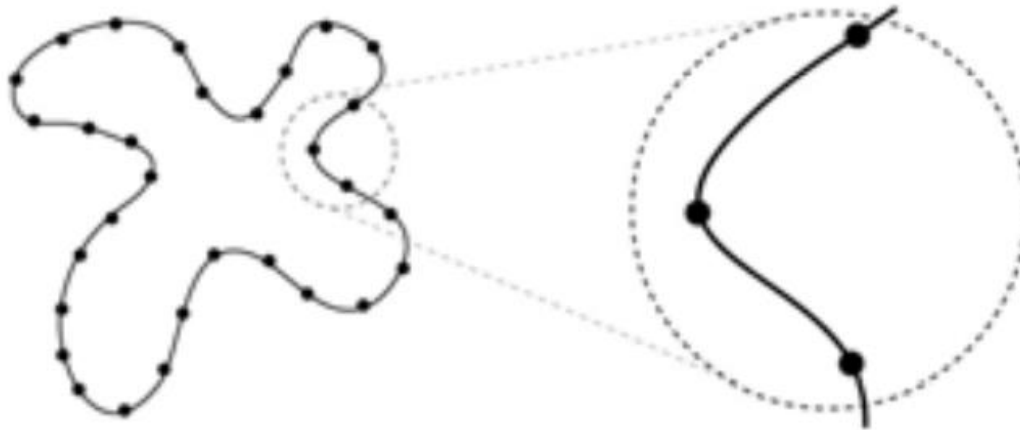
# Boolean Operations

- Classification:
  - Represent smooth surface  $S$  by point cloud  $P$
  - 1. find closest point  $q$  in  $P$
  - 2. classify  $p$  as
    - inside  $V$ , if  $(p-q)' \cdot n < 0$
    - outside  $V$ , if  $(p-q)' \cdot n > 0$



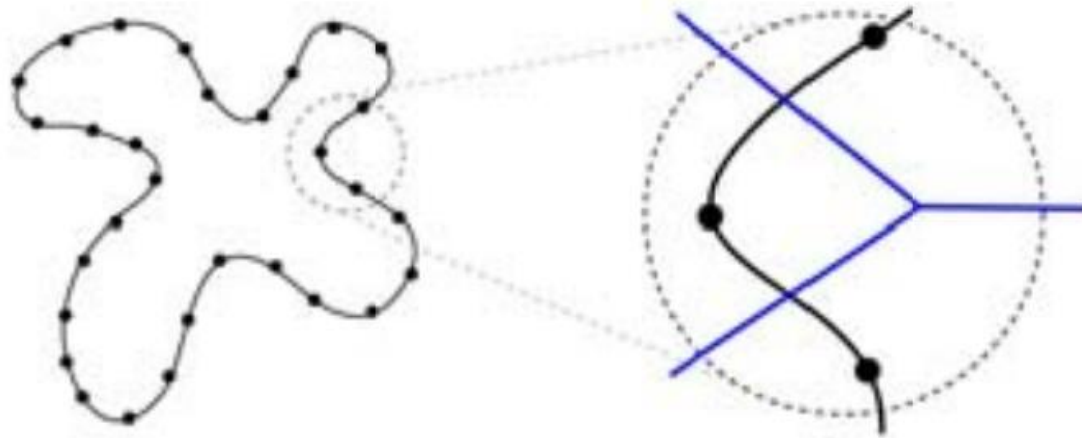
# Boolean Operations

- Classification:
  - piecewise constant surface approximation leads to false classification close to the surface



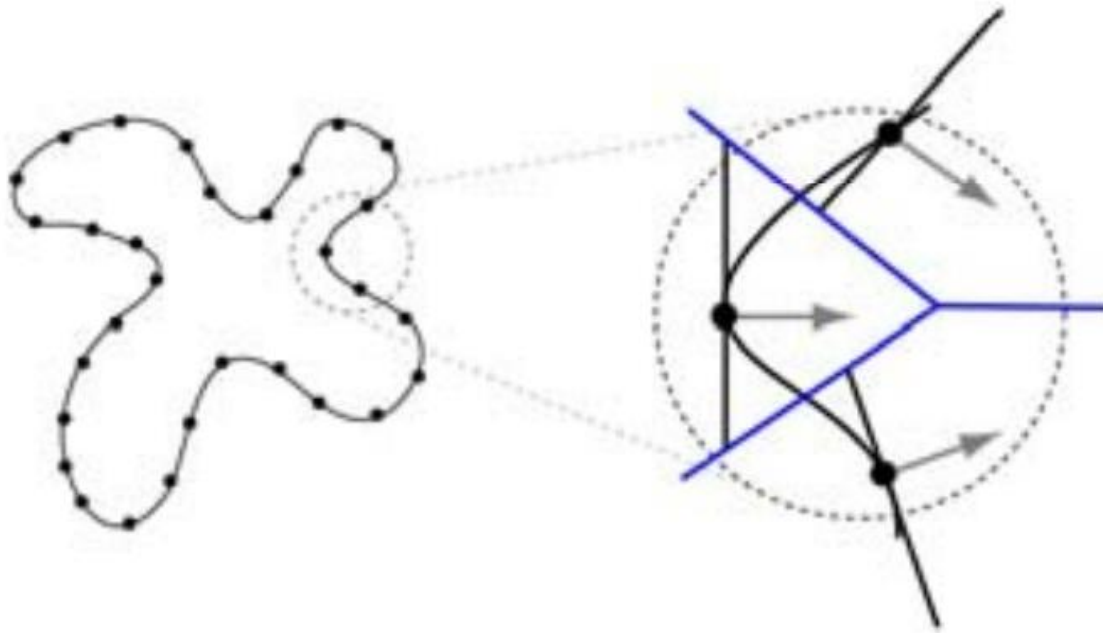
# Boolean Operations

- Classification:
  - piecewise constant surface approximation leads to false classification close to the surface



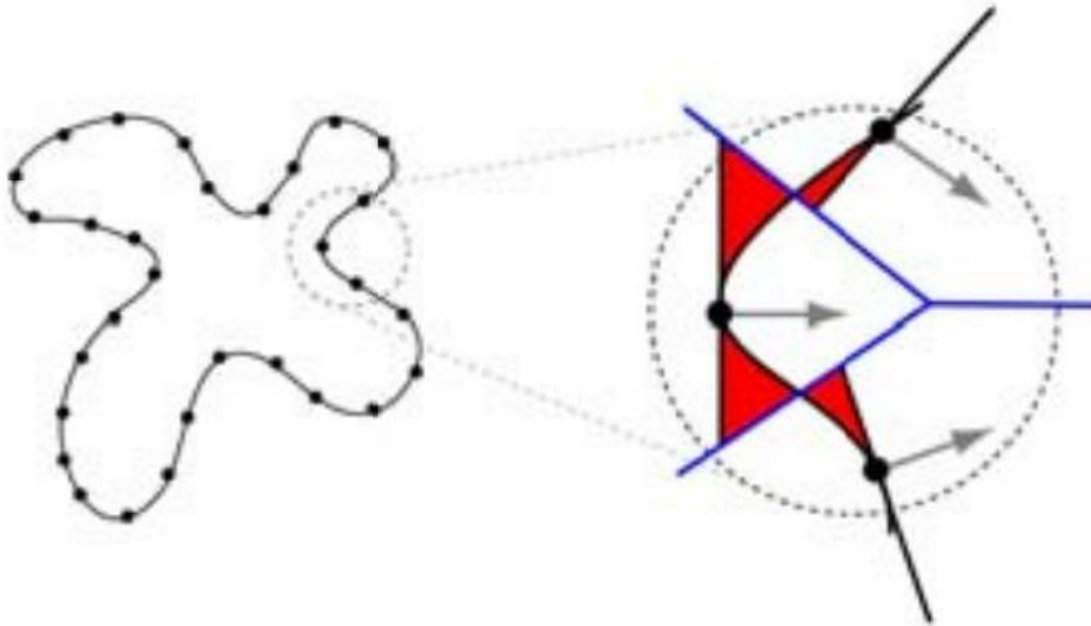
# Boolean Operations

- Classification:
  - piecewise constant surface approximation leads to false classification close to the surface



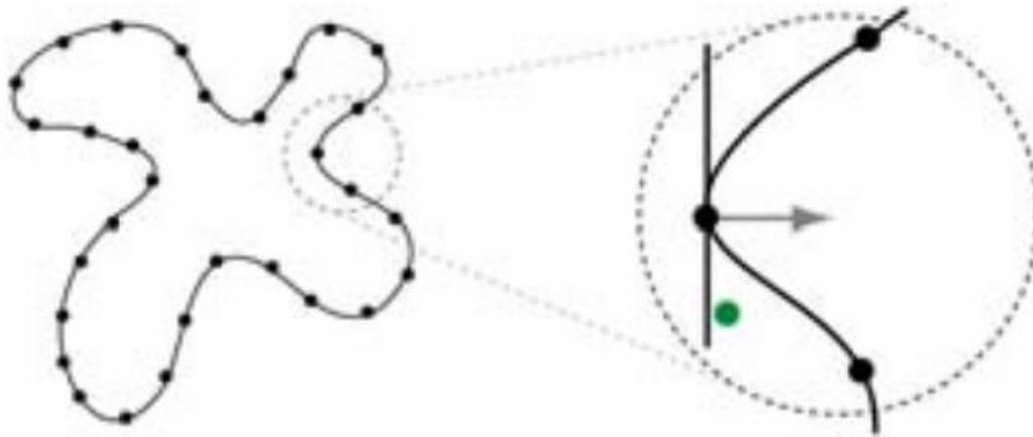
# Boolean Operations

- Classification:
  - piecewise constant surface approximation leads to false classification close to the surface



# Boolean Operations

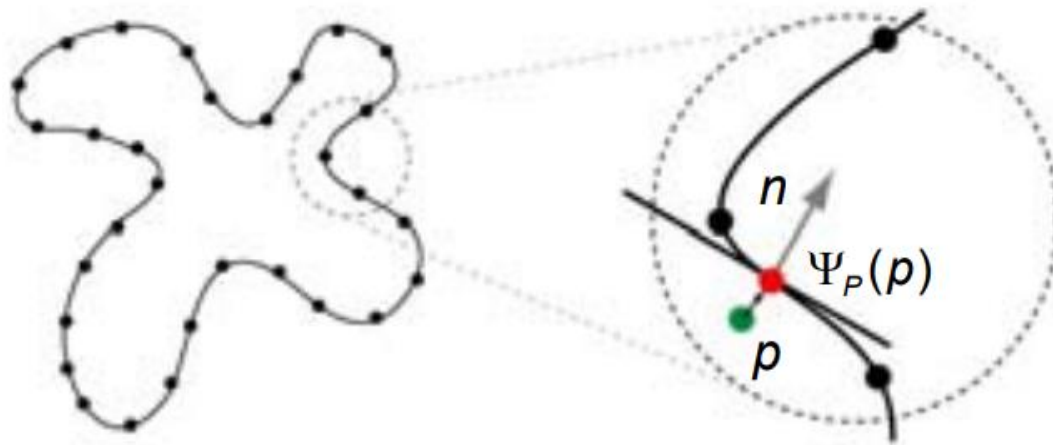
- Classification:
  - piecewise constant surface approximation leads to false classification close to the surface





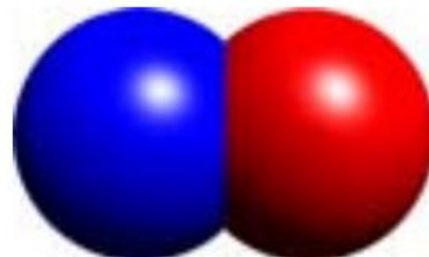
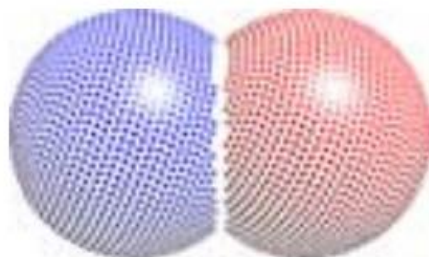
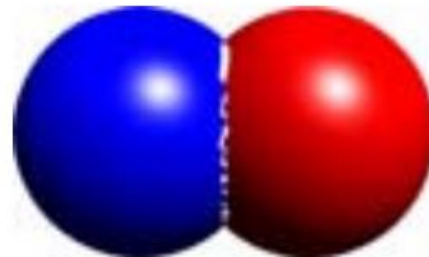
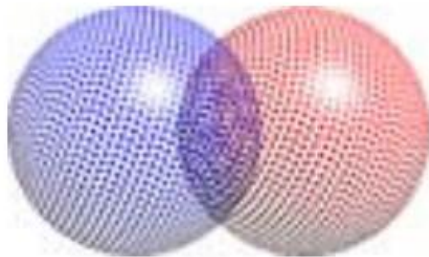
# Boolean Operations

- Classification:
  - use MLS projection of  $p$  for correct classification



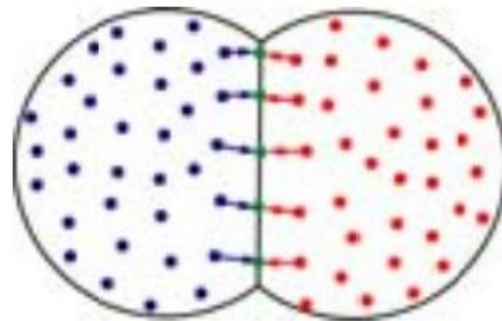
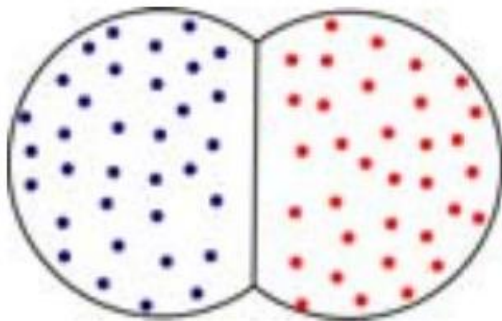
# Boolean Operations

- Sampling the intersection curve



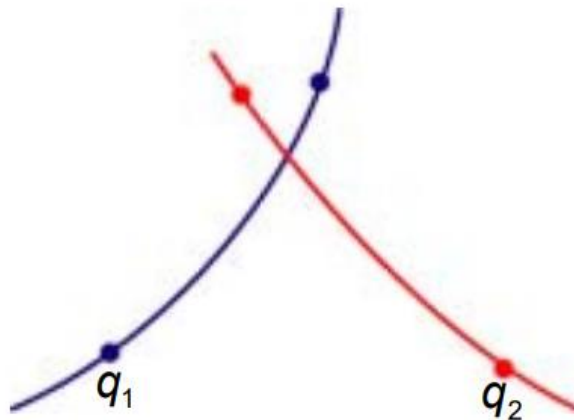
# Boolean Operations

- Newton scheme:
  - Identify pairs of closest points



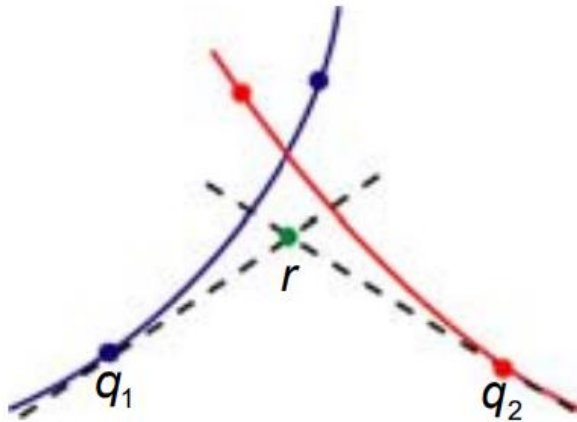
# Boolean Operations

- Newton scheme:
  - Identify pairs of closest points



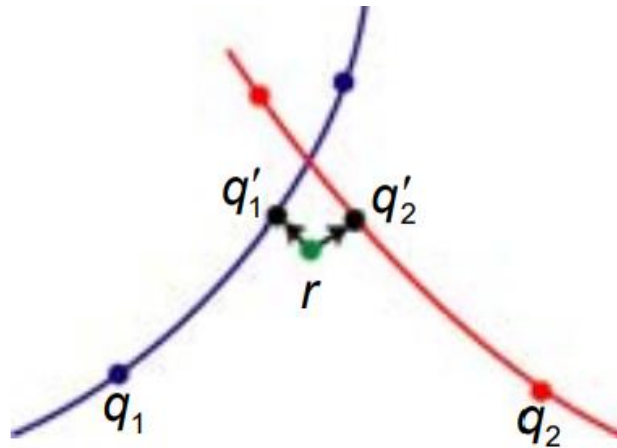
# Boolean Operations

- Newton scheme:
  - Identify pairs of closest points
  - Compute closest point on intersection of tangent spaces



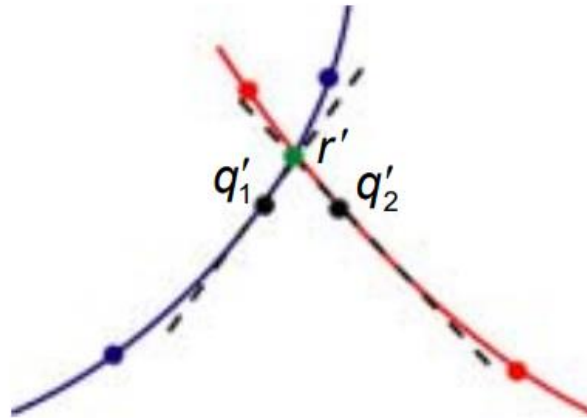
# Boolean Operations

- Newton scheme:
  - Identify pairs of closest points
  - Compute closest point on intersection of tangent spaces
  - Re-project point on both surfaces



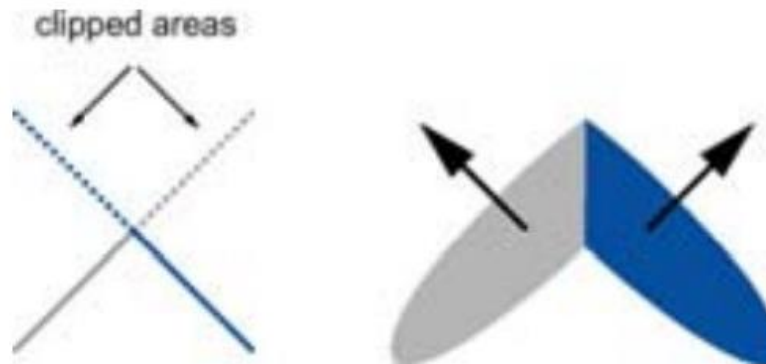
# Boolean Operations

- Newton scheme:
  - Identify pairs of closest points
  - Compute closest point on intersection of tangent spaces
  - Re-project point on both surfaces
  - Iterate



# Boolean Operations

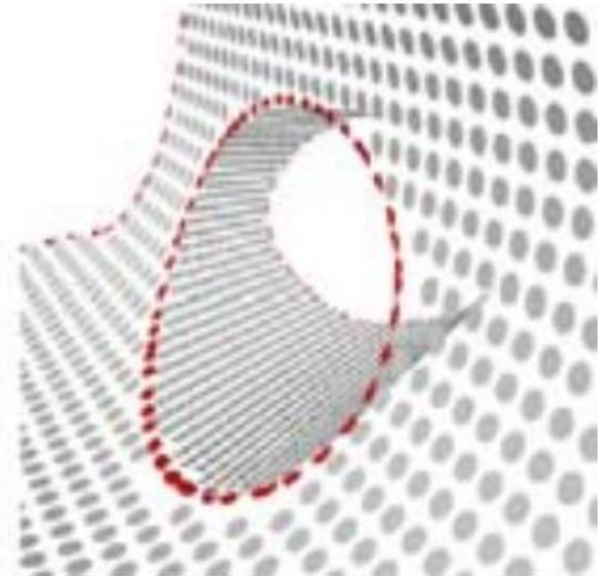
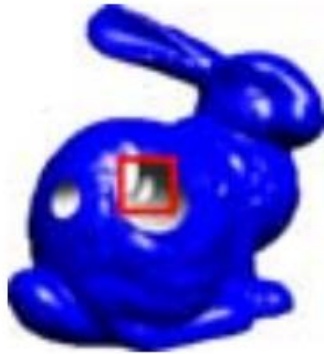
- Rendering sharp creases
  - represent points on intersection curve with two surfels that mutually clip each other





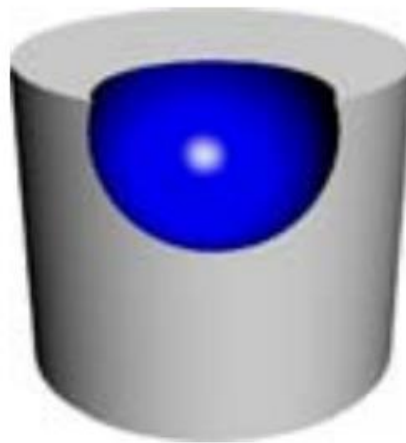
# Boolean Operations

- Rendering sharp creases



# Boolean Operations

- Rendering sharp creases
  - easily extended to handle corners by allowing multiple clipping



# Boolean Operations

- Rendering sharp creases
  - easily extended to handle corners by allowing multiple clipping



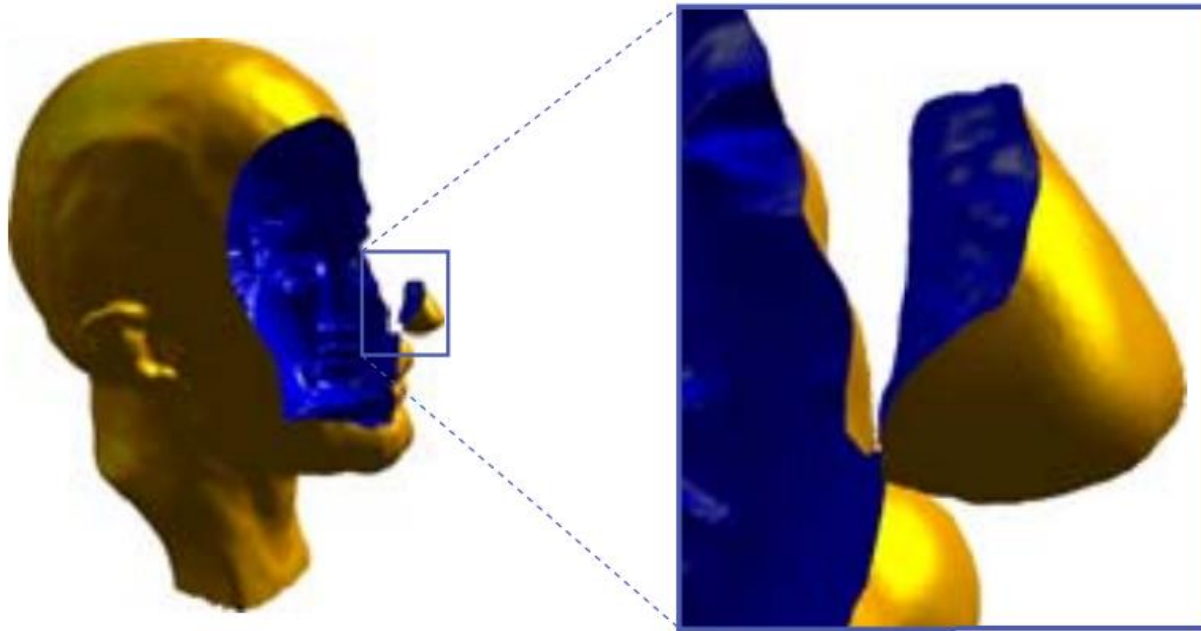
Difference



Union

# Boolean Operations

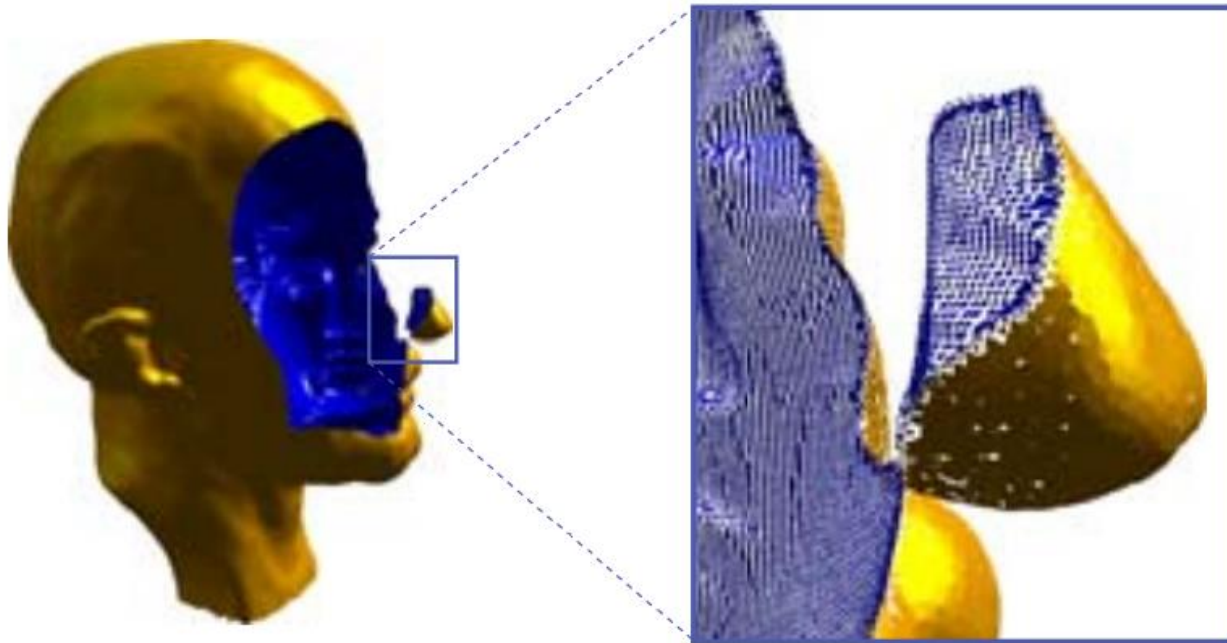
- Rendering sharp creases



Difference

# Boolean Operations

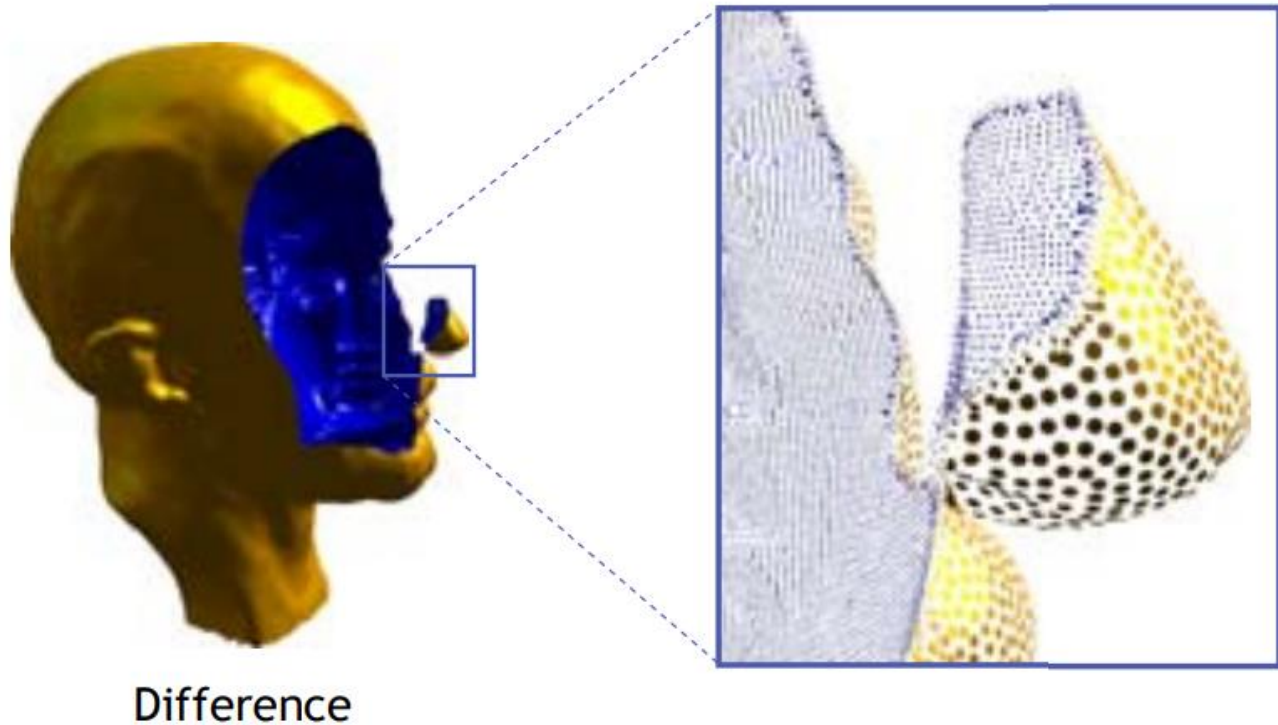
- Rendering sharp creases



Difference

# Boolean Operations

- Rendering sharp creases



# Boolean Operations

- Boolean operations can create intricate shapes with complex topology

$A + B$



$A \cdot B$



$A - B$

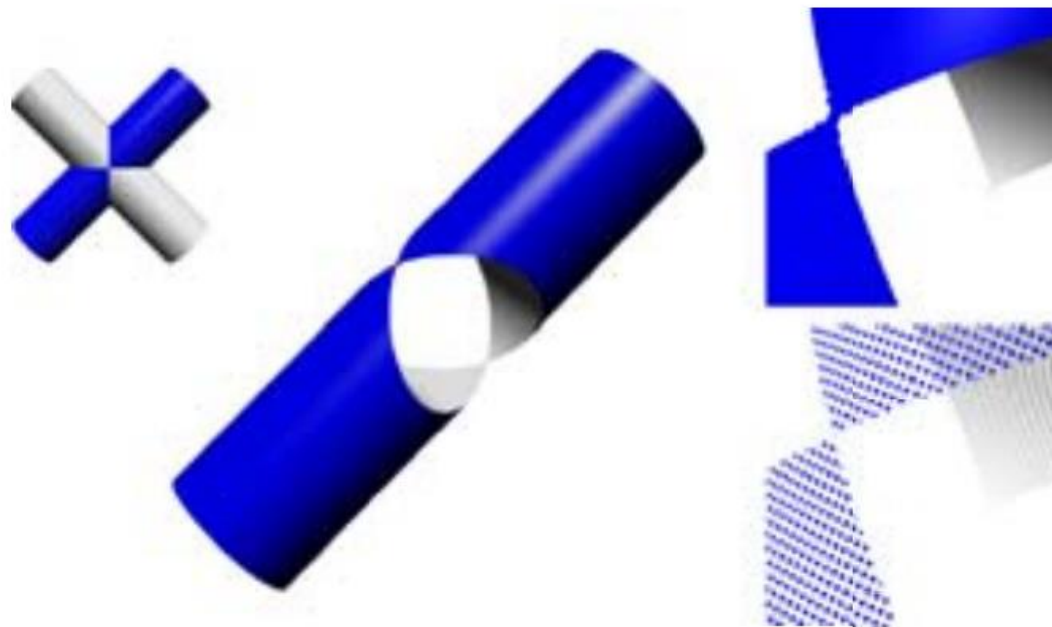


$B - A$



# Boolean Operations

- Singularities lead to numerical instabilities (intersection of almost parallel planes)





# Particle-based Blending

- Boolean operations create sharp intersection curves
- Particle simulation to create smooth transition
  - Repelling force to control particle distribution
  - Normal potentials to control particle orientation

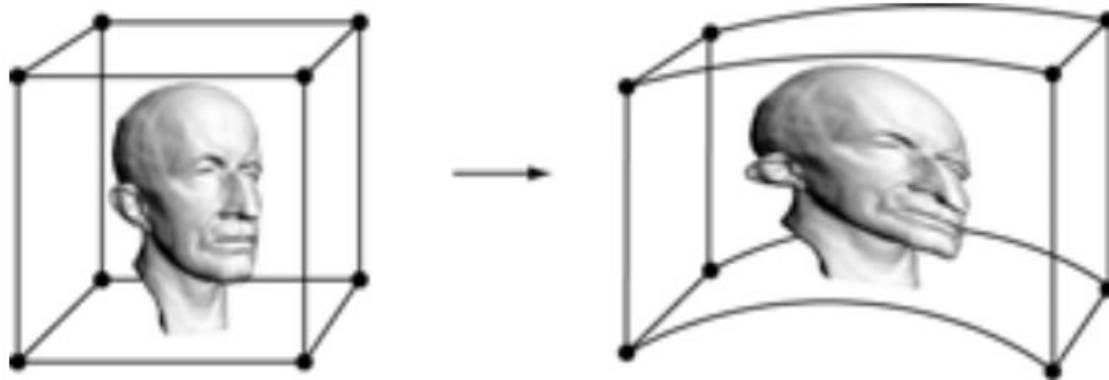


# Free-form deformation



# Free-form Deformation

- Smooth deformation field  $F:\mathbf{R}^3\rightarrow\mathbf{R}^3$  that warps 3D space
- Can be applied directly to point samples



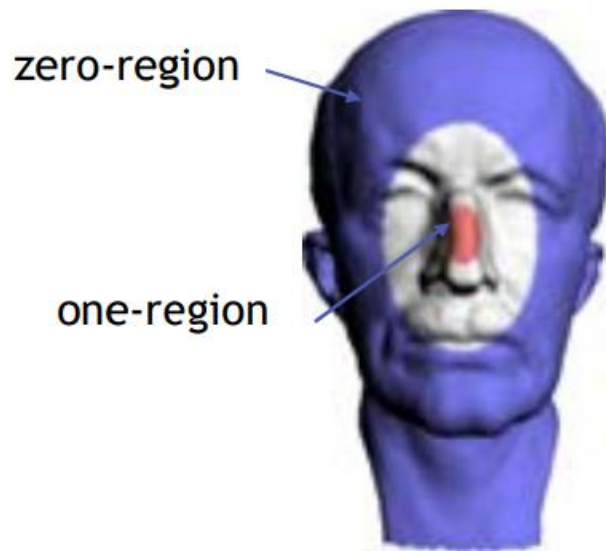
# Free-form Deformation

- How to define the deformation field?
  - Painting metaphor
- How to detect and handle self-intersections?
  - Point-based collision detection, boolean union, particle-based blending
- How to handle strong distortions?
  - Dynamic re-sampling

# Free-form Deformation

- Intuitive editing paradigm using painting metaphor
  - Define rigid surface part (zero-region) and handle (one-region) using interactive painting tool
  - Displace handle using combination of translation and rotation
  - Create smooth blend towards zero-region

# Free-form Deformation



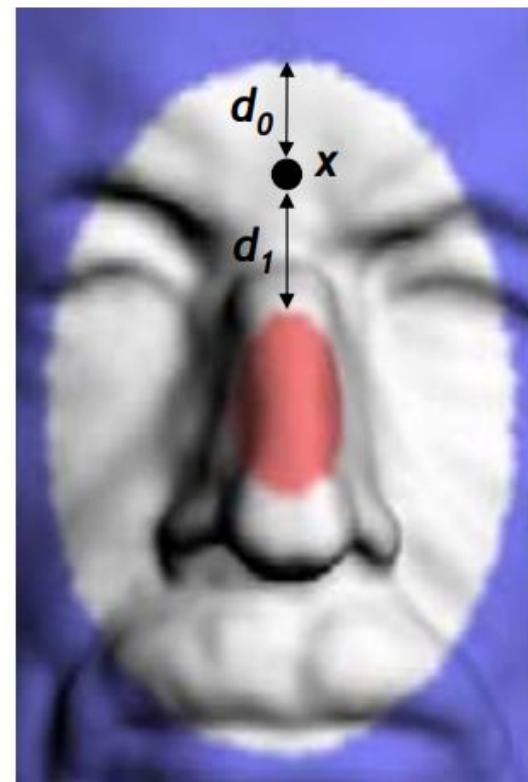
original  
surface



deformed  
surface

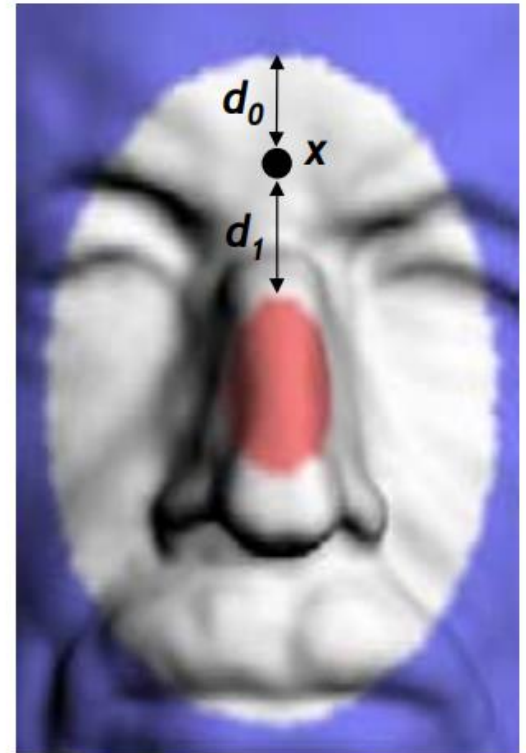
# Free-form Deformation

- Definition of deformation field:
  - Continuous scale parameter  $t_x$ 
    - $t_x = \beta(d_0 / (d_0 + d_1))$
    - $d_0$ : distance of  $x$  to zero-region
    - $d_1$ : distance of  $x$  to one-region
  - Blending function
    - $\beta: [0,1] \rightarrow [0,1]$
    - $\beta \in C^0, \beta(0) = 0, \beta(1) = 1$
  - $t_x = 0$  if  $x$  in zero-region
  - $t_x = 1$  if  $x$  in one-region



# Free-form Deformation

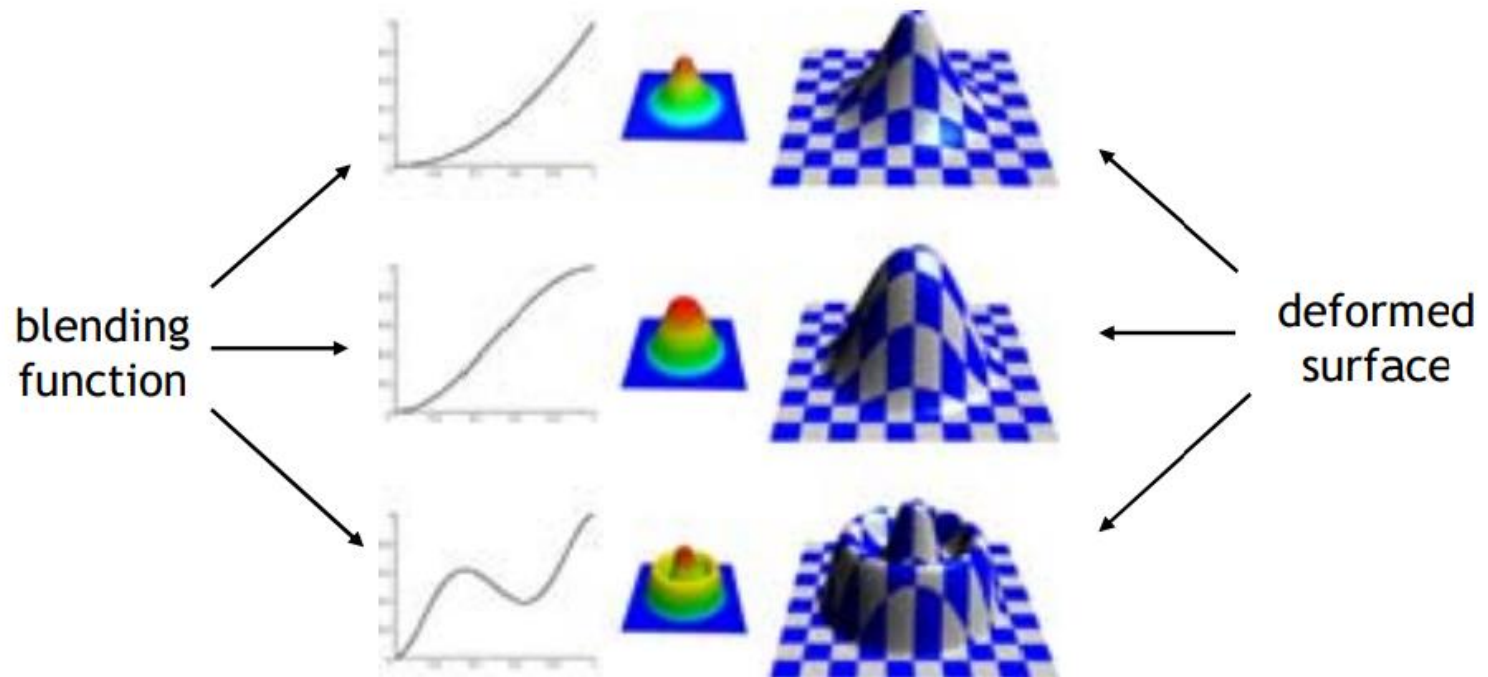
- Definition of deformation field:
  - Deformation function
    - $F(x) = F_T(x) + F_R(x)$
  - Translation
    - $F_T(x) = x + t_x \cdot v$
  - Rotation
    - $F_R(x) = M(t_x) \cdot x$





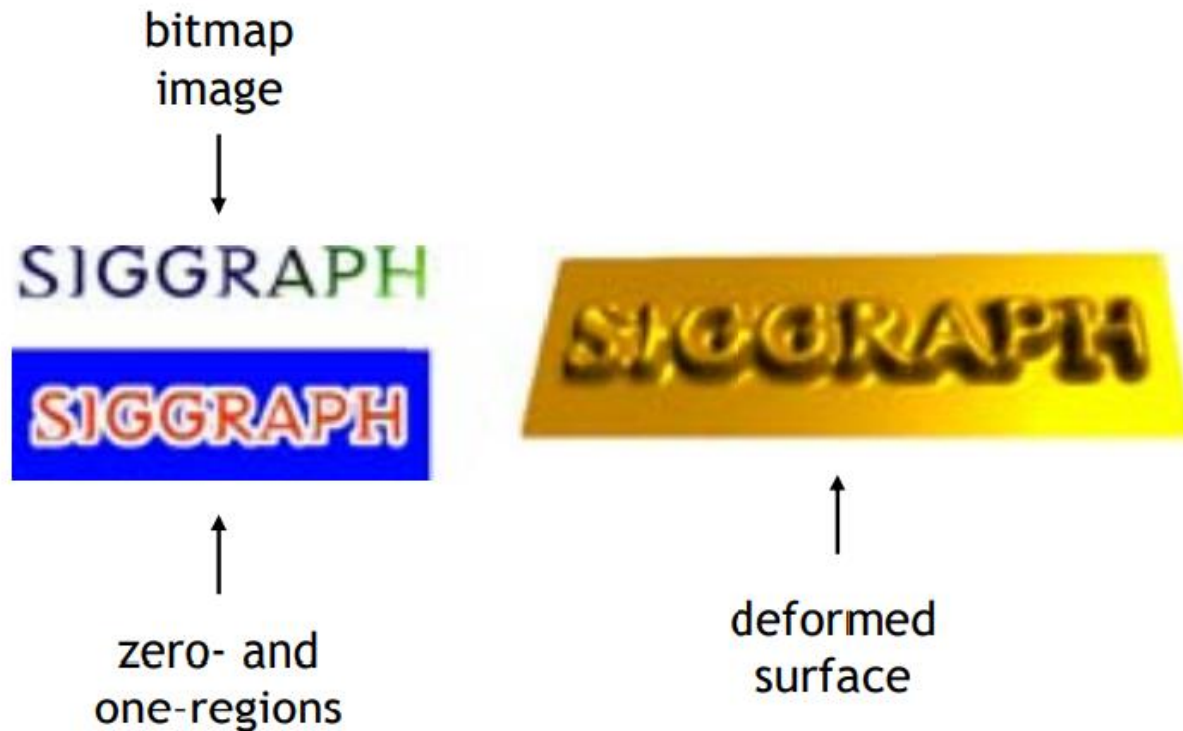
# Free-form Deformation

- Translation for three different blending functions



# Free-form Deformation

- Embossing effect

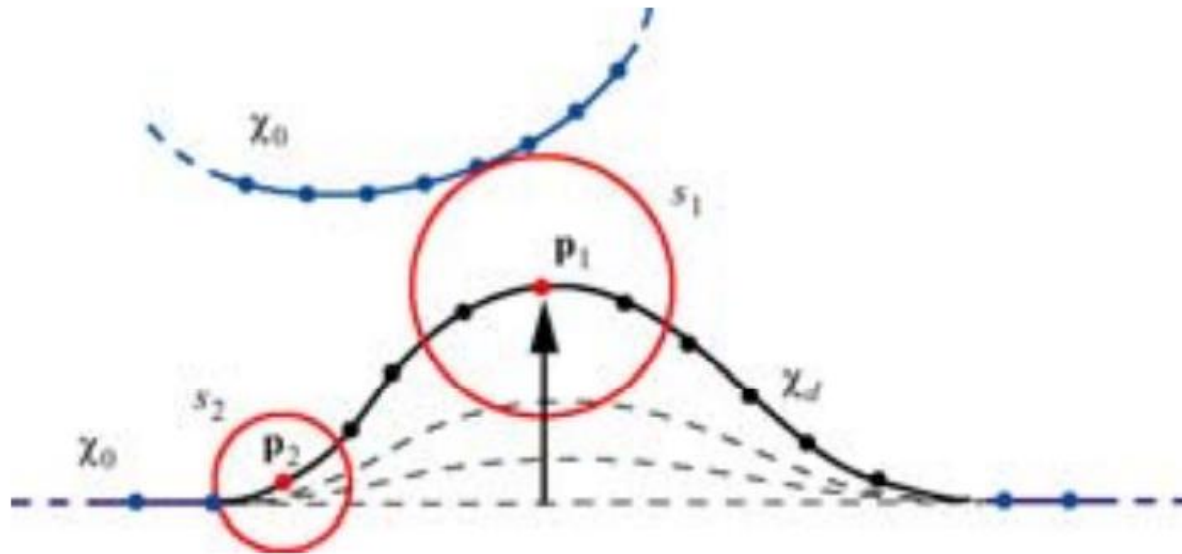


# Collision Detection

- Deformations can lead to self-intersections
- Apply boolean inside/outside classification to detect collisions
- Restricted to collisions between deformable region and zero-region to ensure efficient computations

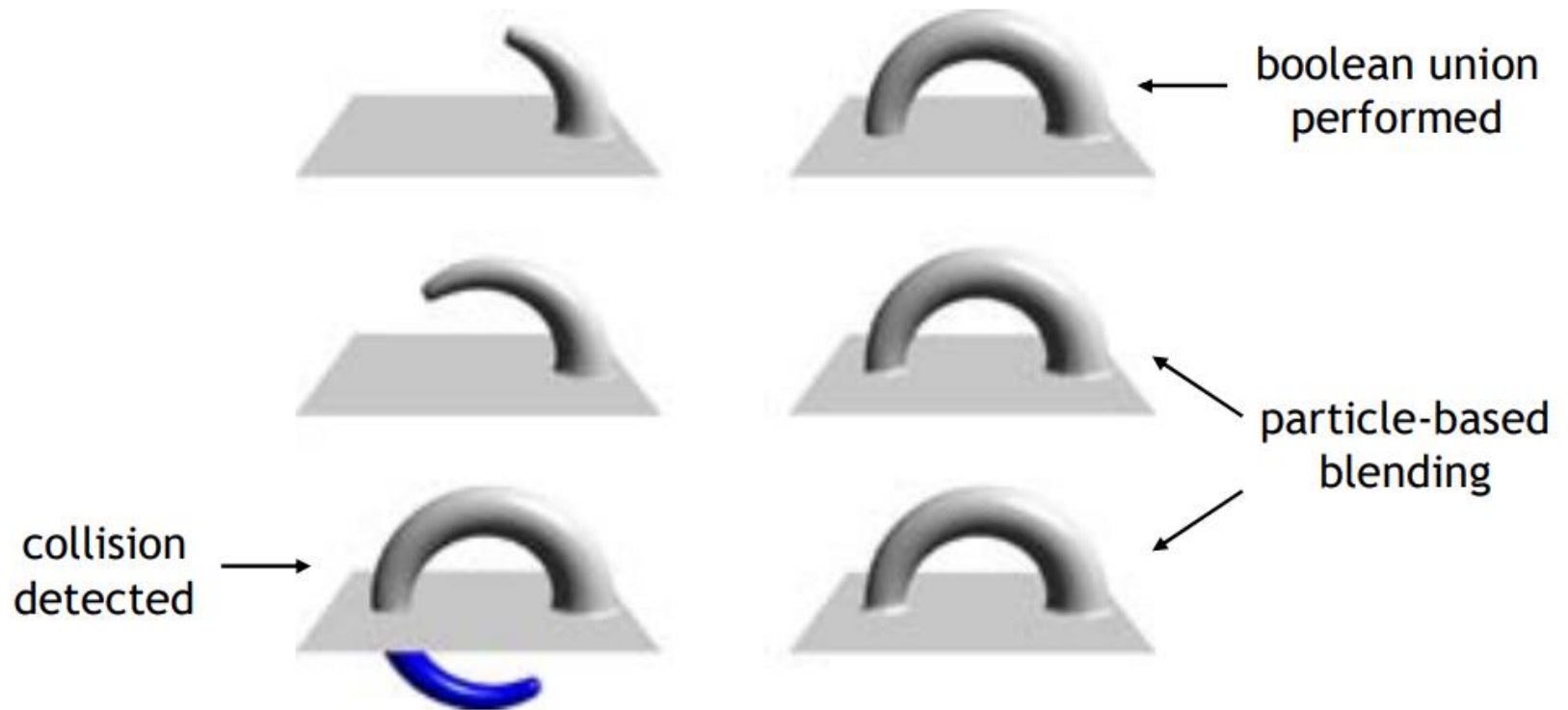
# Collision Detection

- Exploiting temporal coherence



# Collision Detection

- Interactive modeling session



# Dynamic Sampling



10,000 points



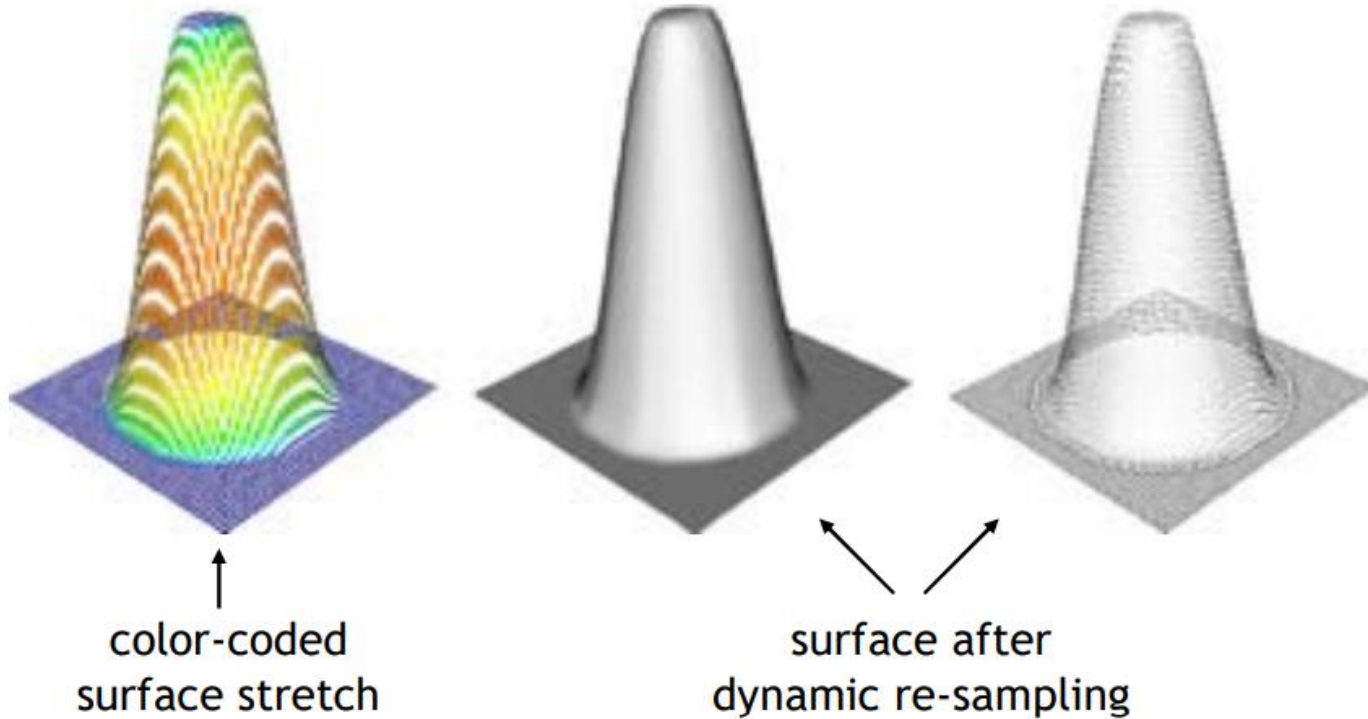
271,743 points

# Dynamic Sampling

- Large model deformations can lead to strong surface distortions
- Requires adaptation of the sampling density
- Dynamic insertion and deletion of point samples

# Dynamic Sampling

- Surface distortion varies locally



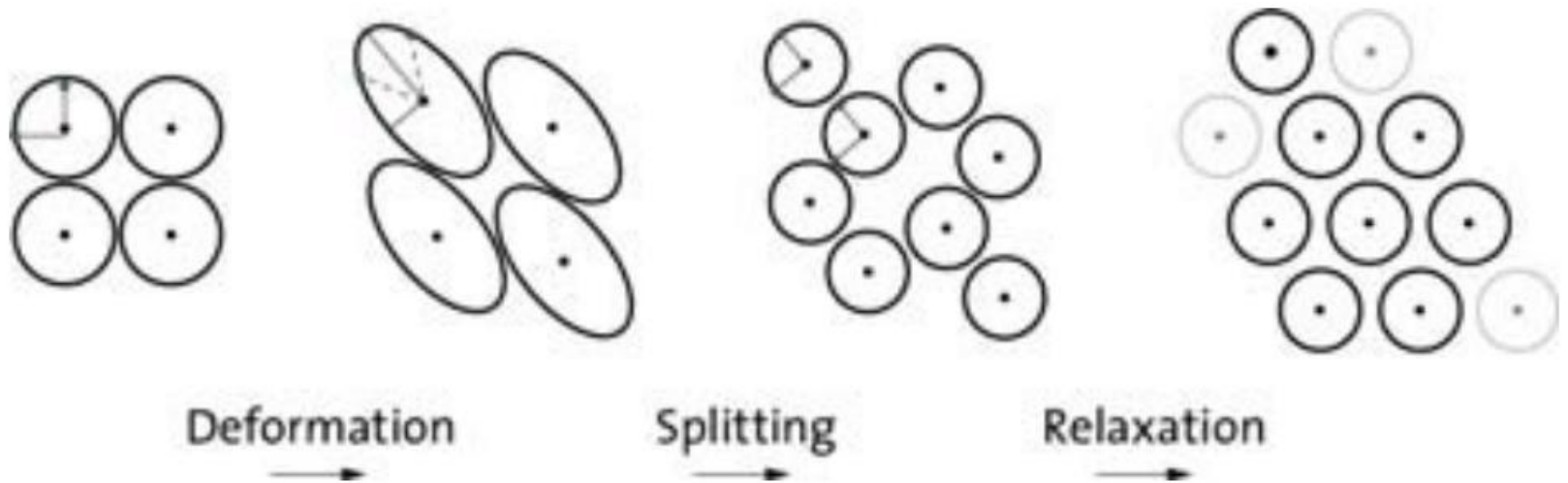


# Dynamic Sampling

- Measure local surface stretch from first fundamental form
- Split samples that exceed stretch threshold
- Regularize distribution by relaxation
- Interpolate scalar attributes

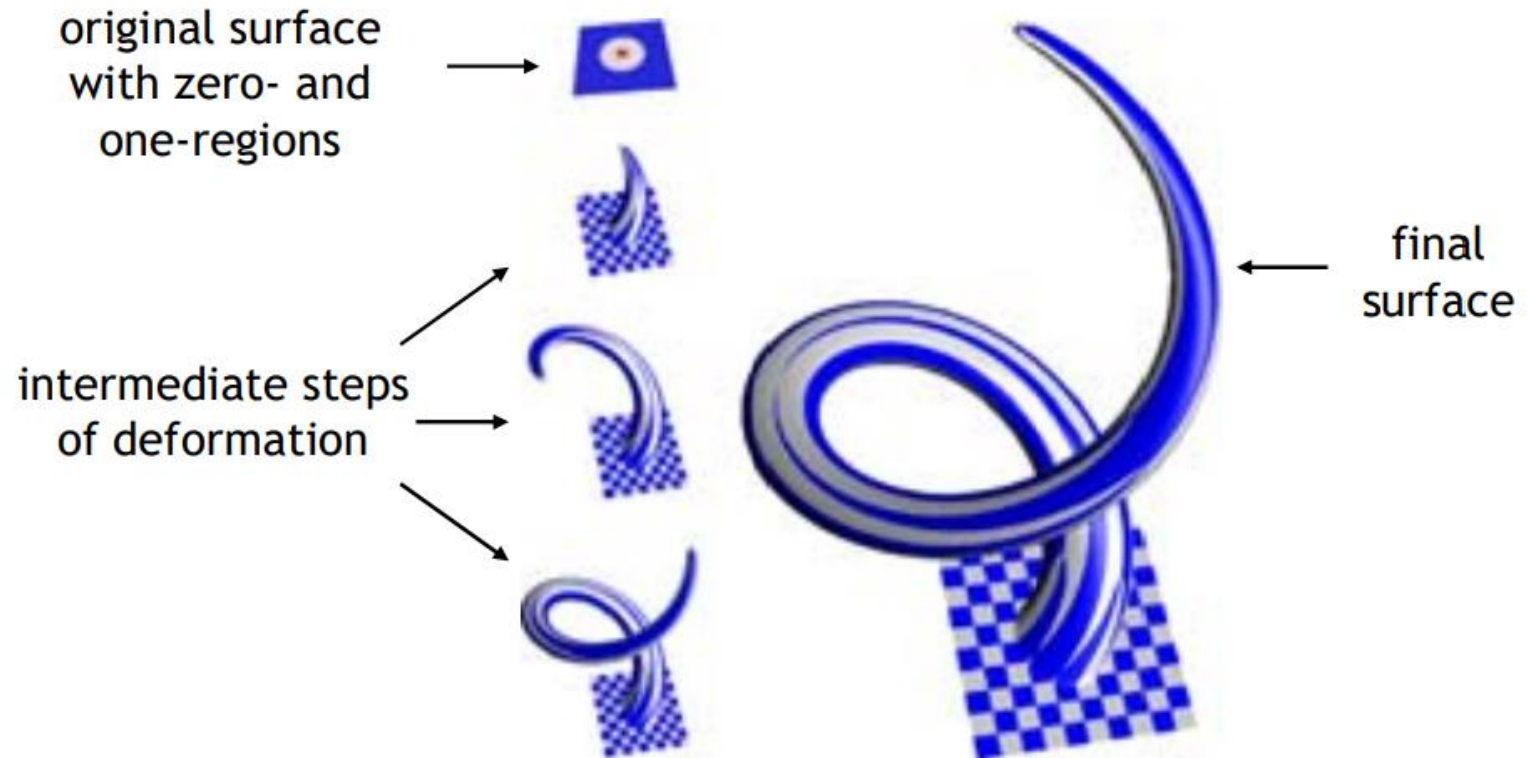
# Dynamic Sampling

- 2D illustration



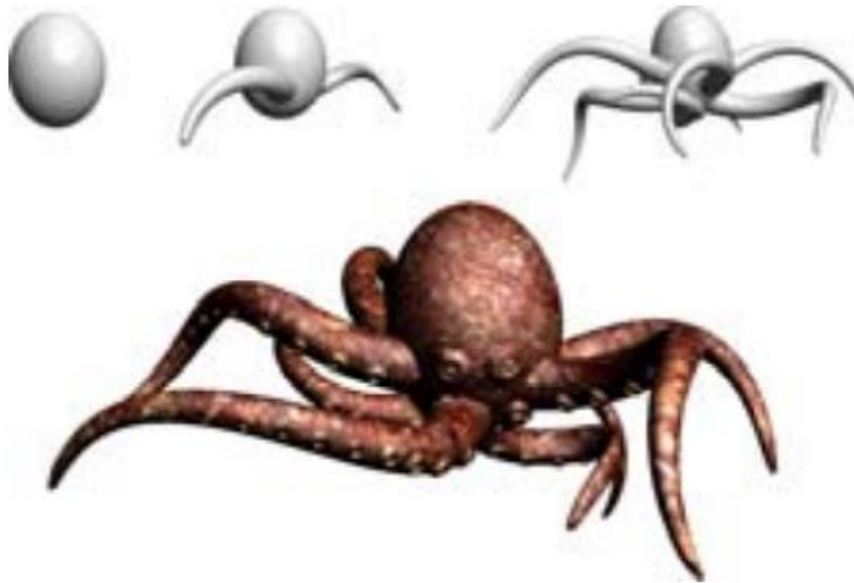
# Free-form Deformation

- Interactive modeling session with dynamic sampling



# Results

- Ab-initio design of an Octopus
  - Free-form deformation with dynamic sampling from 69,706 to 295,222 points



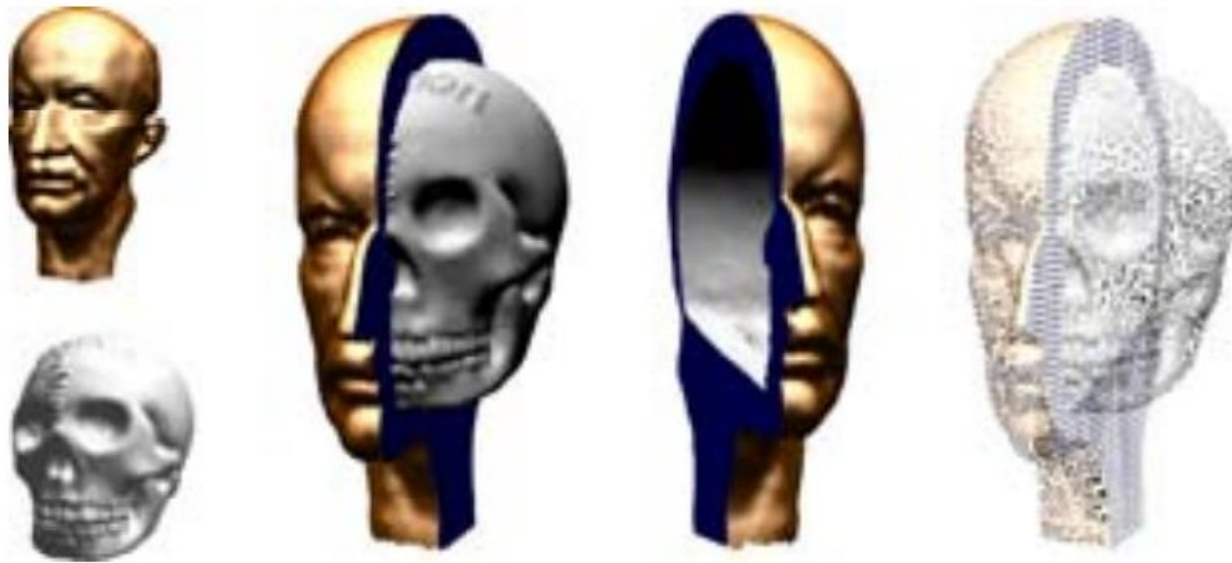
# Results

- Modeling with synthetic and scanned data
  - Combination of free-form deformation with collision detection, boolean operations, particle-based blending, embossing and texturing



# Results

- Boolean operations on scanned data
  - Irregular sampling pattern, low resolution models



# Discussion

- Points are a versatile shape modeling primitive
  - Combines advantages of implicit and parametric surfaces
  - Integrates boolean operations and free-form deformation
  - Dynamic restructuring
  - Time and space efficient implementations

# Discussion

- The power of point clouds as a shape representation for 3D deep learning is not fully utilized
  - Dynamic geometry