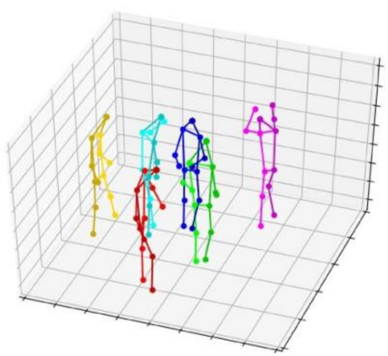
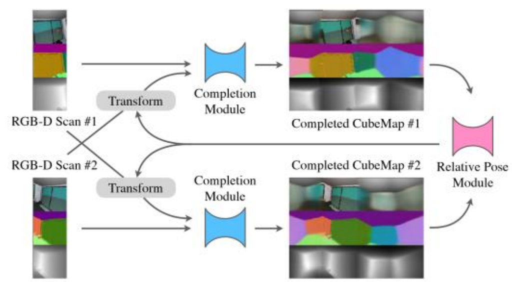
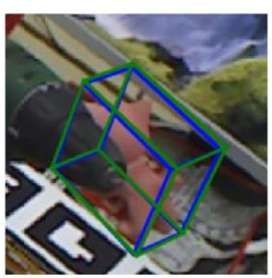
CS376 Computer Vision
Lecture 2: Linear Filters

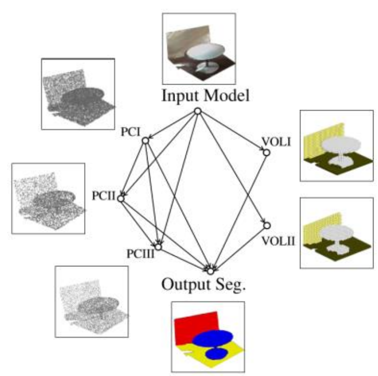




Qixing Huang

January 28th 2019





Announcements

- Piazza for assignment questions
- **A0** due tomorrow. Submit on Canvas.
- Office hours posted on class website

Plan for today

- Image noise
- Linear filters
  - Examples: smoothing filters

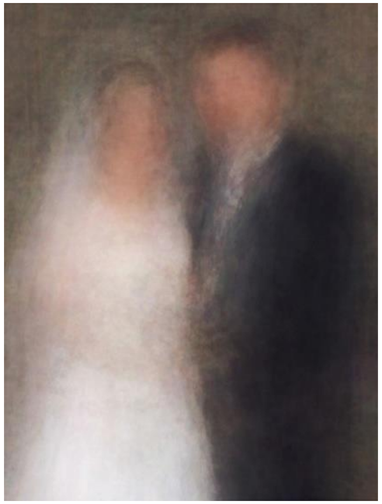
Images as matrices

Result of averaging 100 similar snapshots

Little Leaguer

Kids with Santa

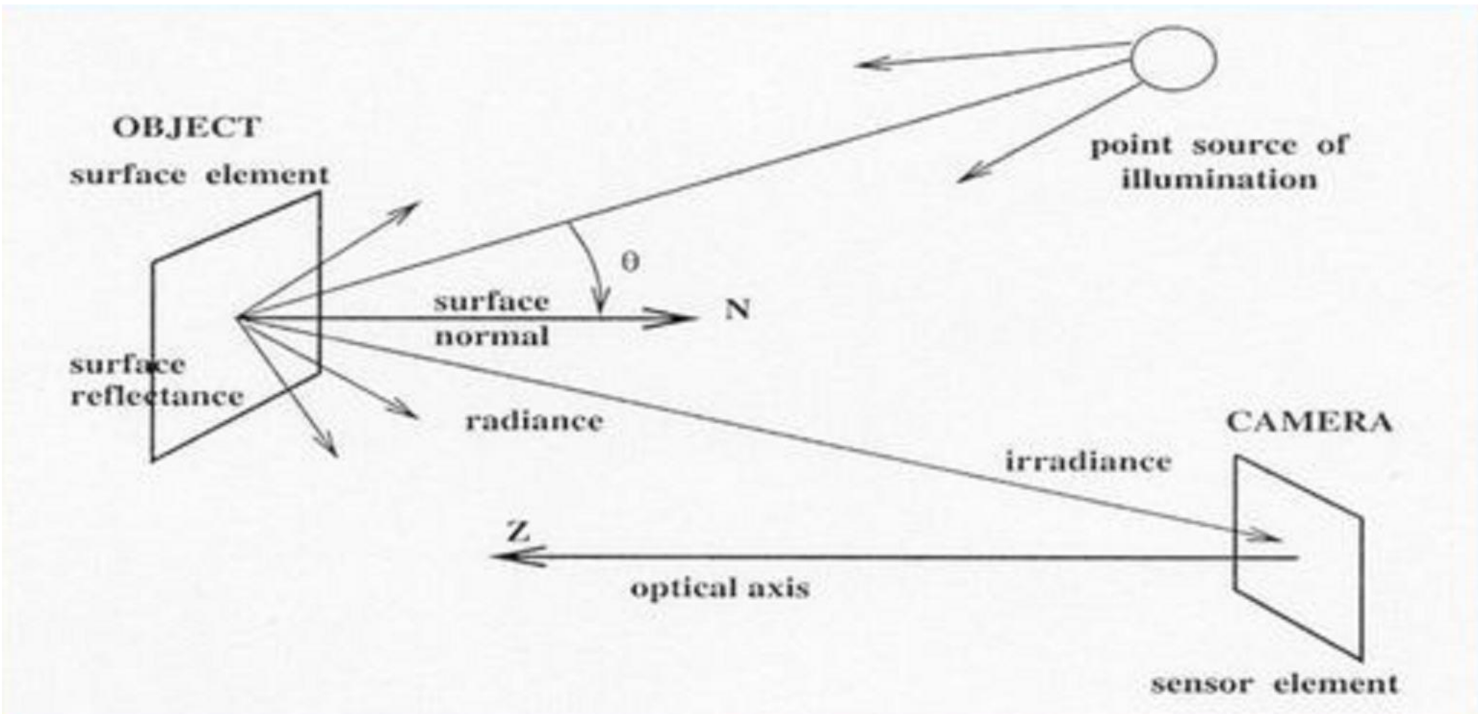
The Graduate



Newlyweds

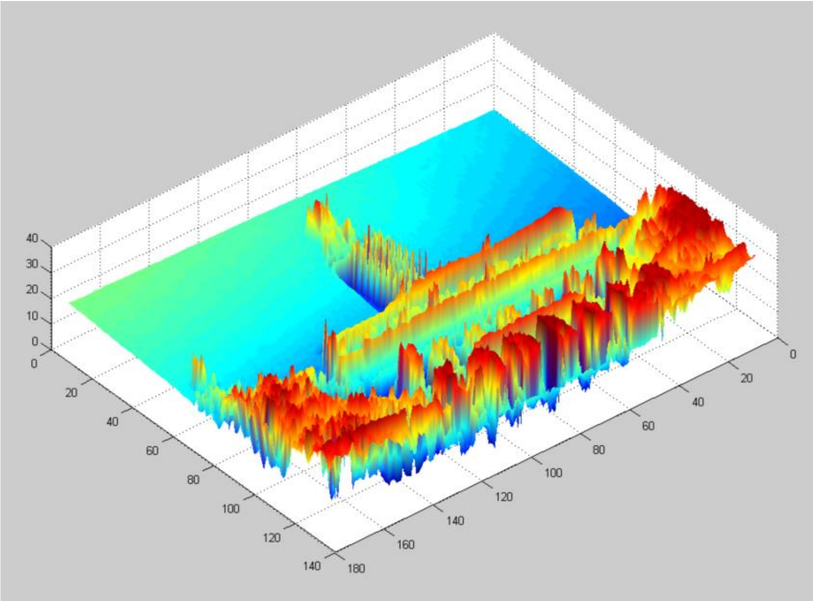
From: *100 Special Moments*, by Jason Salavon (University of Chicago) (2004)
http://salavon.com/SpecialMoments/SpecialMoments.shtml

Image Formation

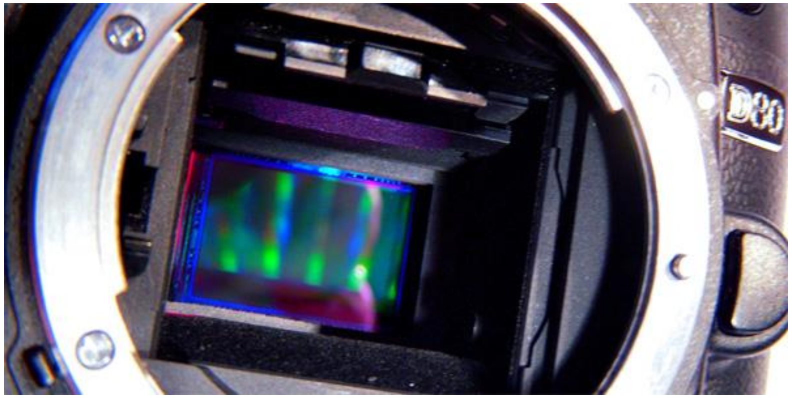


Source: https://slideplayer.com/slide/13474214/

Images as functions



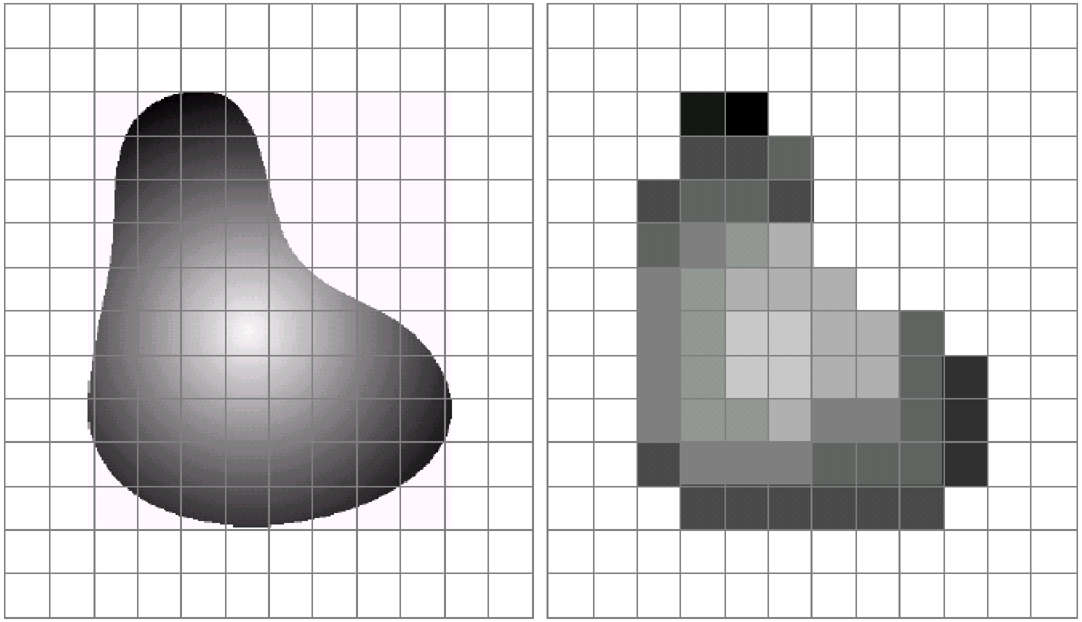
Digital camera

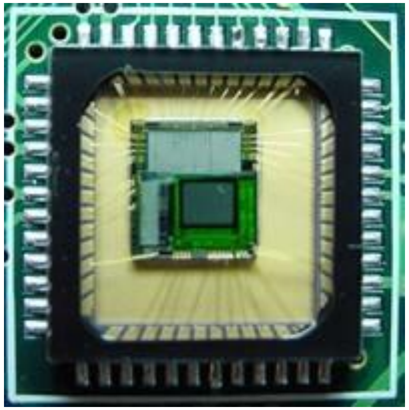


A digital camera replaces film with a sensor array

- Each cell in the array is light-sensitive diode that converts photons to electrons
- http://electronics.howstuffworks.com/digital-camera.htm

Digital images





a b

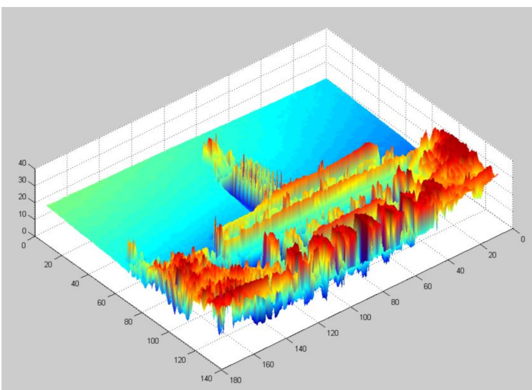
FIGURE 2.17 (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.

Slide credit: Derek Hoiem

Digital images

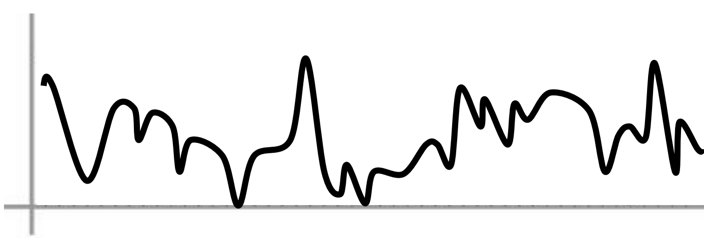
- **Sample** the 2D space on a regular grid
- **Quantize** each sample (e.g., round to nearest integer)

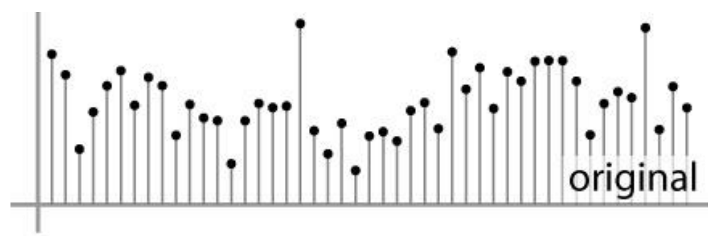
- Image thus represented as a matrix of integer values.



10	10	10	10	24	20
21	21	70	10	30	40
00	05	10	15	30	10
05	15	10	16	17	28
75	50	30	20	25	30
98	78	77	77	75	46

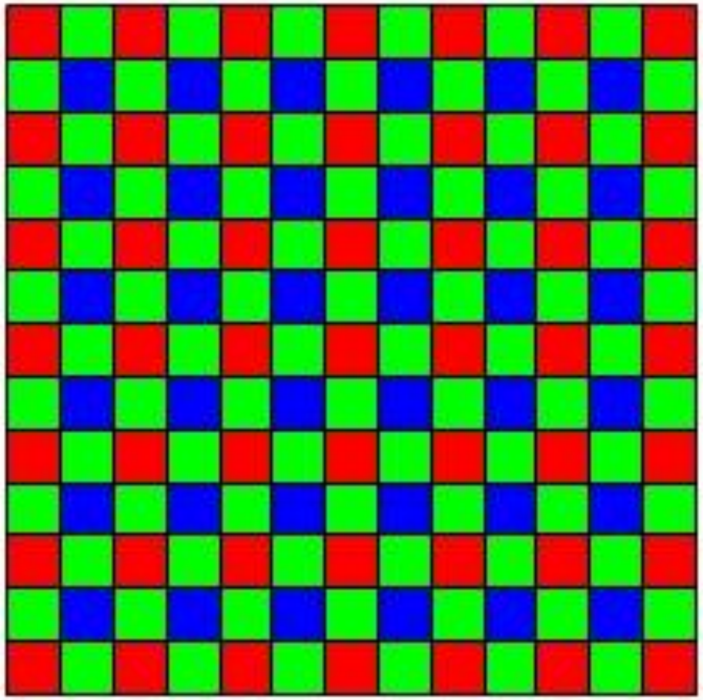
2D





1D

Digital color images

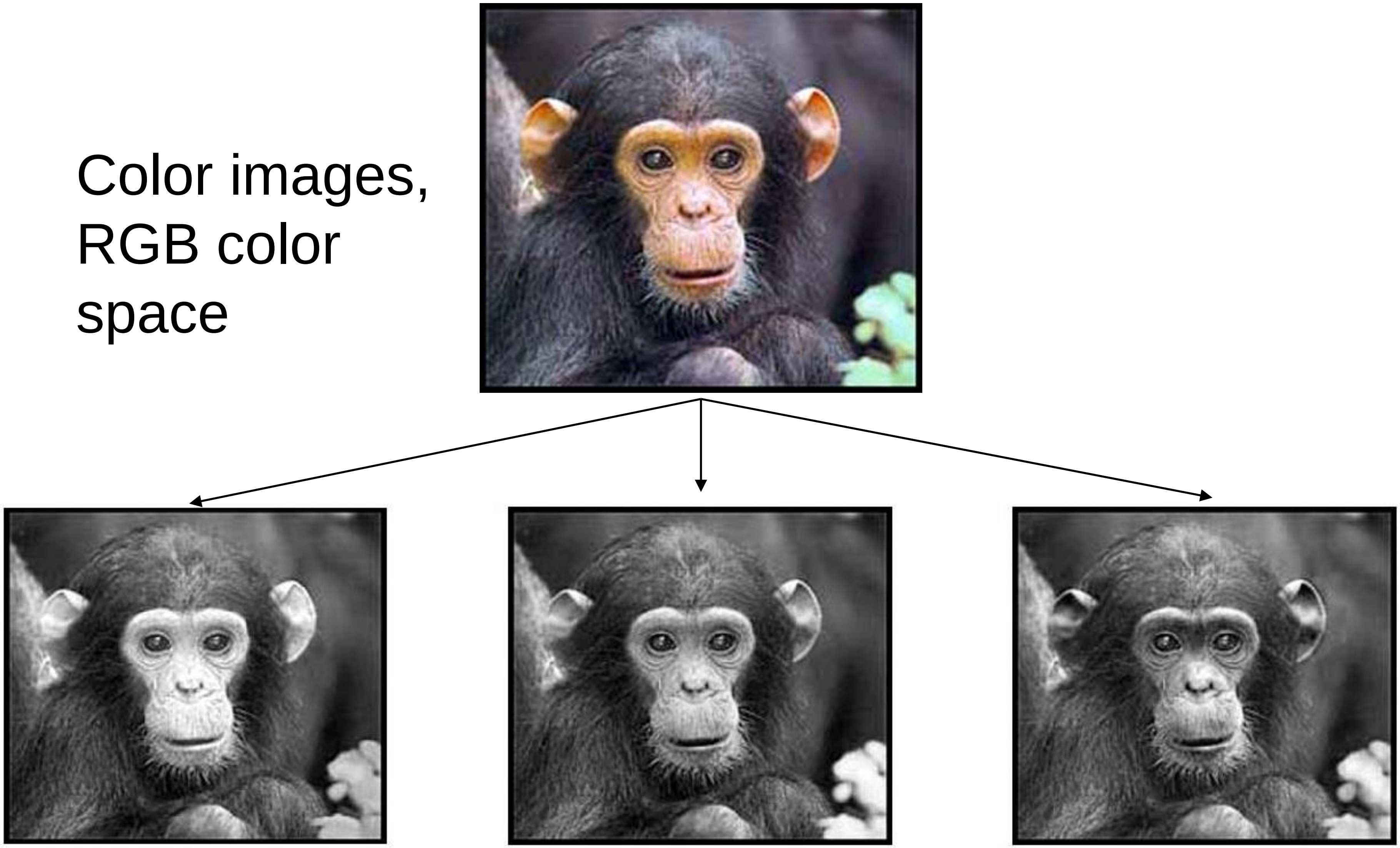


Bayer filter

Digital color images

Slide Credit:
Kristen Grauman

Color images,
RGB color
space



R G B

Images in Matlab

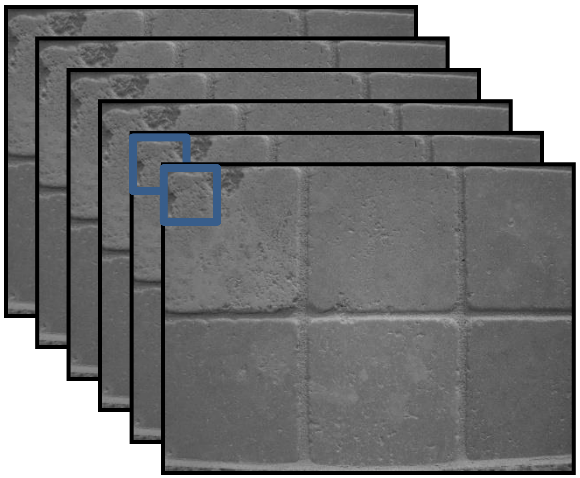
- Images represented as a matrix
- Suppose we have a NxM RGB image called "I"
  - I(1,1,1) = top-left pixel value in R-channel
  - I(y, x, b) = y pixels down, x pixels to right in the b^{th} channel
  - I(N, M, 3) = bottom-right pixel in B-channel
- imread(filename) returns a uint8 image (values 0 to 255)
  - Convert to double format (values 0 to 1) with im2double

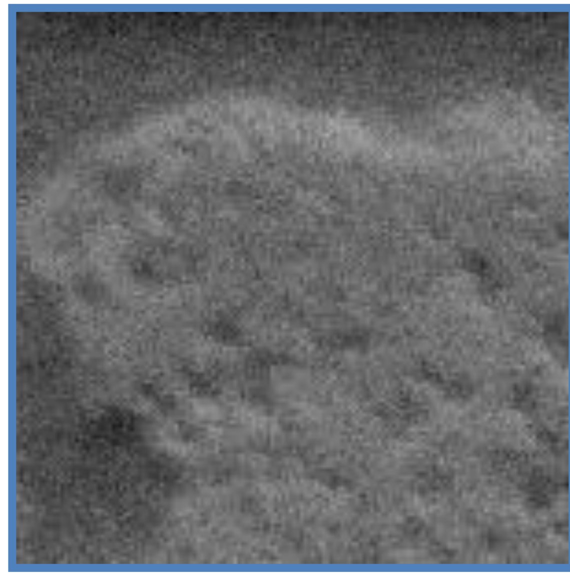
Main idea: image filtering

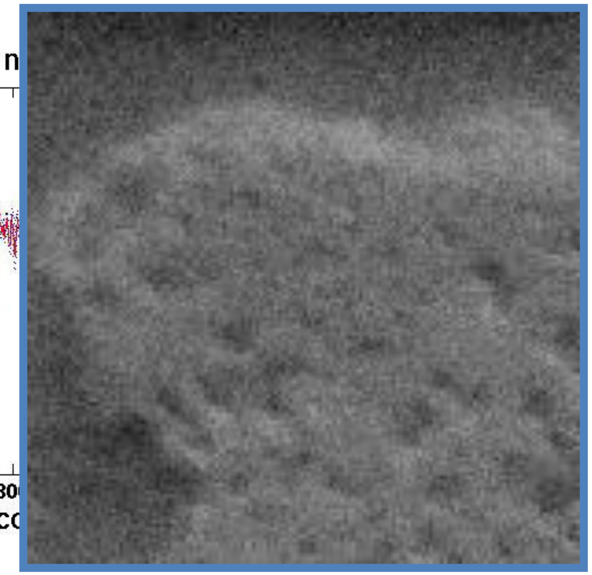
- Aggregate the local neighborhood at each pixel in the image
  – Function specified a pattern saying how to aggregate values from neighbors

- Uses of filtering:
  – Enhance an image (denoise, resize, level-of-details, etc)
  – Extract information (texture, edges, features, etc)
  – Detect patterns (template matching)

Motivation: noise reduction







- Even multiple images of the **same static scene** will not be identical.

Slide credit:
Kristen Grauman

Common types of noise

- **Salt and pepper noise**: random occurrences of black and white pixels
- **Impulse noise:** random occurrences of white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution


Original

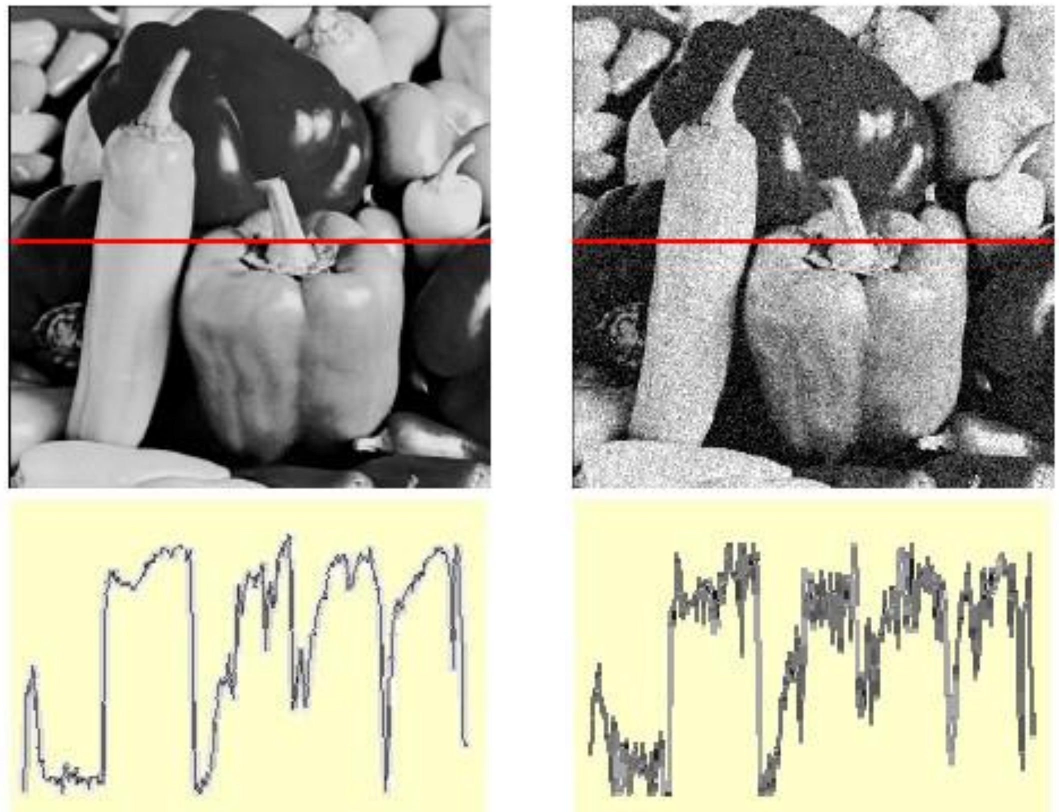

Salt and pepper noise


Impulse noise


Gaussian noise

Source: S. Seitz

Gaussian noise



$$f(x,y) = \overbrace{\bar{f}(x,y)}^{\text{Ideal Image}} + \overbrace{\eta(x,y)}^{\text{Noise process}}$$

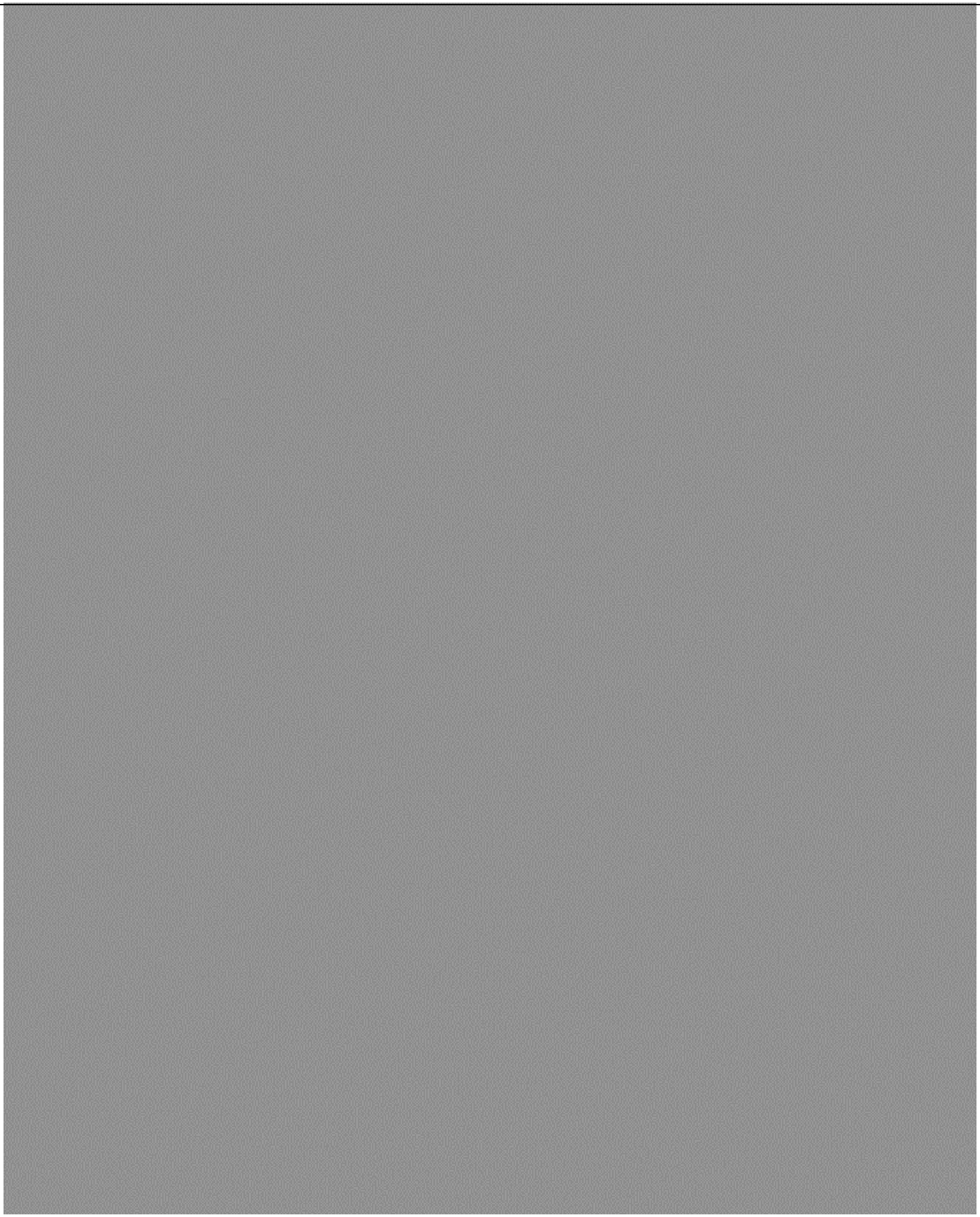
Gaussian i.i.d. ("white") noise:
$\eta(x,y) \sim \mathcal{N}(\mu, \sigma)$

```
>> noise = randn(size(im)).*sigma;
>> output = im + noise;
```

What is impact of the sigma?

Fig: M. Hebert

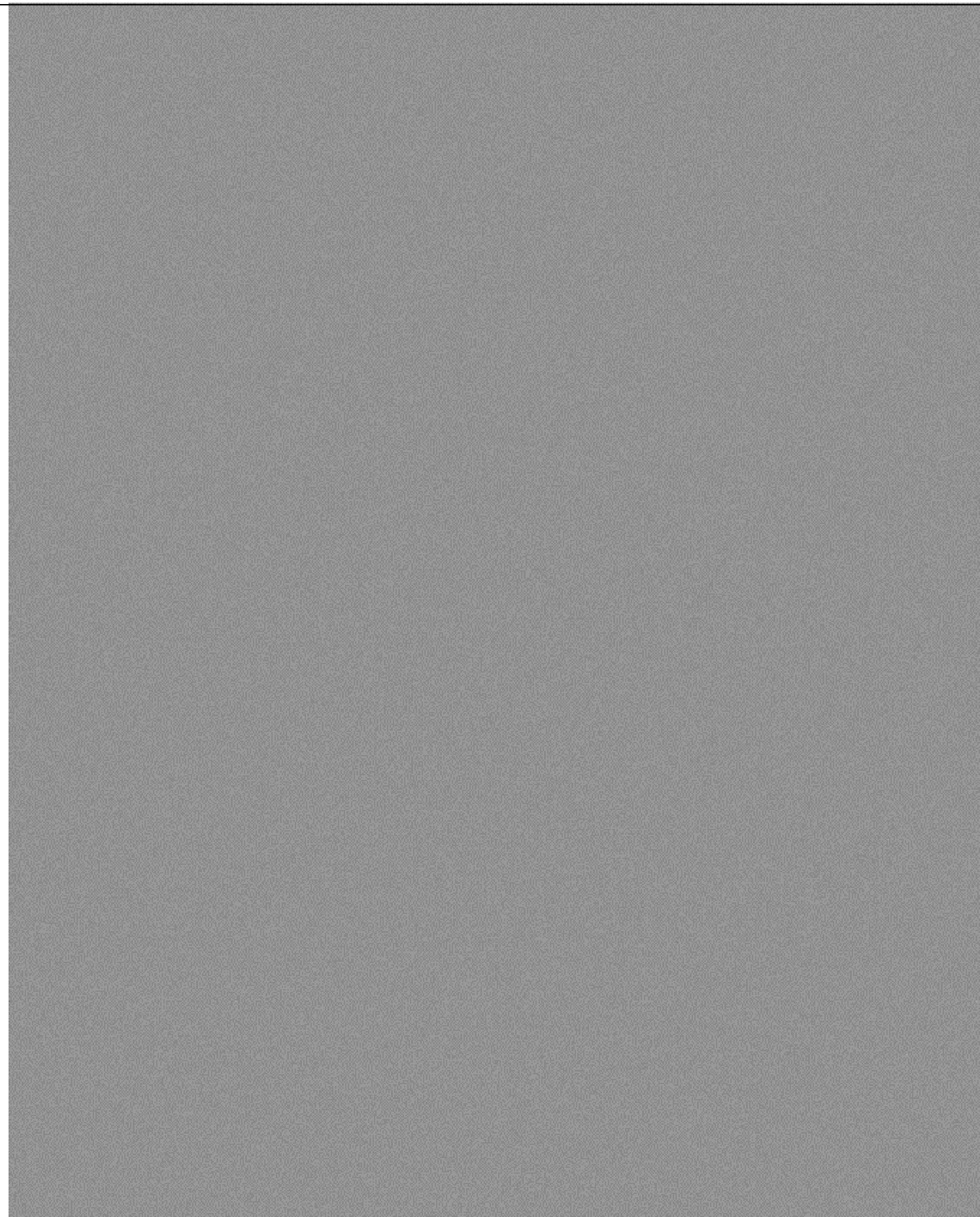
sigma=1



Effect of sigma on Gaussian noise:

Image shows the noise values themselves.

sigma=4



Effect of sigma on Gaussian noise:

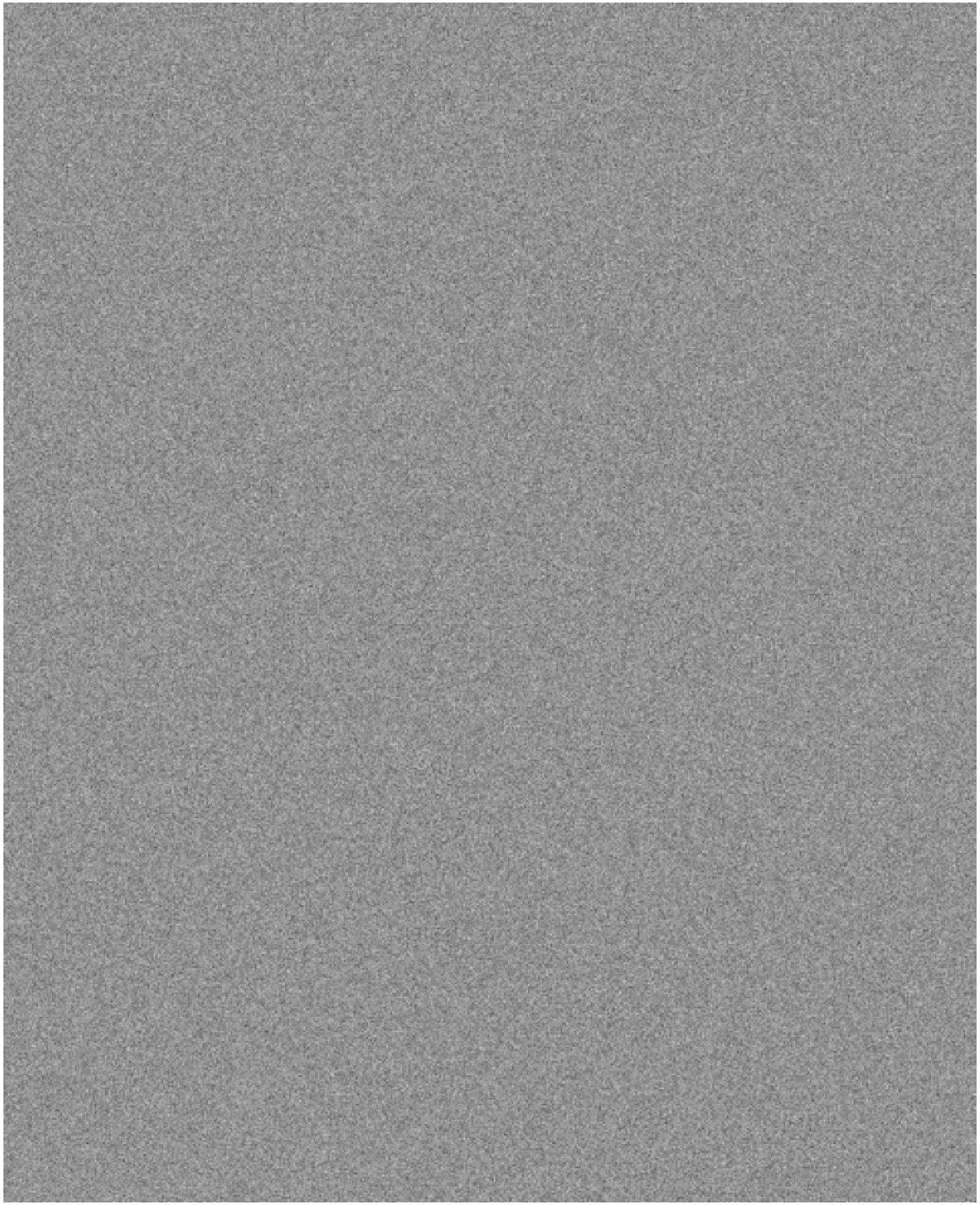
Image shows the noise values themselves.

sigma=1

Effect of sigma on Gaussian noise:

This shows the noise values added to the raw intensities of an image.

sigma=
16



Effect of sigma on Gaussian noise:

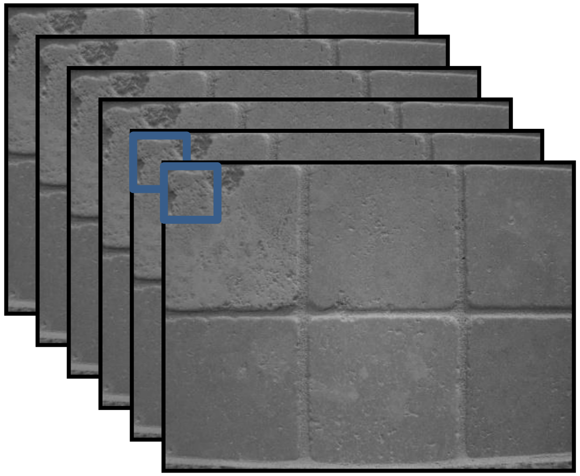
Image shows the noise values themselves.

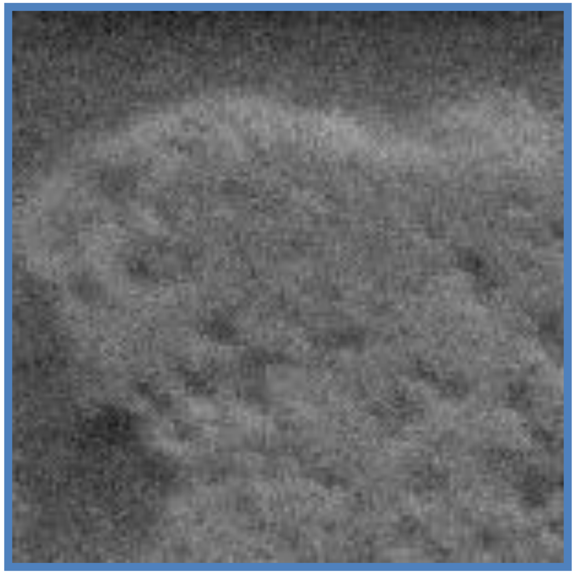
sigma=16

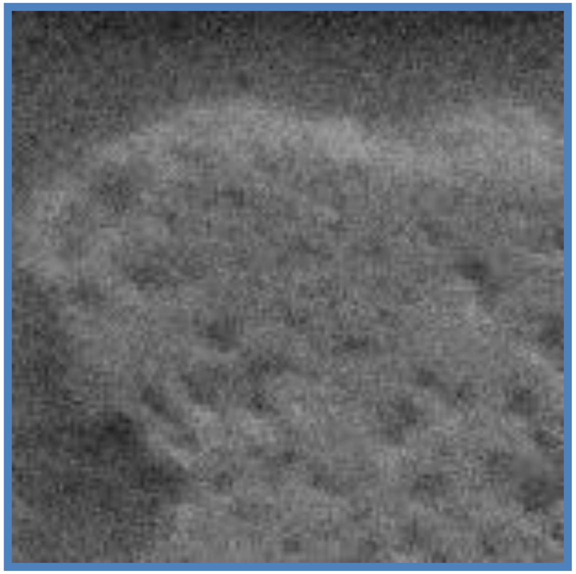
Effect of sigma on Gaussian noise

This shows the noise values added to the raw intensities of an image.

Motivation: noise reduction







- Even multiple images of the same static scene will not be identical.
- How could we reduce the noise, i.e., give an estimate of the true intensities?
- **What if there's only one image?**

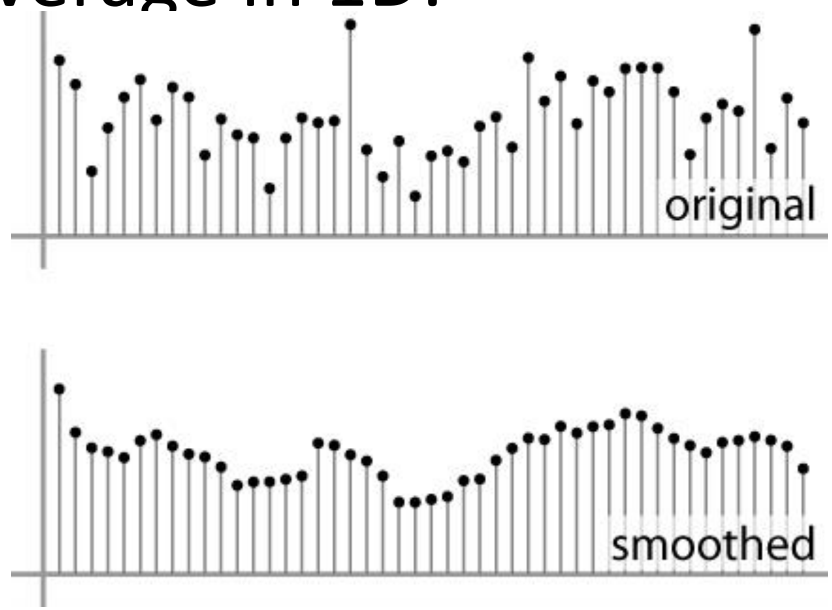
Slide credit:
Kristen Grauman

First attempt at a solution

- Let's replace each pixel with an average of all the values in its neighborhood
- Assumptions:
  - Expect pixels to be like their neighbors
  - Expect noise processes to be independent from pixel to pixel

First attempt at a solution

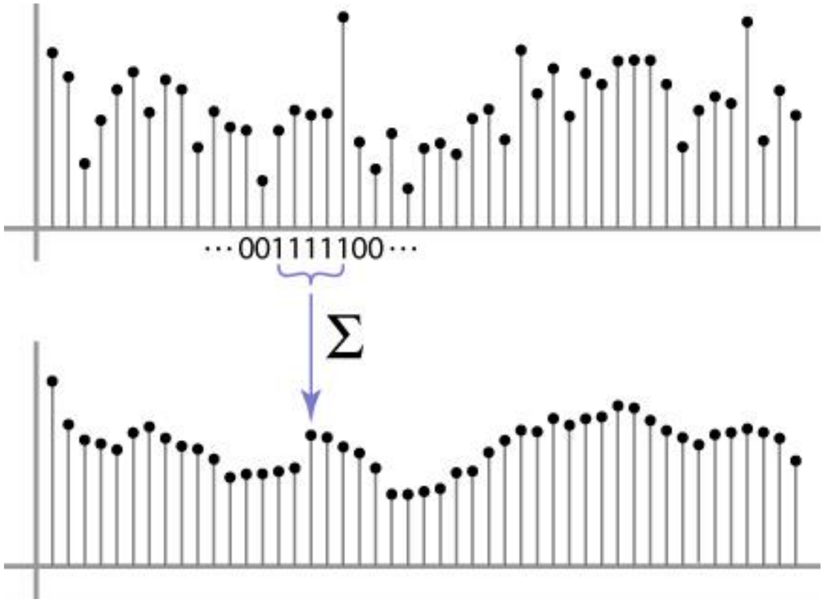
- Let’s replace each pixel with an average of all the values in its neighborhood
- Moving average in 1D:



Source: S. Marschner

Weighted Moving Average

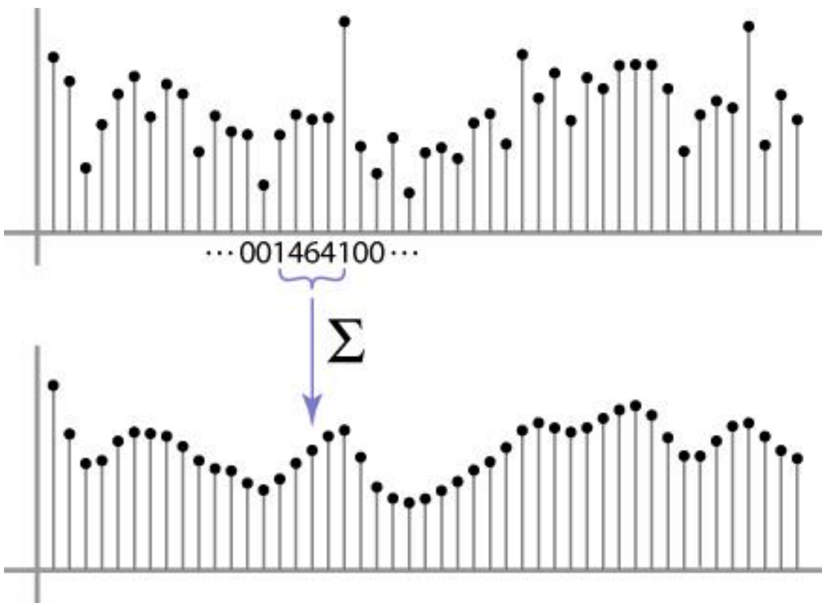
- Can add weights to our moving average
- *Weights* [1, 1, 1, 1, 1] / 5



Source: S. Marschner

Weighted Moving Average

- Non-uniform weights [1, 4, 6, 4, 1] / 16



Source: S. Marschner

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0								

Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10							

Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20						

Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30					

Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30	30				

Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

Source: S. Seitz

Correlation filtering

Say the averaging window size is 2k+1 x 2k+1:

$$G[i,j] = \underbrace{\frac{1}{(2k+1)^2}}_{\text{Attribute uniform weight to each pixel}} \underbrace{\sum_{u=-k}^{k} \sum_{v=-k}^{k} F[i+u, j+v]}_{\text{Loop over all pixels in neighborhood around image pixel } F[i,j]}$$

Attribute uniform weight to each pixel

Loop over all pixels in neighborhood around image pixel F[i,j]

Now generalize to allow different weights depending on neighboring pixel's relative position:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} \underbrace{H[u,v]}_{\text{Non-uniform weights}} F[i+u, j+v]$$

Non-uniform weights

Correlation filtering

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

This is called cross-correlation, denoted $G = H \otimes F$

Filtering an image: replace each pixel with a linear combination of its neighbors.

The filter "kernel" or "mask" $H[u,v]$ is the prescription for the weights in the linear combination.

Averaging filter

- What values belong in the kernel *H* for the moving average example?

$F[x, y]$ $\otimes$ $H[u, v]$ $G[x, y]$

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$H[u, v]$ = $\frac{1}{9}$ ×

1	1	1
1	1 ?	1
1	1	1

“box filter”

$G[x, y]$

	0	20	40	60	60				

$$G = H \otimes F$$

Boundary Issues

- What is the size of the output?

Padding Options	
numeric scalar, X	Input array values outside the bounds of the array are assigned the value X. When no padding option is specified, the default is 0.
'symmetric'	Input array values outside the bounds of the array are computed by mirror-reflecting the array across the array border.
'replicate'	Input array values outside the bounds of the array are assumed to equal the nearest array border value.
'circular'	Input array values outside the bounds of the array are computed by implicitly assuming the input array is periodic.
Output Size	
'same'	The output array is the same size as the input array. This is the default behavior when no output size options are specified.
'full'	The output array is the full filtered result, and so is larger than the input array.
Correlation and Convolution Options	
'corr'	imfilter performs multidimensional filtering using correlation, which is the same way that filter2 performs filtering. When no correlation or convolution option is specified, imfilter uses correlation.
'conv'	imfilter performs multidimensional filtering using convolution.

imfilter

Boundary Issues

- What is the size of the output?

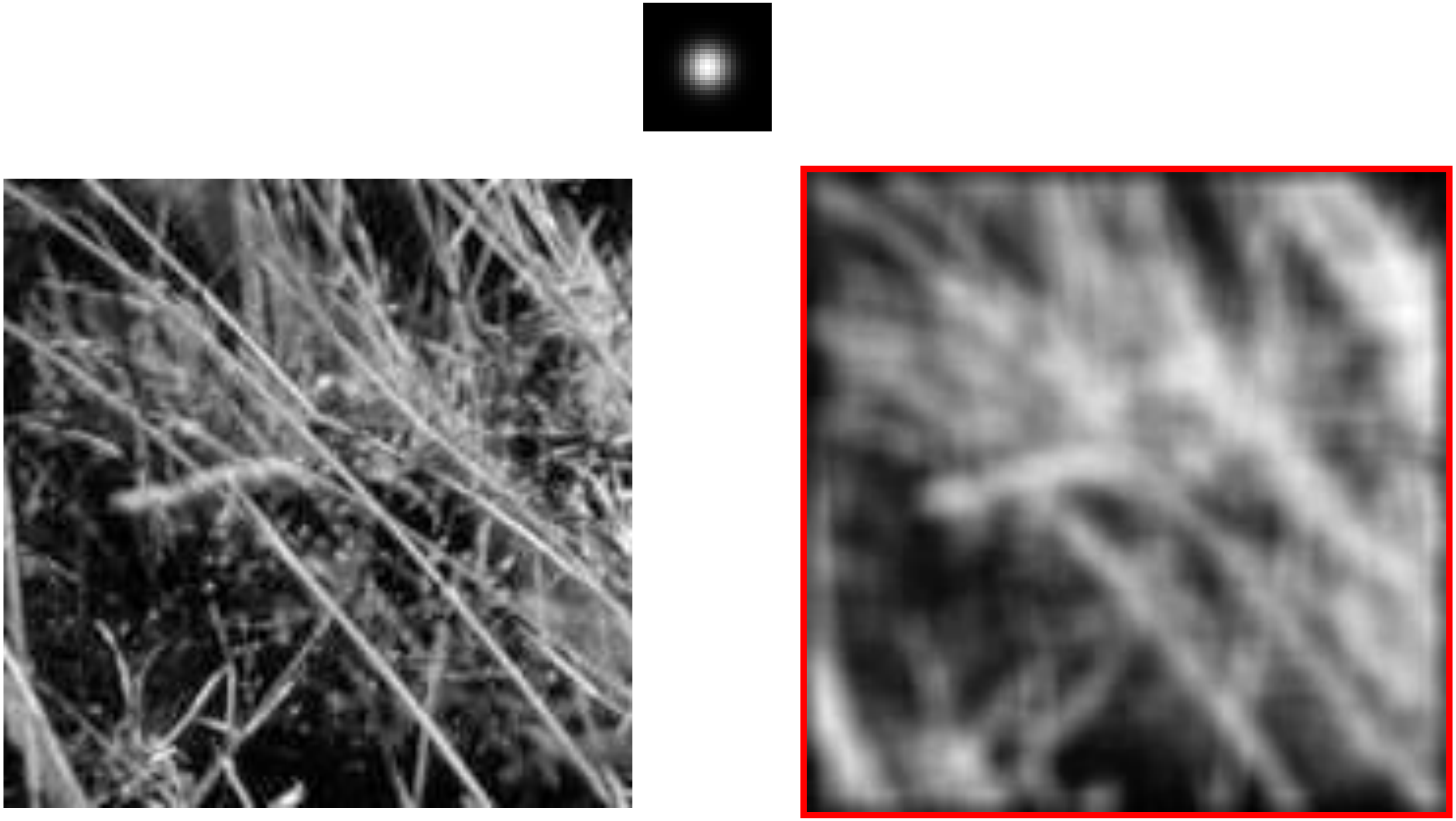
shape — Subsection of filtered data
`'same'` (default) | `'full'` | `'valid'`

Subsection of the filtered data, specified as one of these values:

- `'same'` — Return the central part of the filtered data, which is the same size as `X`.
- `'full'` — Return the full 2-D filtered data.
- `'valid'` — Return only parts of the filtered data that are computed without zero-padded edges.

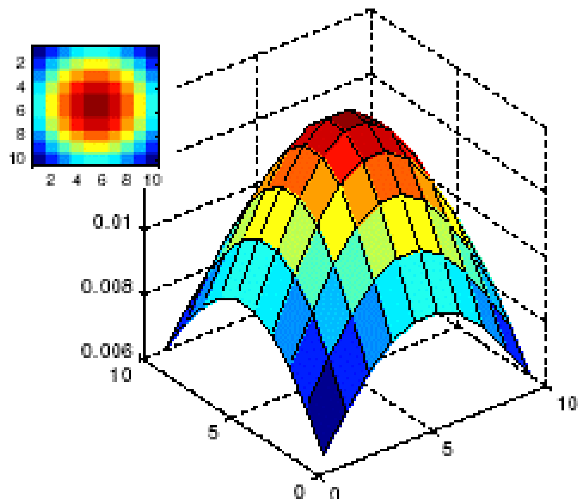
filter2

Smoothing with a Gaussian

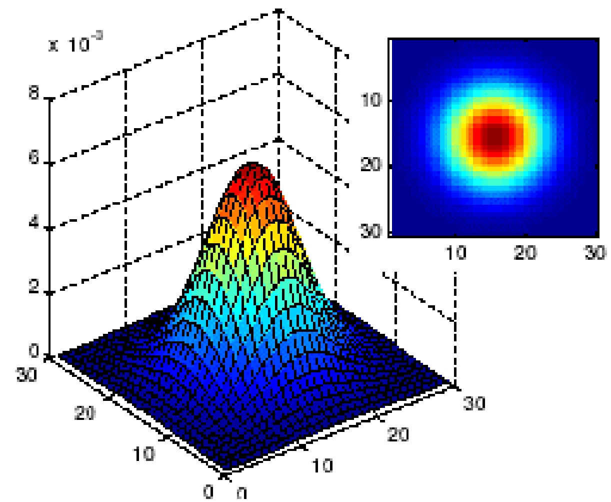


Gaussian filters

- What parameters matter here?
- **Size** of kernel or mask
  – Note, Gaussian function has infinite support, but discrete filters use finite kernels



σ = 5 with
10 x 10
kernel

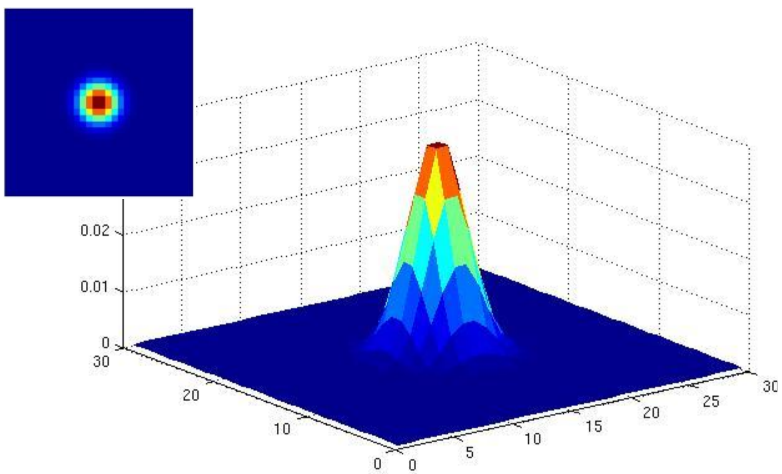


σ = 5 with
30 x 30
kernel

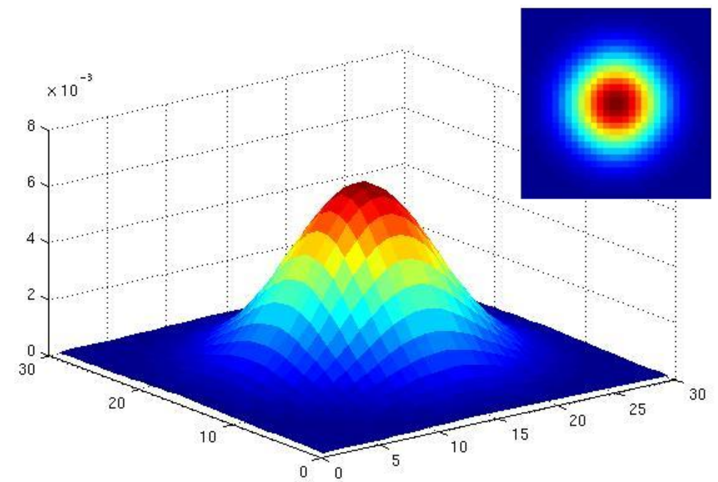
Slide credit:
Kristen Grauman

Gaussian filters

- What parameters matter here?
- **Variance** of Gaussian: determines extent of smoothing



σ = 2 with
30 x 30
kernel



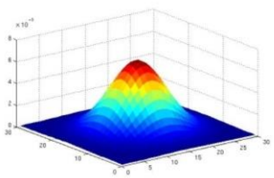
σ = 5 with
30 x 30
kernel

Slide credit:
Kristen Grauman

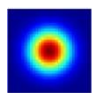
Matlab

```
>> hsize = 10;
>> sigma = 5;
>> h = fspecial('gaussian' hsize, sigma);
```

```
>> mesh(h);
```



```
>> imagesc(h);
```



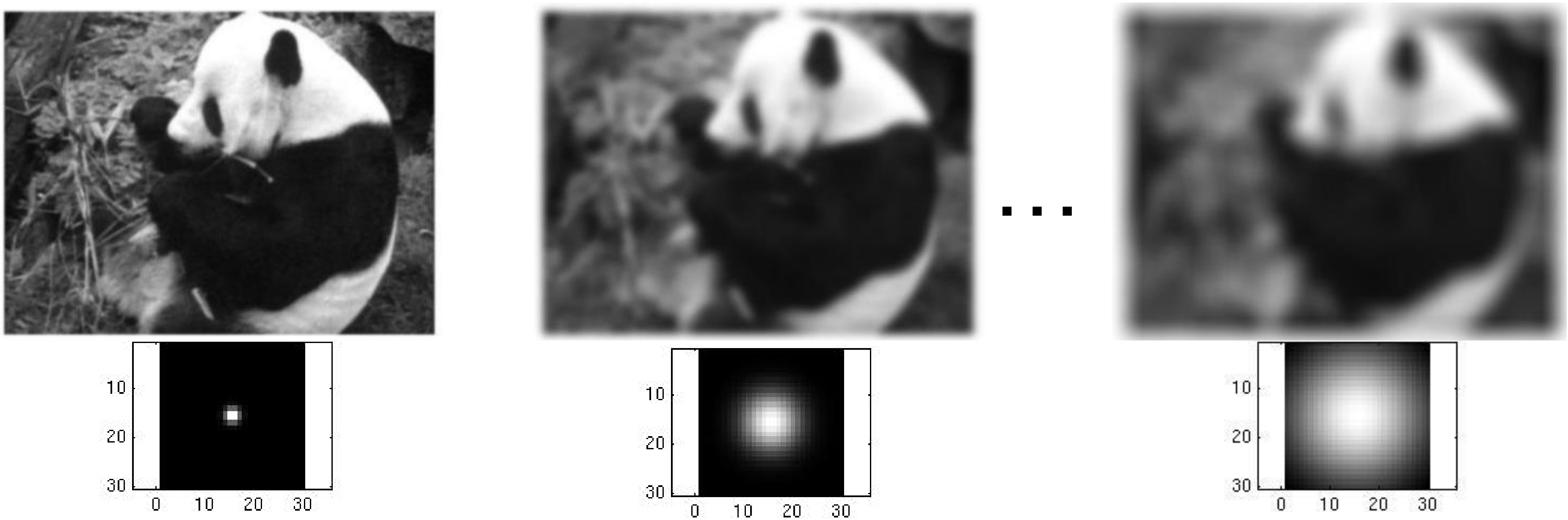
```
>> outim = imfilter(im, h); % correlation
>> imshow(outim);
```

outim

Slide credit:
Kristen Grauman

Smoothing with a Gaussian

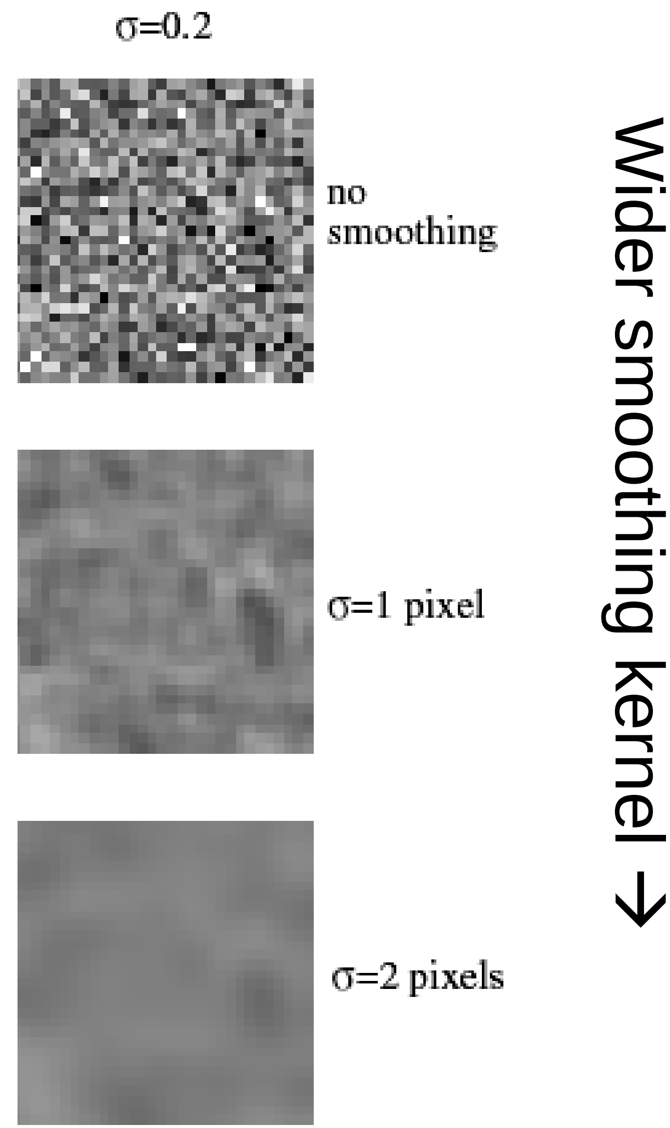
Parameter σ is the “scale” / “width” / “spread” of the Gaussian kernel, and controls the amount of smoothing.




```
for sigma=1:3:10
   h = fspecial('gaussian', fsize, sigma);
   out = imfilter(im, h);
   imshow(out);
   pause;
end
```

Slide credit:
Kristen Grauman

Keeping the two Gaussians in play straight…



Properties of smoothing filters

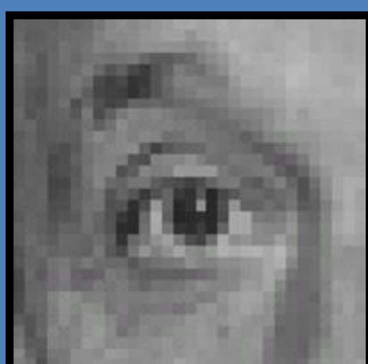
- Smoothing
  - Values positive
  - Sum to 1 → constant regions same as input
  - Amount of smoothing proportional to mask size
  - Remove “high-frequency” components; “low-pass” filter

Slide credit:
Kristen Grauman

Predict the outputs using correlation filtering

$$* \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = ?$$

$$* \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = ?$$



$$* \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = ?$$

Slide credit:
Kristen Grauman

Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

?

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

Filtered
(no change)

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	0	1
0	0	0

?

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	0	1
0	0	0

Shifted left by 1 pixel with correlation

Source: D. Lowe

Practice with linear filters

Original

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

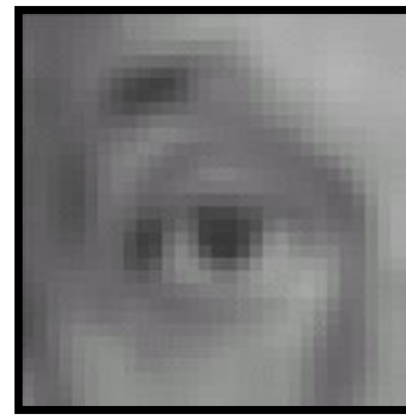
?

Source: D. Lowe

Practice with linear filters

Original

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$



Blur (with a box filter)

Source: D. Lowe

Practice with linear filters

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

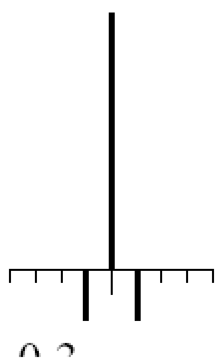
?

Source: D. Lowe

Practice with linear filters

Original

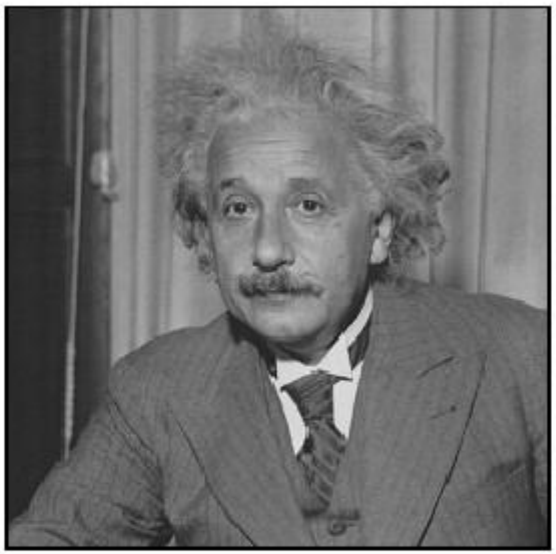
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



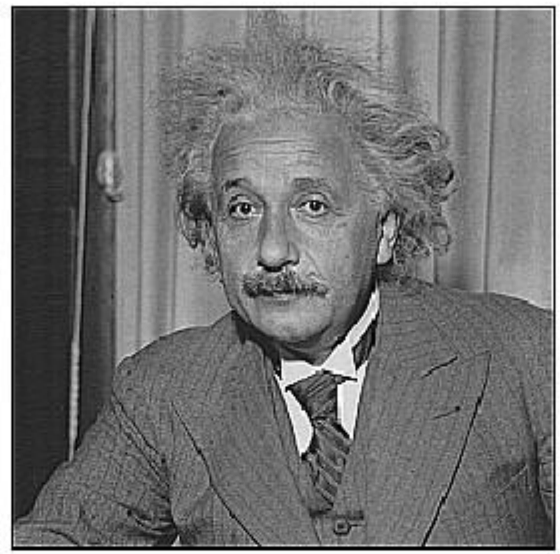
Sharpening filter:
accentuates differences
with local average

Source: D. Lowe

Filtering examples: sharpening



before



after

More Examples

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

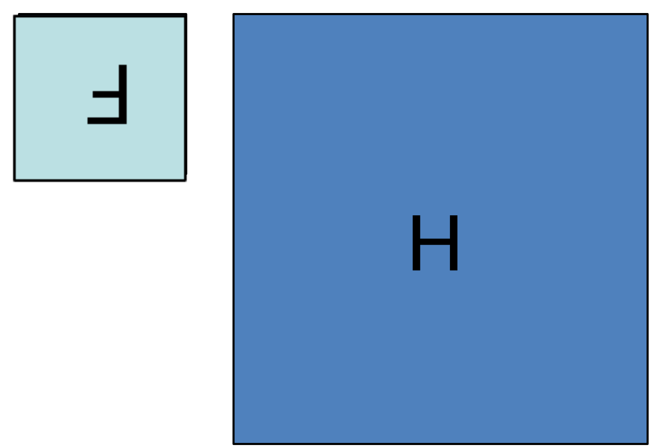
Convolution

- Convolution:
  - Flip the filter in both dimensions (bottom to top, right to left)
  - Then apply cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

$$G = H \star F$$

Notation for convolution operator



Slide Credit: Kristen Grauman

Properties of convolution

- **Shift invariant:**
  - Operator behaves the same everywhere, i.e. the value of the output depends on the pattern in the image neighborhood, not the position of the neighborhood.
- **Superposition:**
  - $h * (f1 + f2) = (h * f1) + (h * f2)$

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Properties of convolution

- Commutative:

  $f * g = g * f$

- Associative

  $(f * g) * h = f * (g * h)$

- Distributes over addition

  $f * (g + h) = (f * g) + (f * h)$

- Scalars factor out

  $kf * g = f * kg = k(f * g)$

- Identity:

  unit impulse e = [..., 0, 0, 1, 0, 0, ...]. $f * e = f$

Slide Credit: Kristen Grauman

Separability

- In some cases, filter is separable, and we can factor into two steps:
  - Convolve all rows with a 1D filter
  - Convolve all columns with a 1D filter

$$\begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 \end{bmatrix} \circ \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 \end{bmatrix}$$

Slide Credit: Kristen Grauman

Effect of smoothing filters

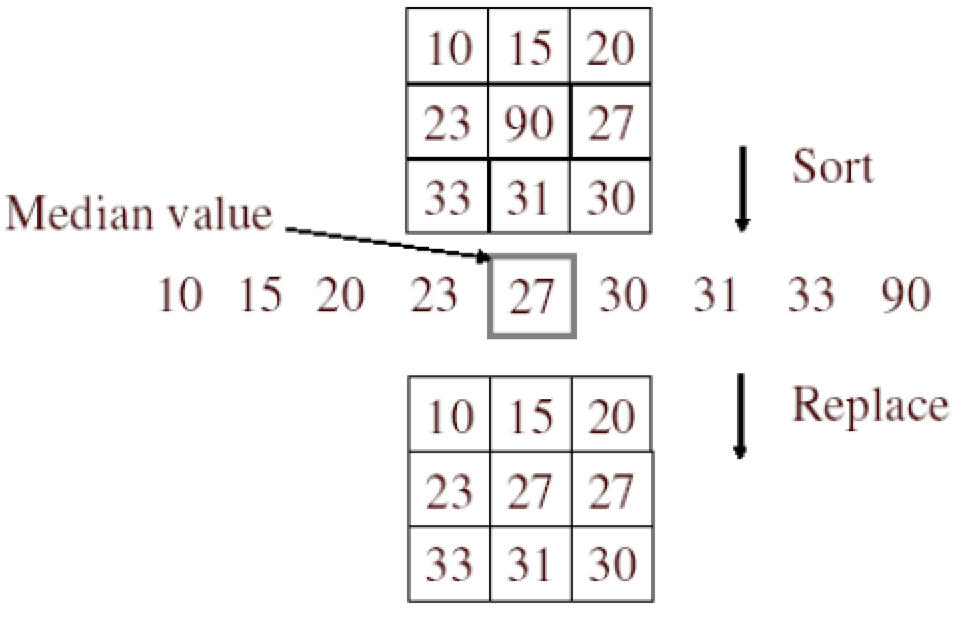
5x5

Additive Gaussian noise

Salt and pepper noise

Slide Credit: Kristen Grauman

Median filter



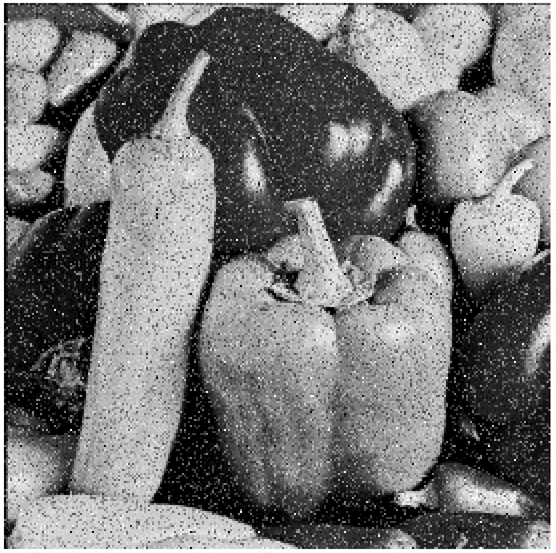
- No new pixel values introduced
- Removes spikes: good for impulse, salt & pepper noise
- Non-linear filter

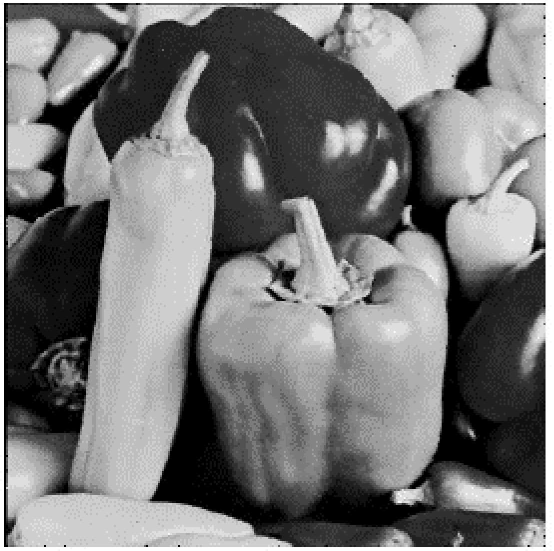
Slide Credit: Kristen Grauman

Slide Credit: Kristen Grauman

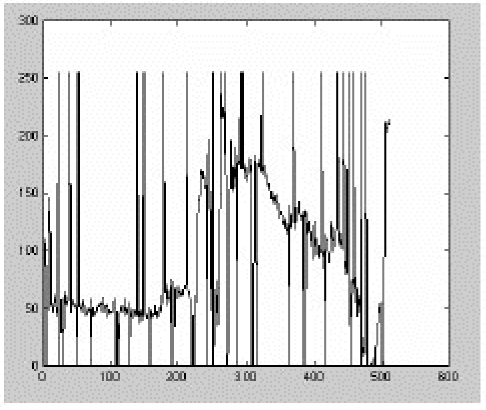
Median filter

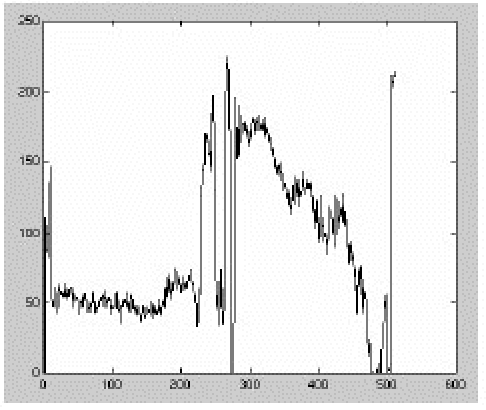
Salt and pepper noise





Median filtered





Plots of a row of the image

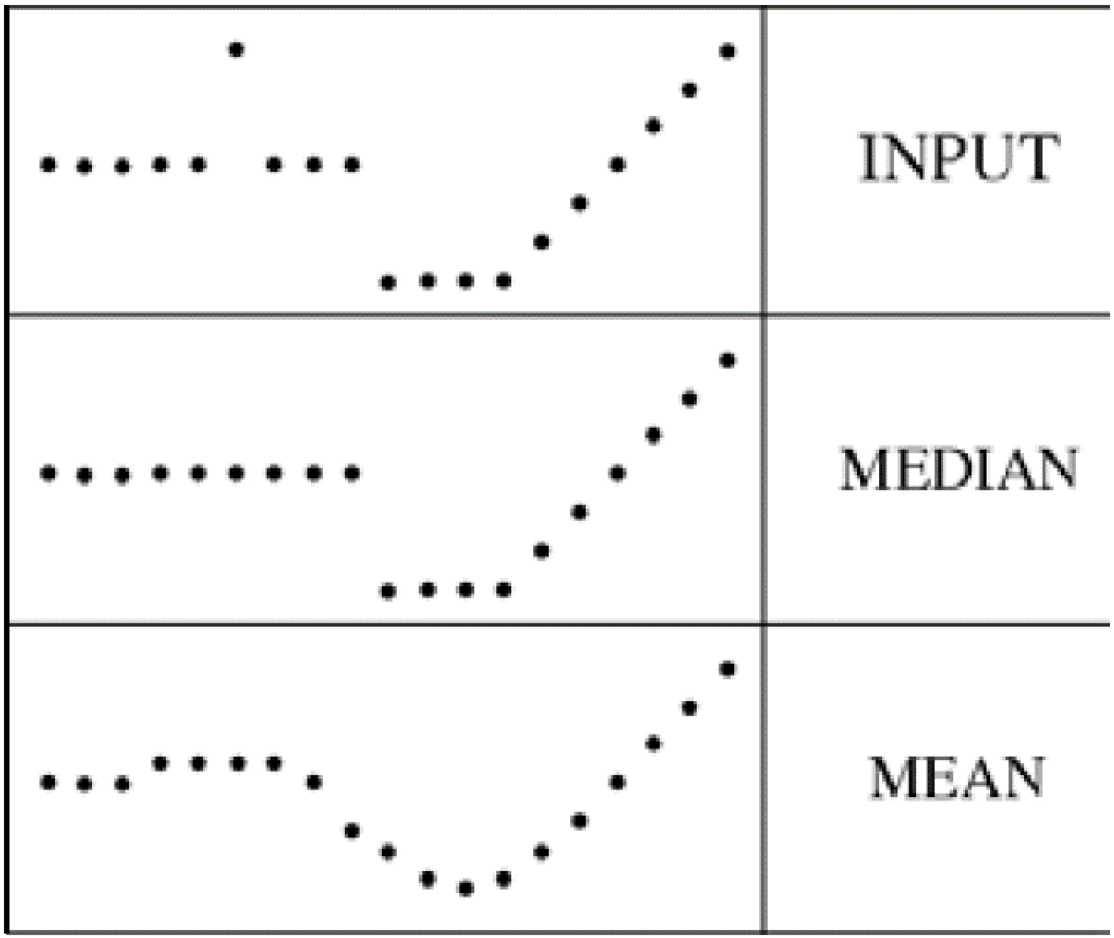
Matlab: output im = medfilt2(im, [h w]);

Source: M. Hebert

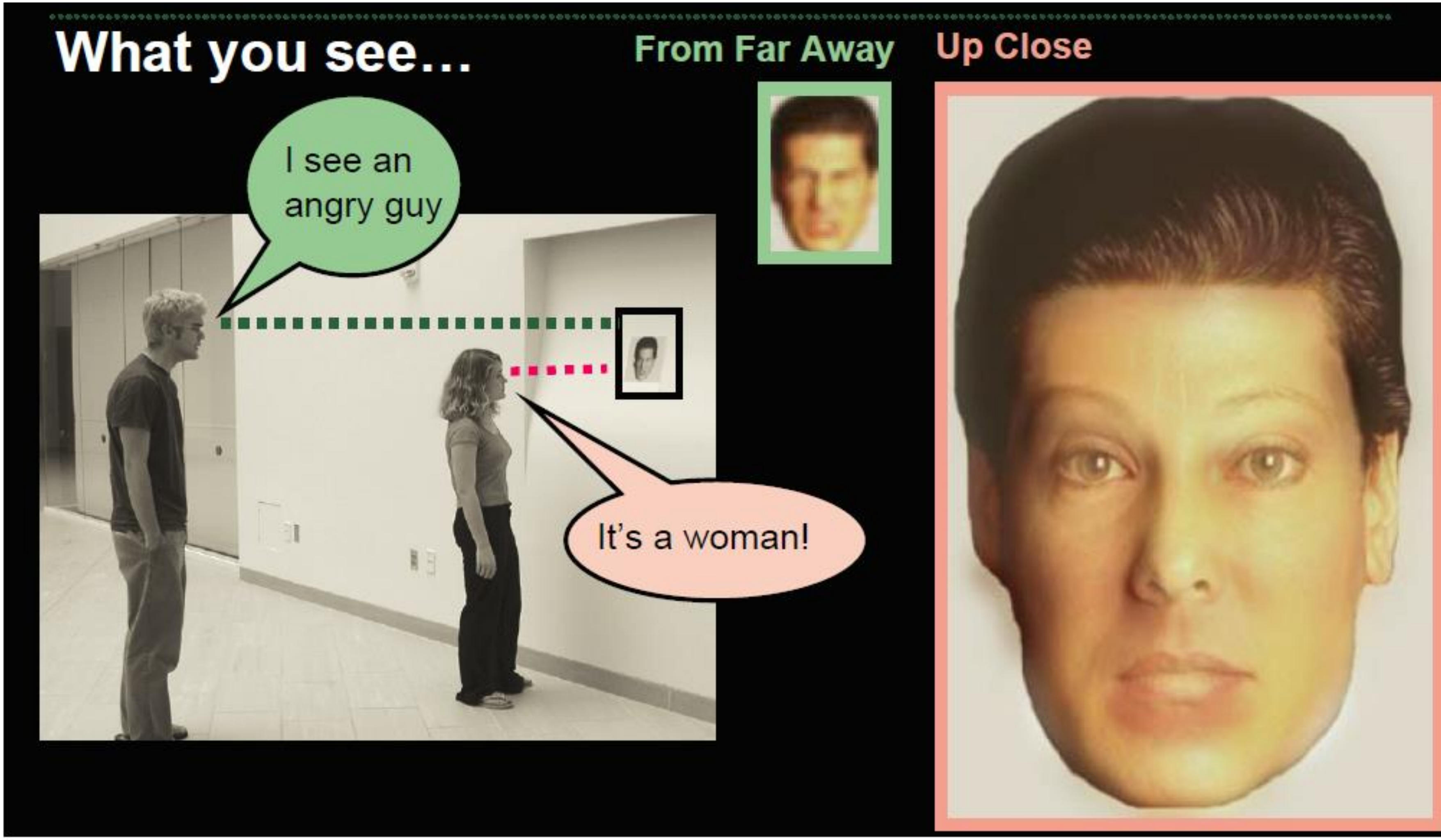
Median filter

Slide Credit: Kristen Grauman

- Median filter is edge preserving



Filtering application: Hybrid Images

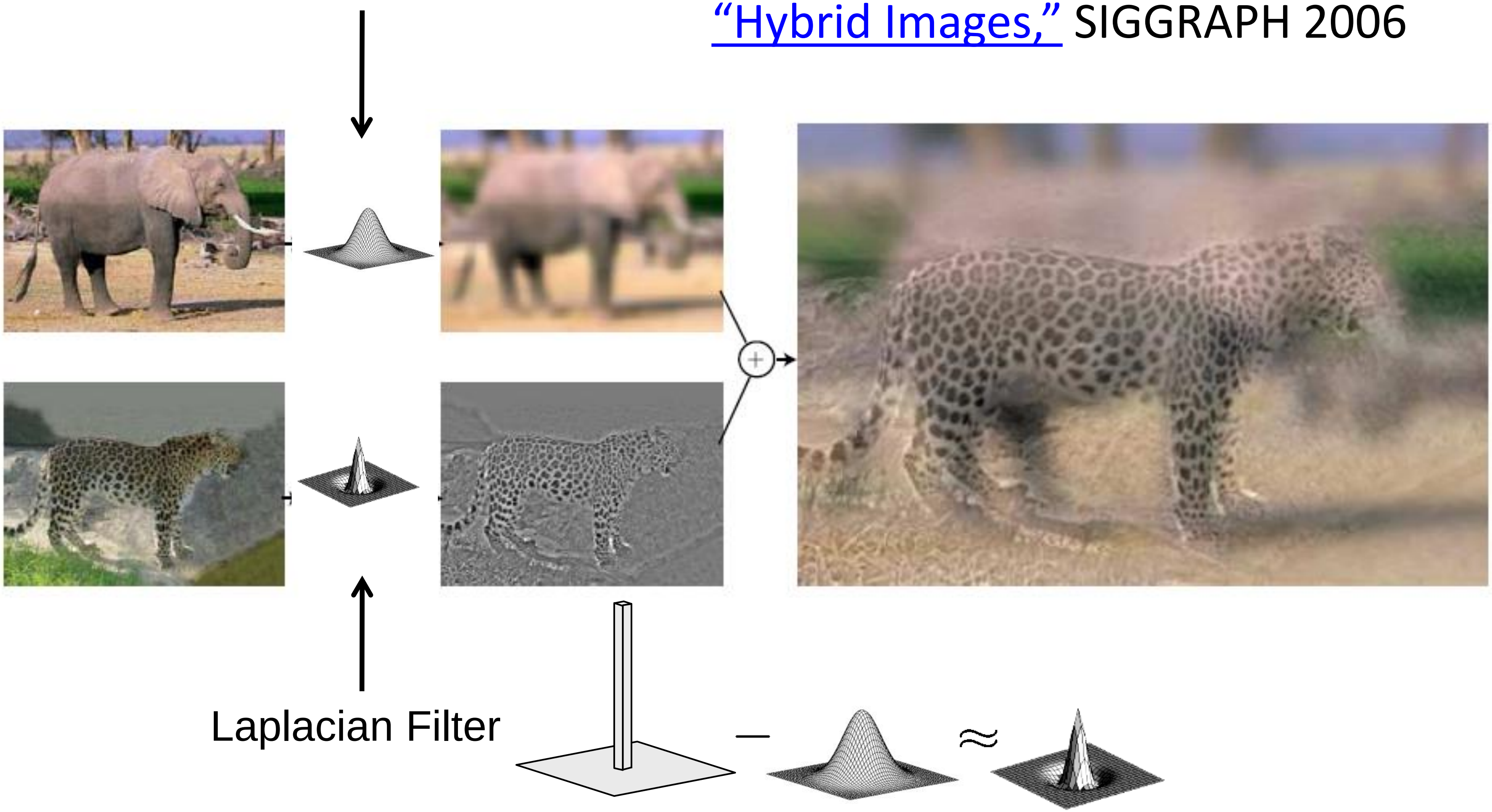


Aude Oliva & Antonio Torralba & Philippe G Schyns, SIGGRAPH 2006

Application: Hybrid Images

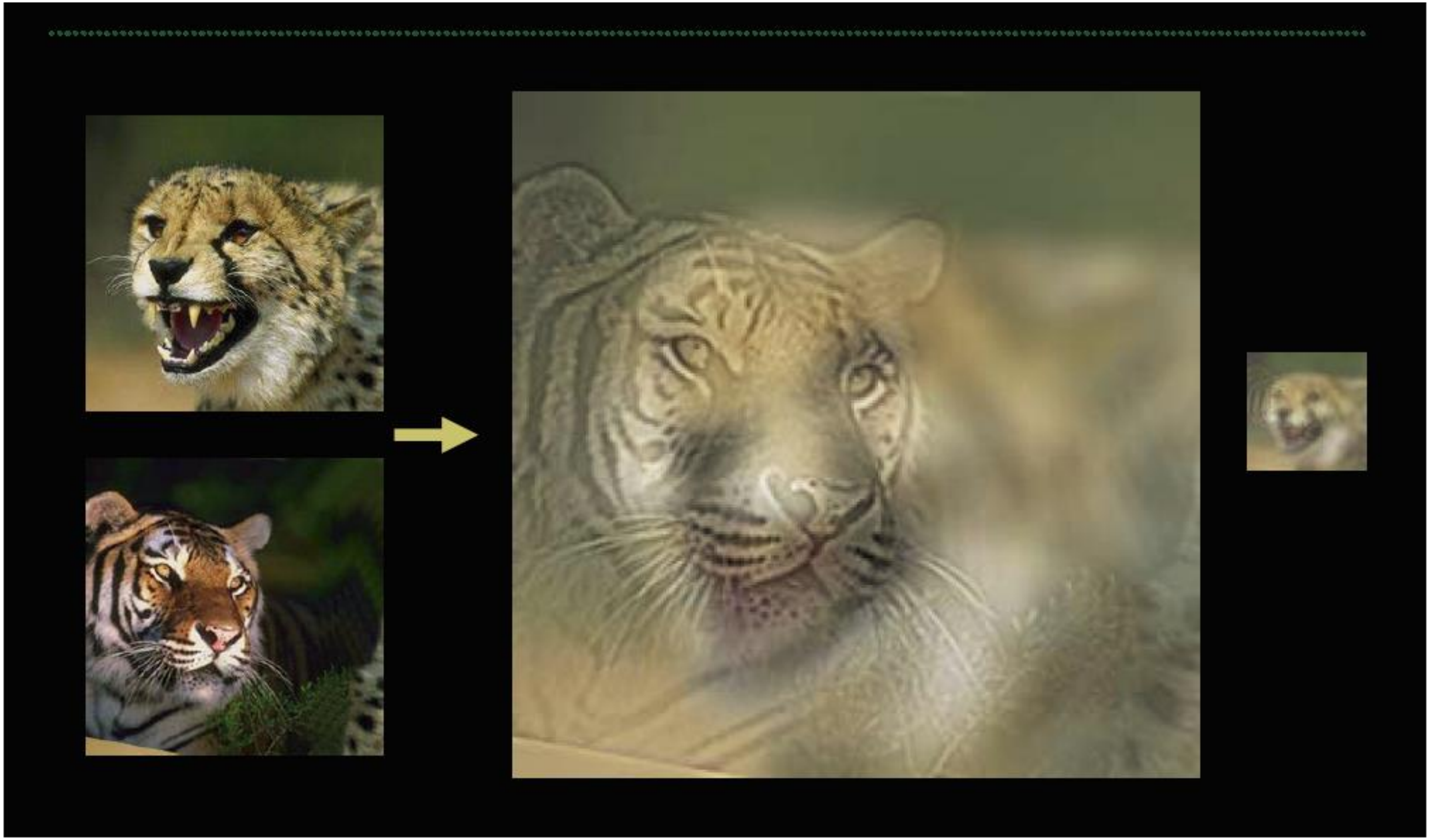
A. Oliva, A. Torralba, P.G. Schyns, "Hybrid Images," SIGGRAPH 2006

Gaussian Filter



Laplacian Filter

unit impulse — Gaussian ≈ Laplacian of Gaussian



Aude Oliva & Antonio Torralba & Philippe G Schyns, SIGGRAPH 2006

Changing expression

SIGGRAPH2006

Sad ⟷ Surprised

Aude Oliva & Antonio Torralba & Philippe G Schyns, SIGGRAPH 2006

Summary

- Image “noise”
- Linear filters and convolution useful for
  - Enhancing images (smoothing, removing noise)
    - Box filter
    - Gaussian filter
    - Impact of scale / width of smoothing filter
  - Detecting features (next time)
- Separable filters more efficient
- Median filter: a non-linear filter, edge-preserving

Coming up

- **Wednesday:**
  - Filtering part 2: filtering for features (edges, gradients, seam carving application)

- **Tomorrow:**
  - Assignment 0 is due on Canvas 11:59 PM

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