Slide Credit: Mirela Ben-Chen

Mesh Simplification

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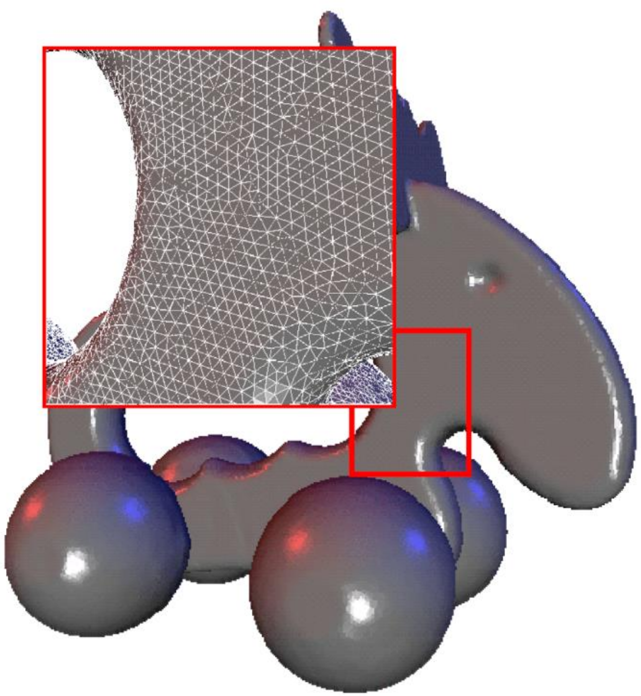
Feb. 23th 2017

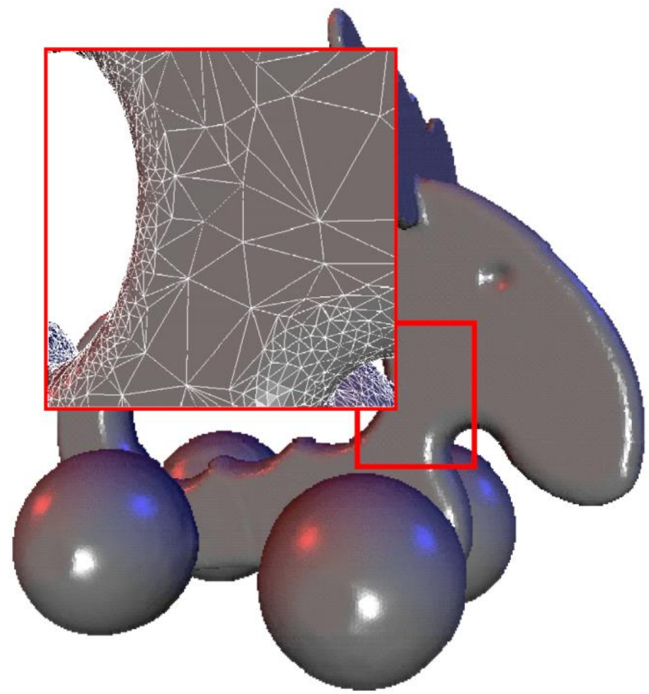
Multiple Simplification

Applications

- Oversampled 3D scan data



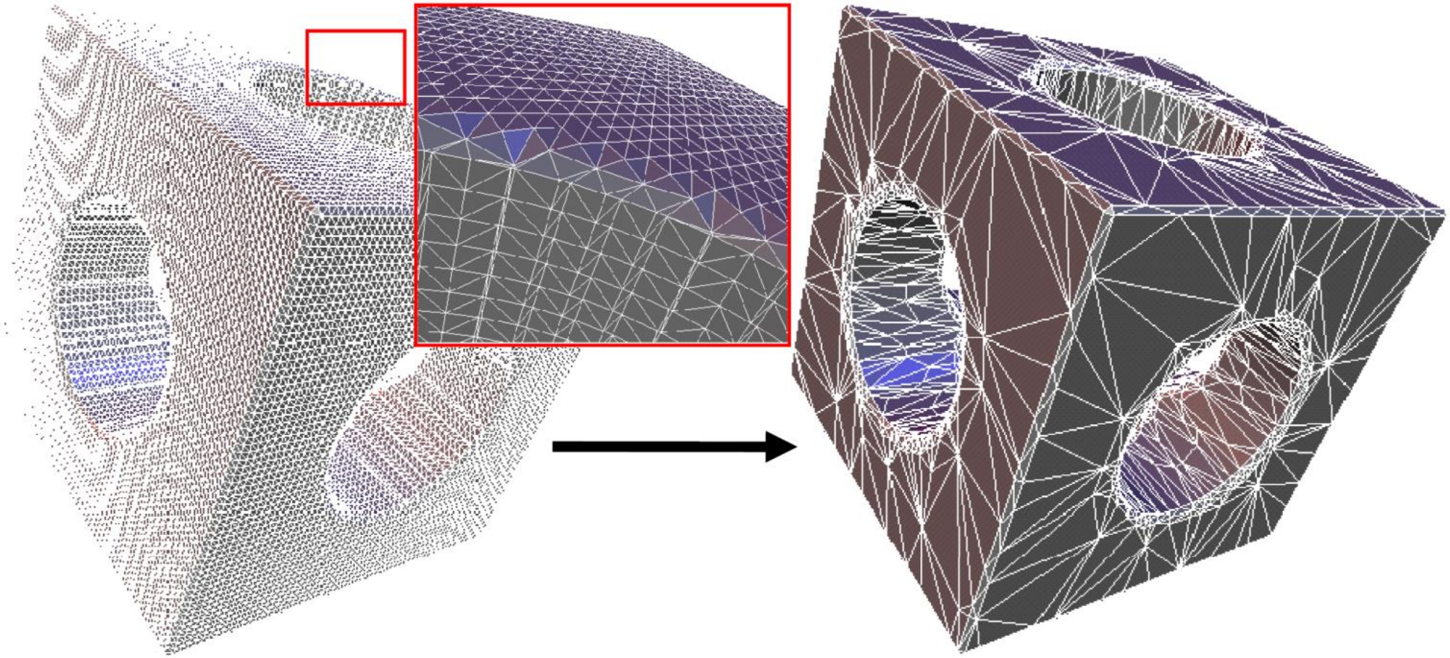
~150k triangles



~80k triangles

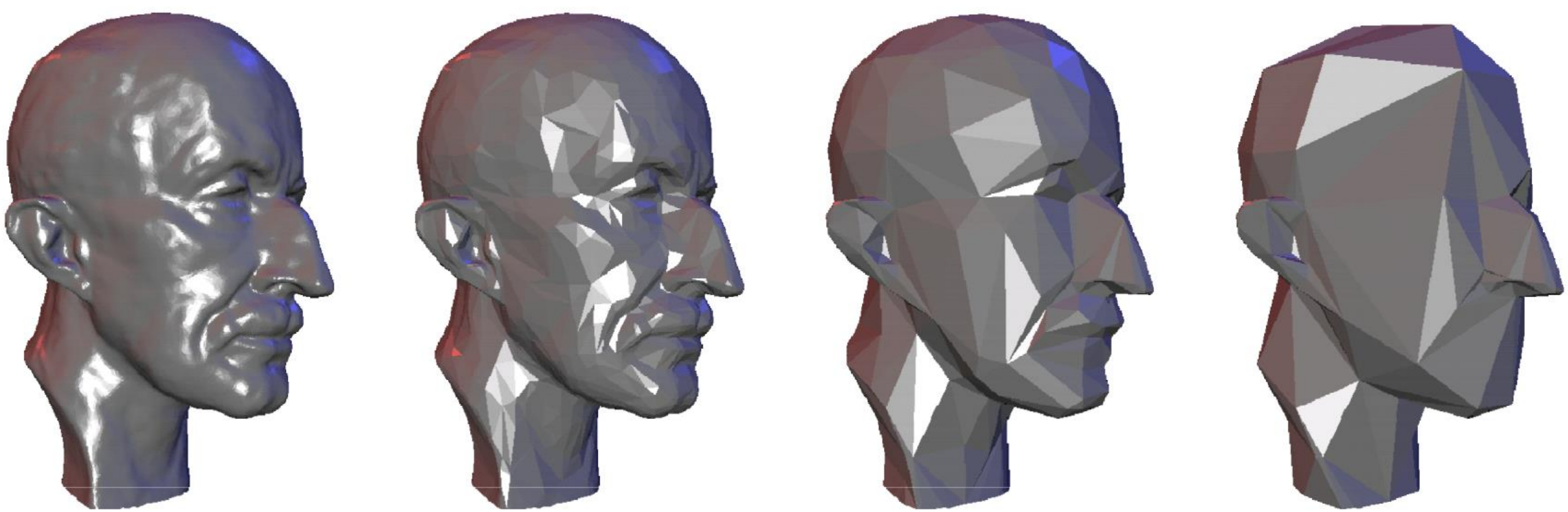
Applications

- Overtessellation: E.g. iso-surface extraction



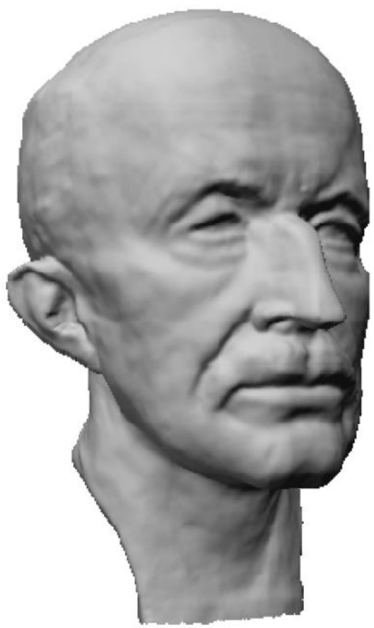
Applications

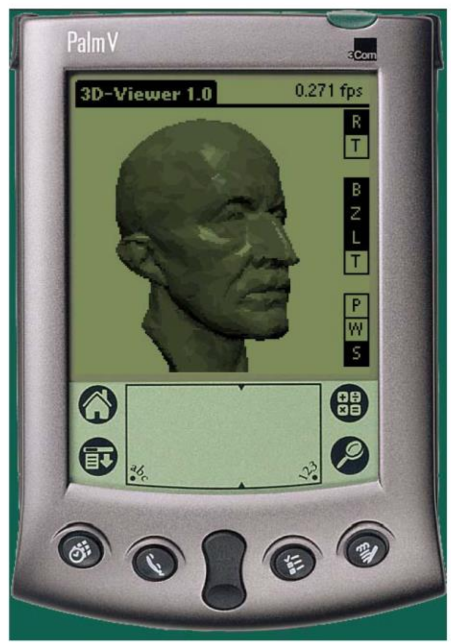
- Multi-resolution hierarchies for
  - efficient geometry processing
  - level-of-detail (LOD) rendering



Applications

- Adaptation to hardware capabilities

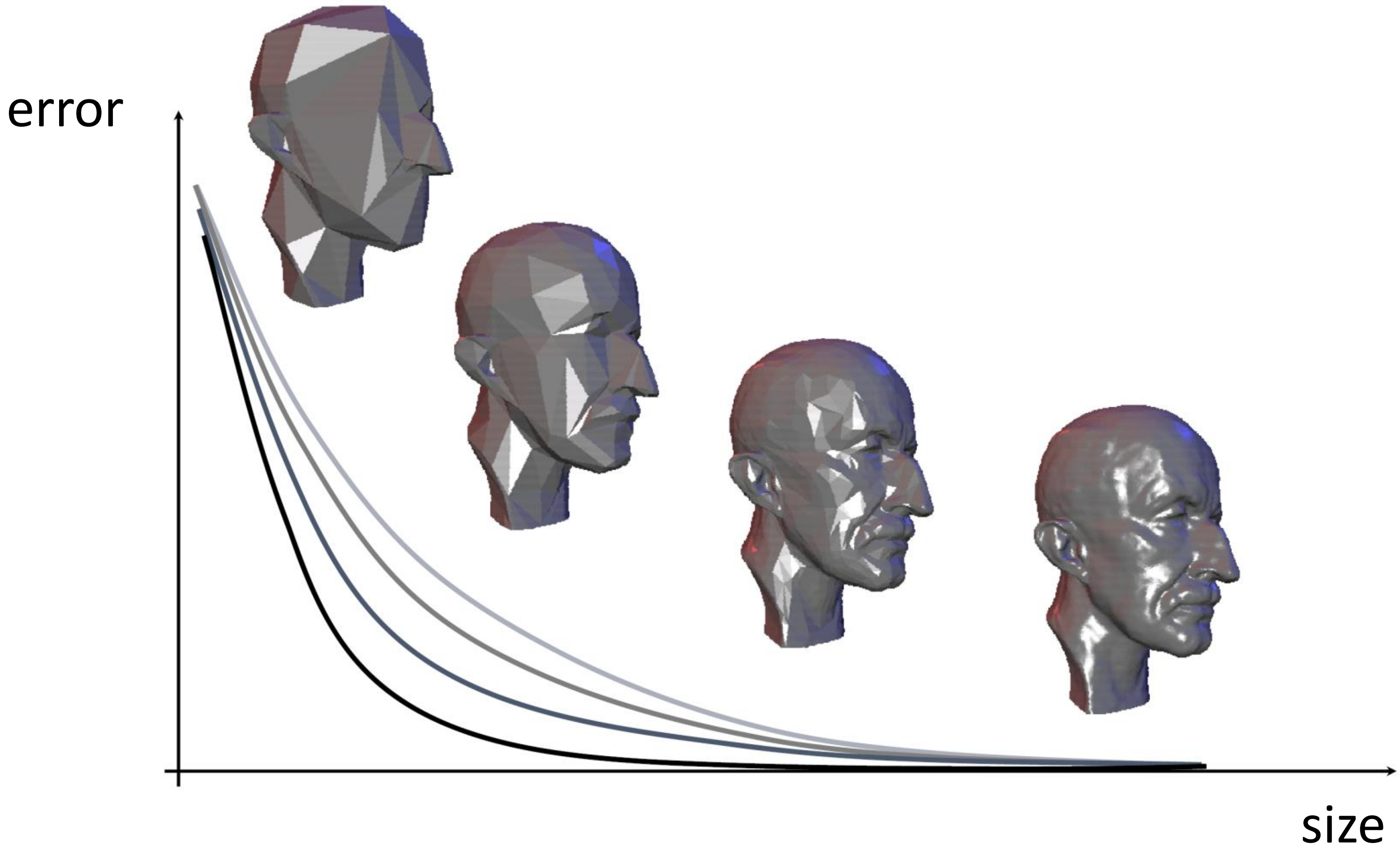




1999

2012

Size-Quality Tradeoff



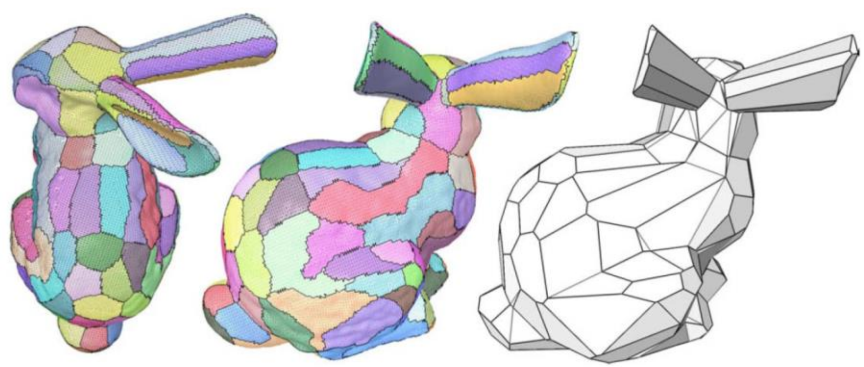
Problem Statement

- Given: $M = (V,F)$
- Find: $M' = (V',F')$ such that
  - $|V'| = n < |V|$ and $d(M,M')$ is minimal, or
  - $d(M,M') < eps$ and $|V'|$ is minimal

- Respect additional fairness criteria
  - Normal deviation, triangle shape, scalar attributes, etc.

Mesh Decimation Methods

- **Vertex clustering**
- **Incremental decimation**
- Resampling
- Mesh approximation

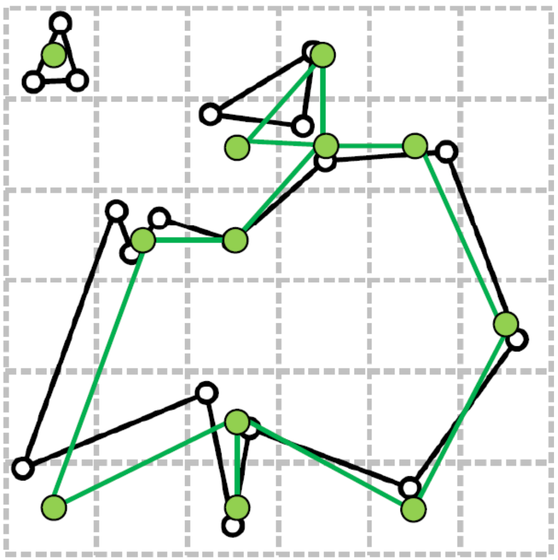


Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation
- Topology changes

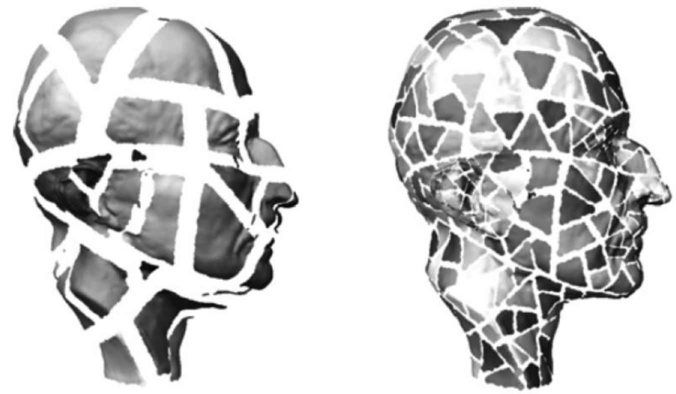
Vertex Clustering

- Cluster Generation
  - Uniform 3D grid
  - Map vertices to cluster cells
- Computing a representative
- Mesh generation
- Topology changes



Vertex Clustering

- Cluster Generation
  - Hierarchical approach
  - Top-down or bottom-up



- Computing a representative
- Mesh generation
- Topology changes

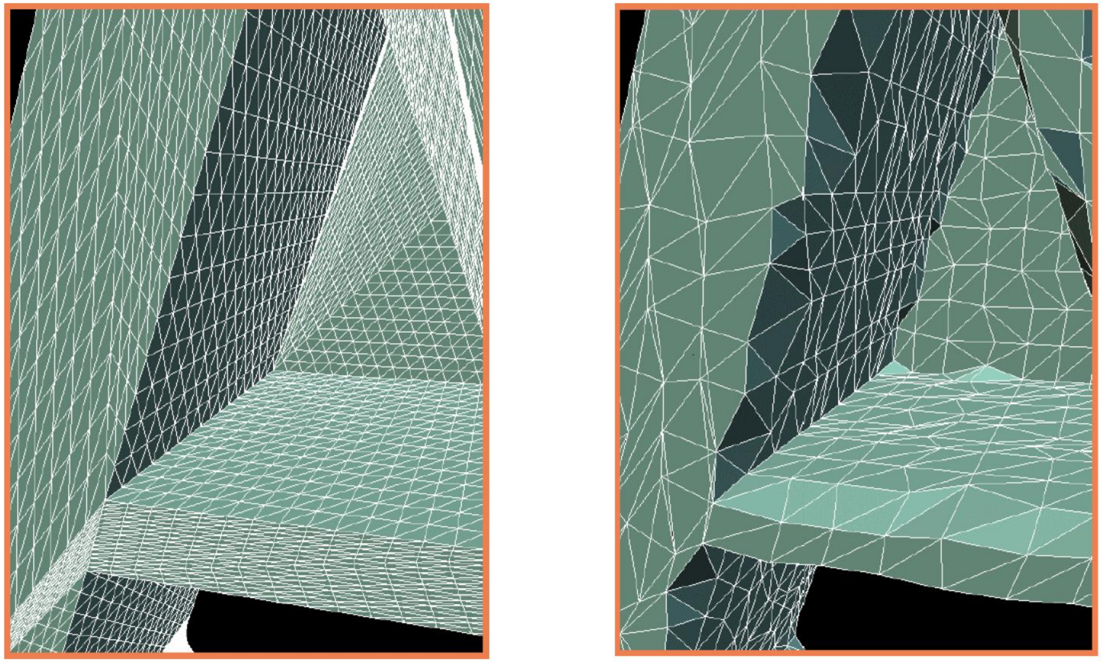
Vertex Clustering

- Cluster Generation

- Computing a representative
  - Average/median vertex position
  - Error quadrics

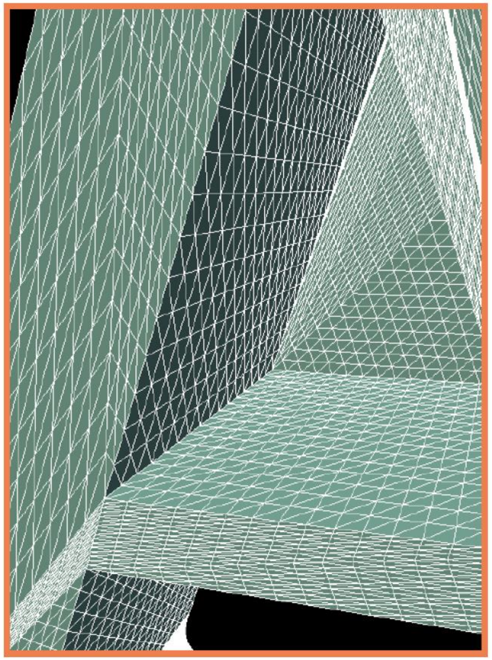
- Mesh generation
- Topology changes

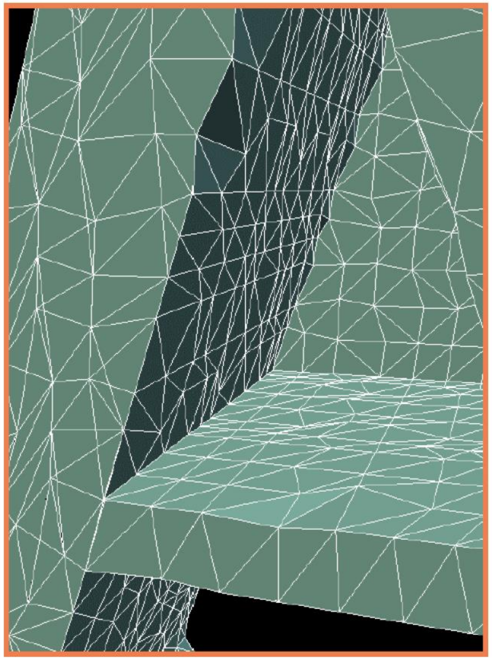
Computing a Representative



Average vertex position

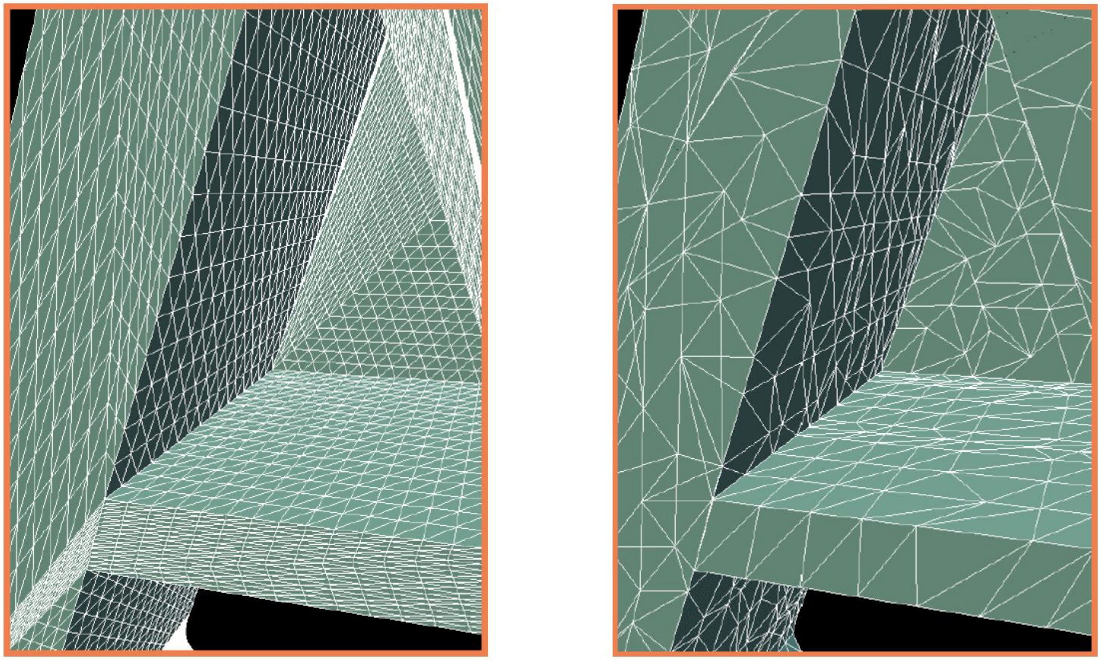
Computing a Representative





Median vertex position

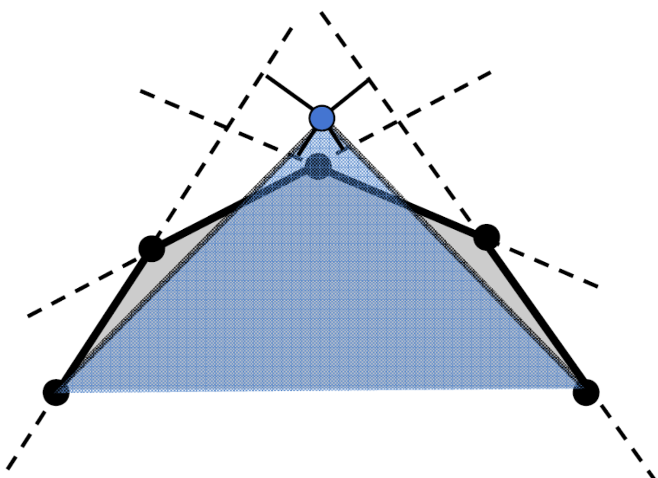
Computing a Representative



Error quadrics

Error Quadrics

- Patch is expected to be piecewise flat
- Minimize distance to neighboring triangles' planes



Error Quadrics

- Squared distance of point *p* to plane *q*

$$p = (x, y, z, 1)^T, \quad q = (a, b, c, d)^T$$

$$dist(q, p)^2 = (q^T p)^2 = p^T (q q^T) p =: p^T Q_q p$$

$$Q_q = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix}$$

Error Quadrics

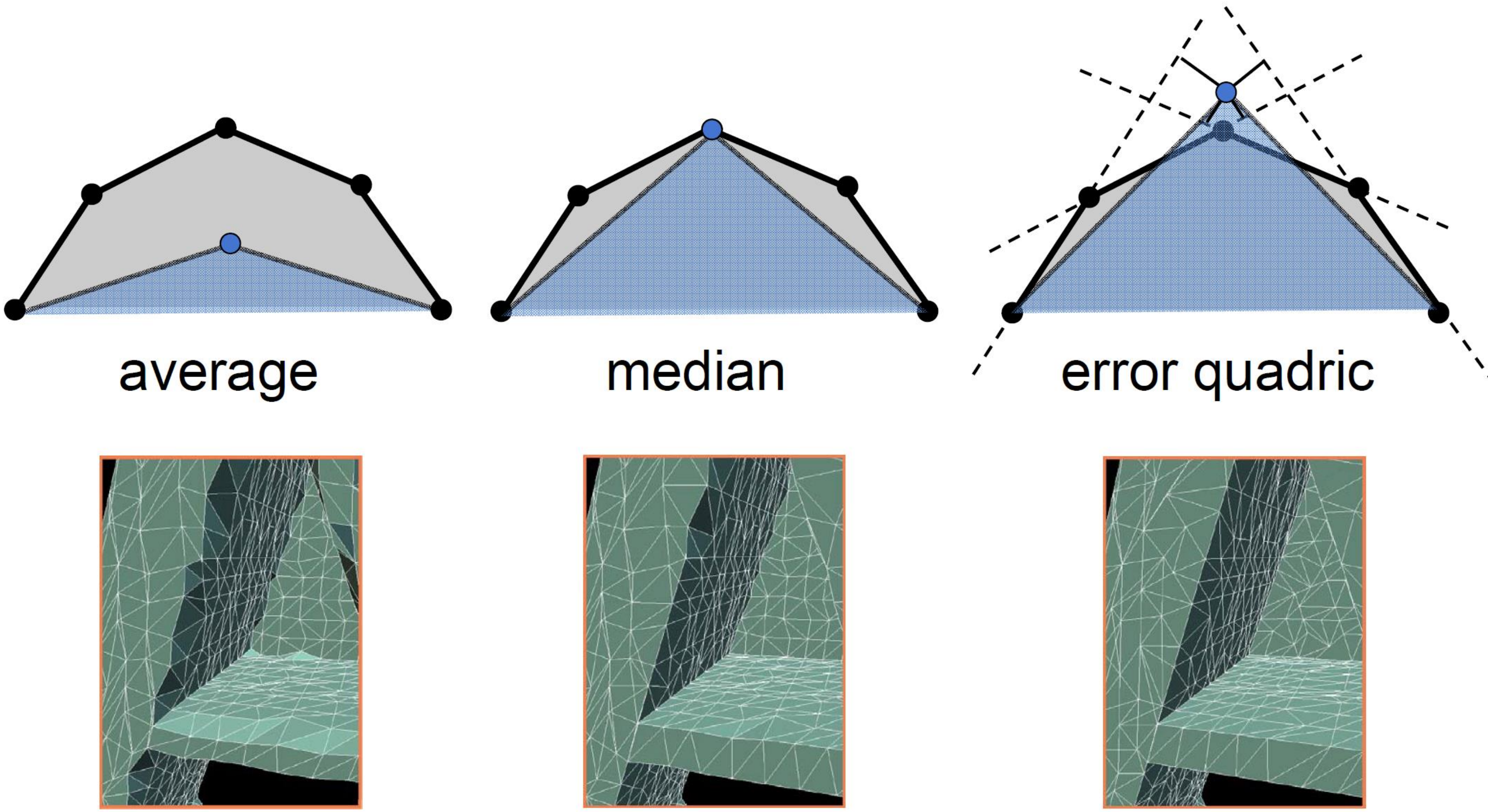
- Sum distances to planes q_i of vertex' neighboring triangles

$$\sum_i dist(q_i, p)^2 = \sum_i p^T Q_{q_i} p = p^T \left(\sum_i Q_{q_i} \right) p =: p^T Q_p p$$

- Point p* that minimizes the error satisfies:

$$\begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} p^* = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Comparison



Vertex Clustering

- Cluster Generation
- Computing a representative

- Mesh generation
  - Clusters p<-> $\{p_0,\ldots, p_n\}$, q<-> $\{q_0,\ldots, q_m\}$

- Topology changes

Vertex Clustering

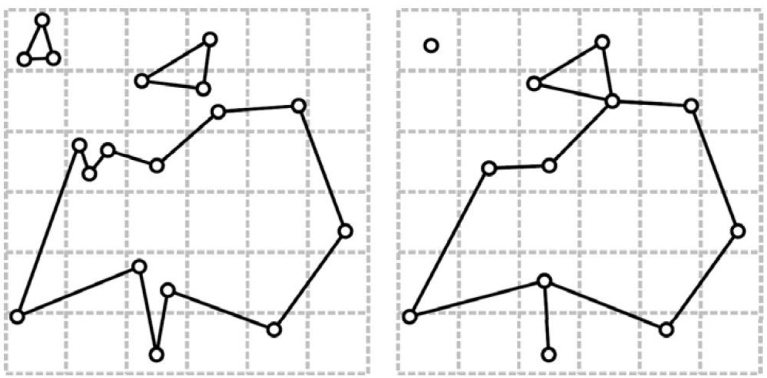
- Cluster Generation
- Computing a representative

- Mesh generation
  - Clusters p<-> $\{p_0,\ldots, p_n\}$, q<-> $\{q_0,\ldots, q_m\}$
  - Connect (p,q) if there was an edge (p_i, q_i)
- Topology changes

Vertex Clustering

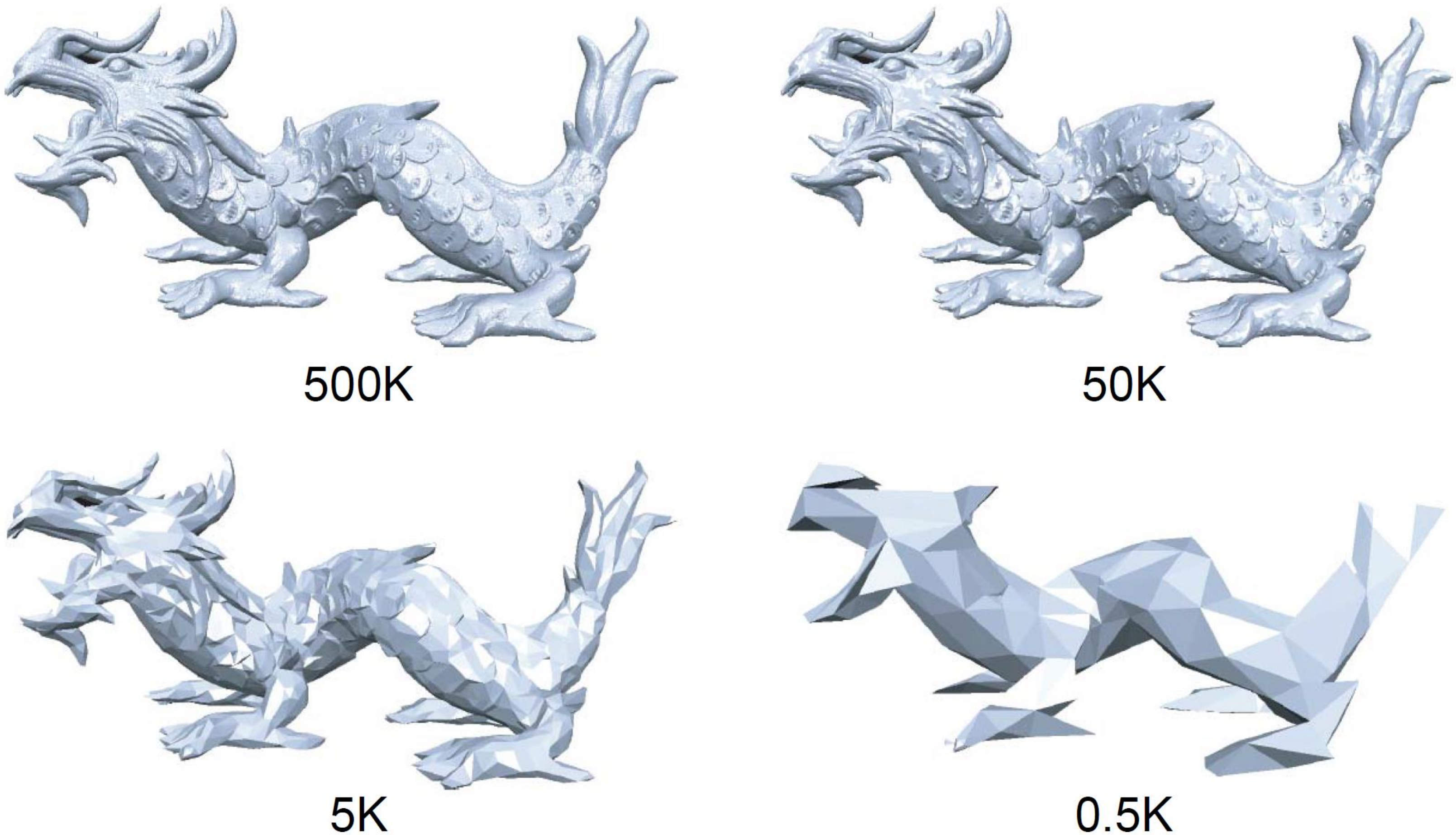
- Cluster Generation
- Computing a representative
- Mesh generation

- Topology changes
  - If different sheets pass through one cell
  - Can be non-manifold



Incremental Decimation

Incremental Decimation



Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria

General Setup

- Repeat:
  - Pick mesh region
  - Apply decimation operator
- Until no further reduction possible

Greedy Optimization

- For each region
  - evaluate quality after decimation
  - enqeue(quality, region)

- Repeat:
  - get best mesh region from queue
  - apply decimation operator
  - update queue
- Until no further reduction possible

Global Error Control

- For each region
  - evaluate quality after decimation
  - enqeue(quality, region)

- Repeat:
  - get best mesh region from queue
  - If error < eps
    - Apply decimation operator
    - Update queue
- Until no further reduction possible

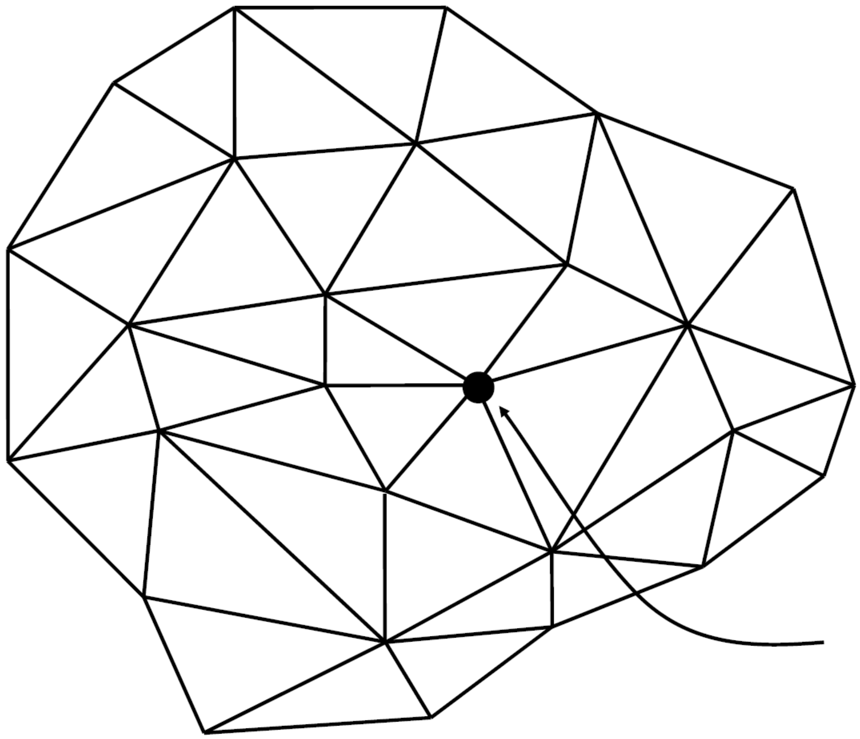
Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria

Decimation Operators

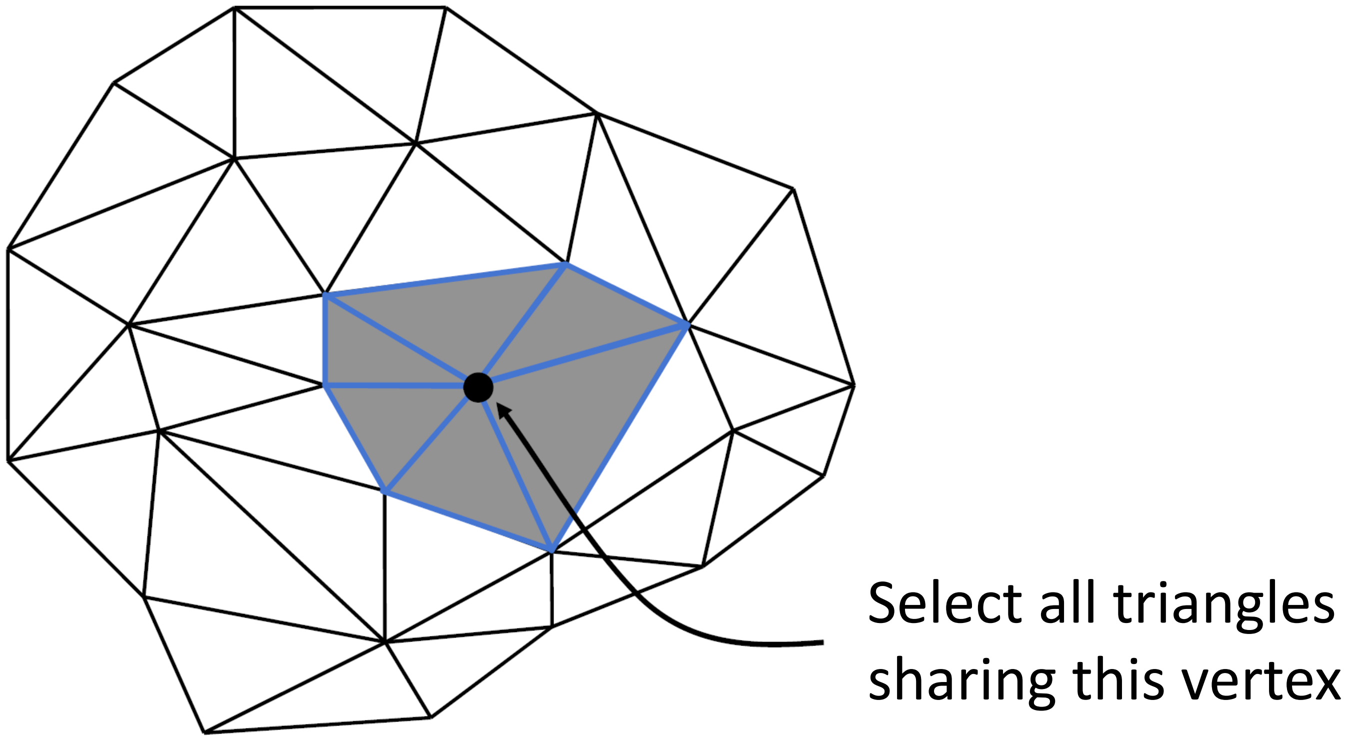
- What is a “region”?
- What are the DOF for re-triangulation?
- Classification
  - Topology-changing vs. topology-preserving
  - Subsampling vs. filtering
  - Inverse operation -> progressive meshes [Hoppe et al….]

Vertex Removal

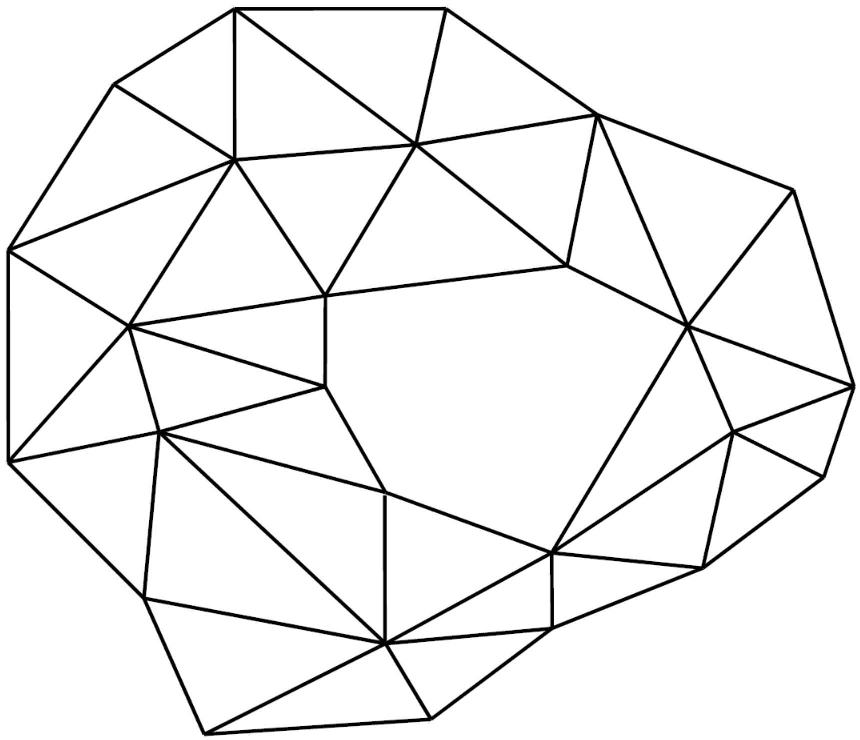


Select a vertex to be eliminated

Vertex Removal

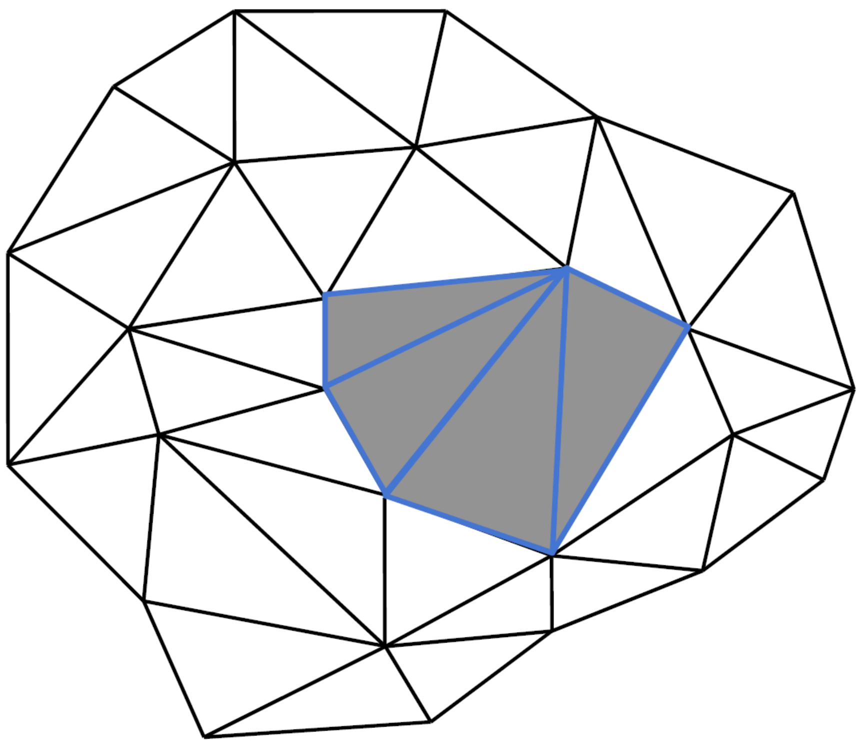


Vertex Removal



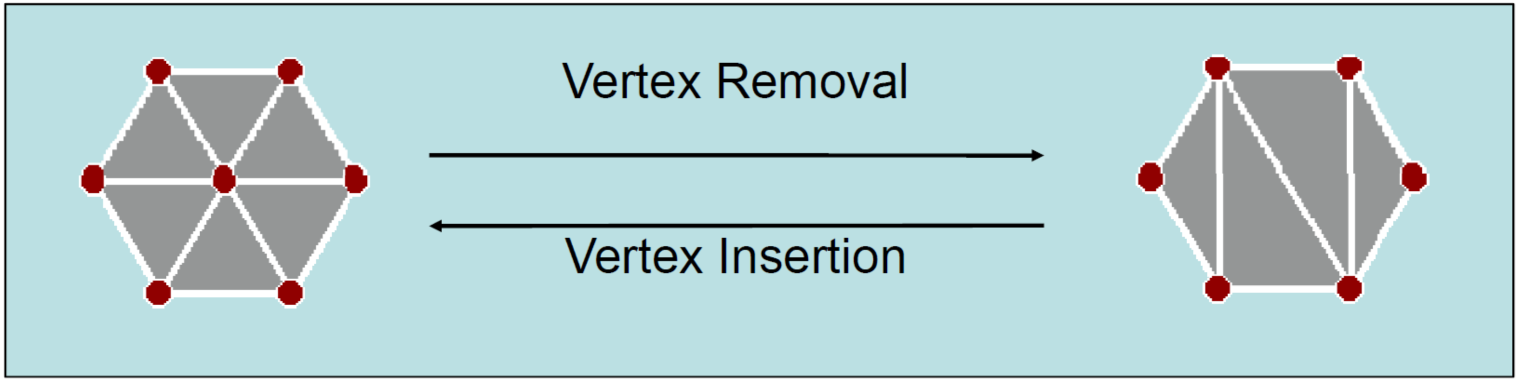
Remove the selected triangles, creating the hole

Vertex Removal



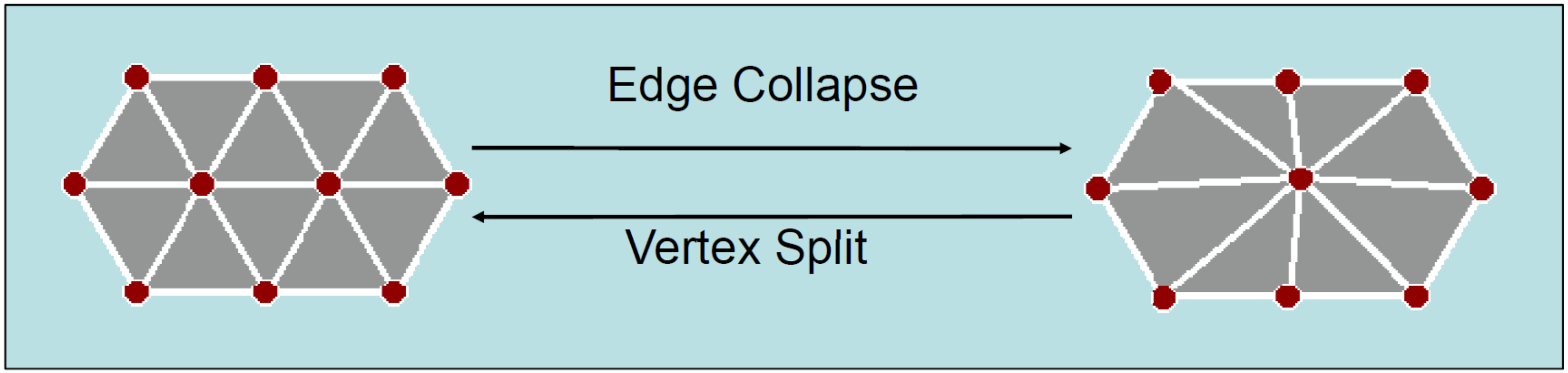
Fill the hole with new triangles

Decimation Operators



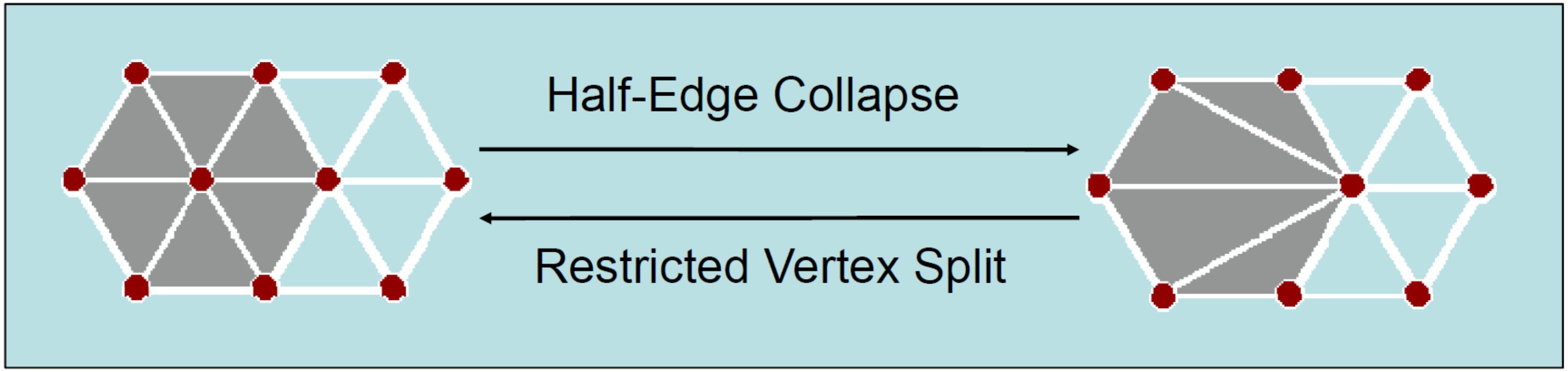
- Remove vertex
- Re-triangulate hole
  - Combinatorial degrees of freedom

Decimation Operators



- Merge two adjacent vertices
- Define new vertex position
  - Continuous degrees of freedom

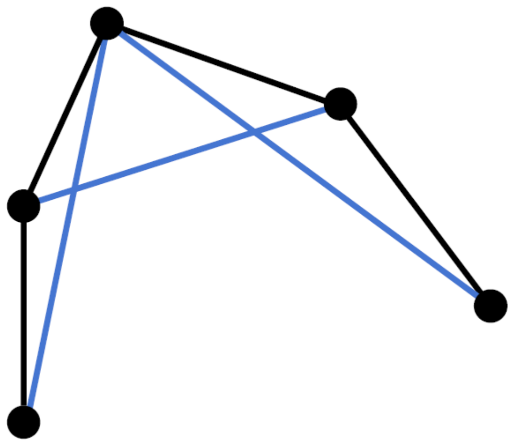
Decimation Operators



- Collapse edge into one end point
  - Special case of vertex removal
  - Special case of edge collapse
- No degrees of freedom

Fairness Criteria

- Rate quality of decimation operation
  - Approximation error
  - Triangle shape
  - Dihedral angles
  - Valence balance
  - …



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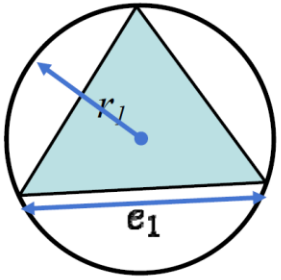
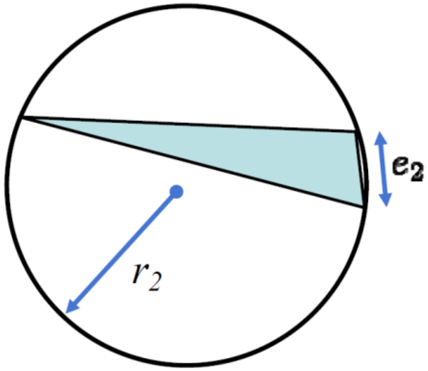


$$\frac{r_1}{e_1} < \frac{r_2}{e_2}$$

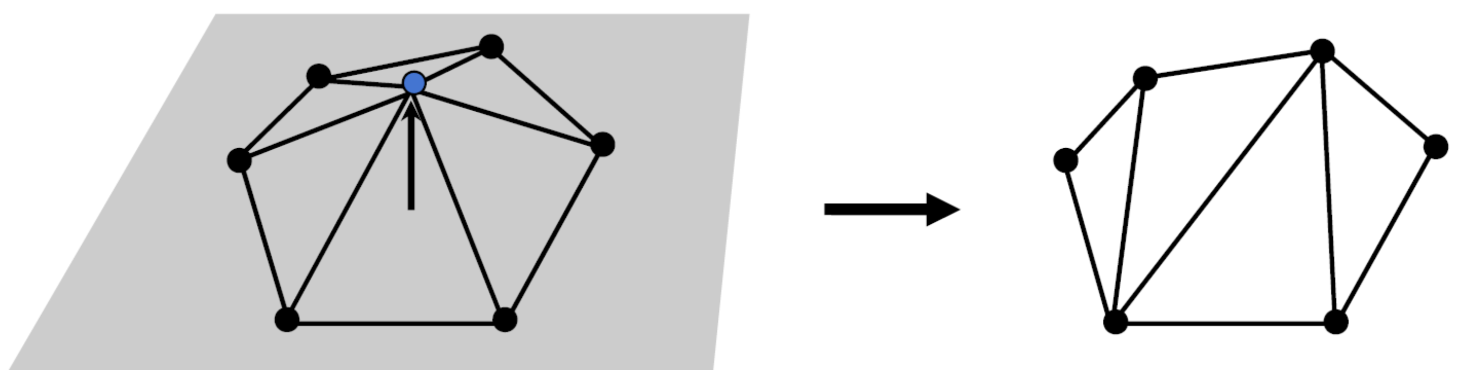


Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria

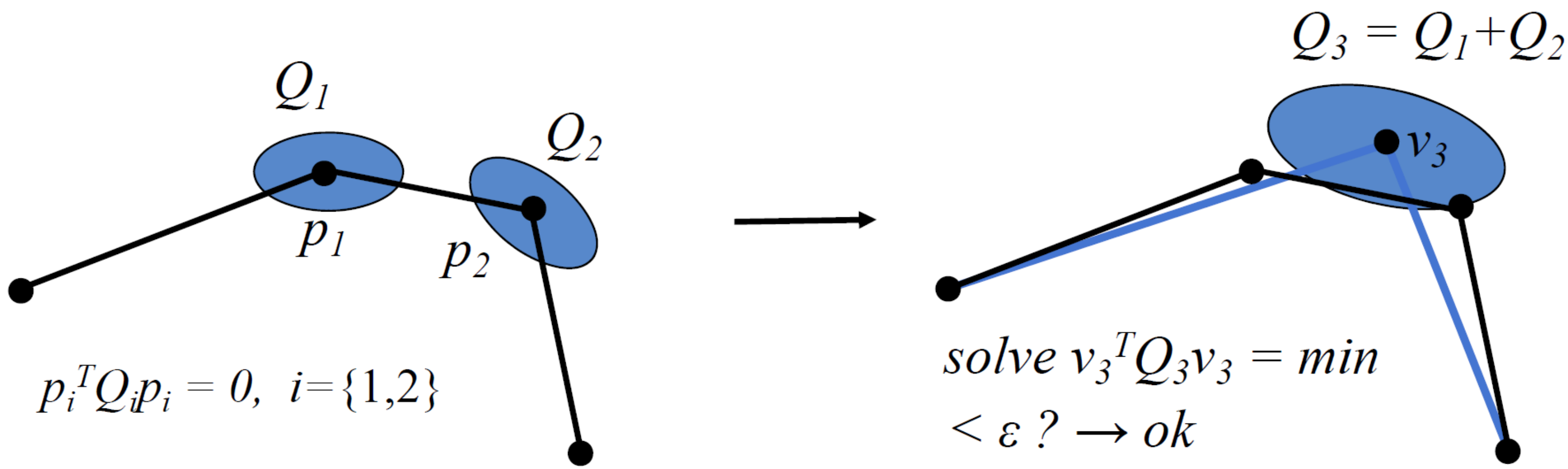
Local Error Metrics

- Local distance to mesh
  - Compute average plane
  - No comparison to *original* geometry



Global Error Metrics

- Error quadrics
  - Squared distance to planes at vertex
  - No bound on true error



Incremental Decimation

- General Setup
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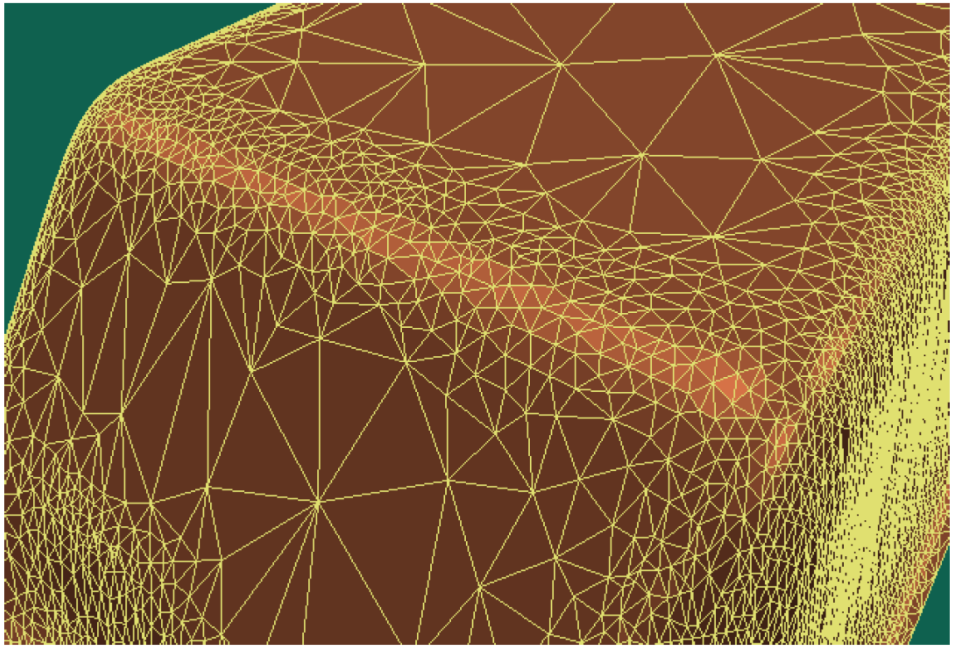

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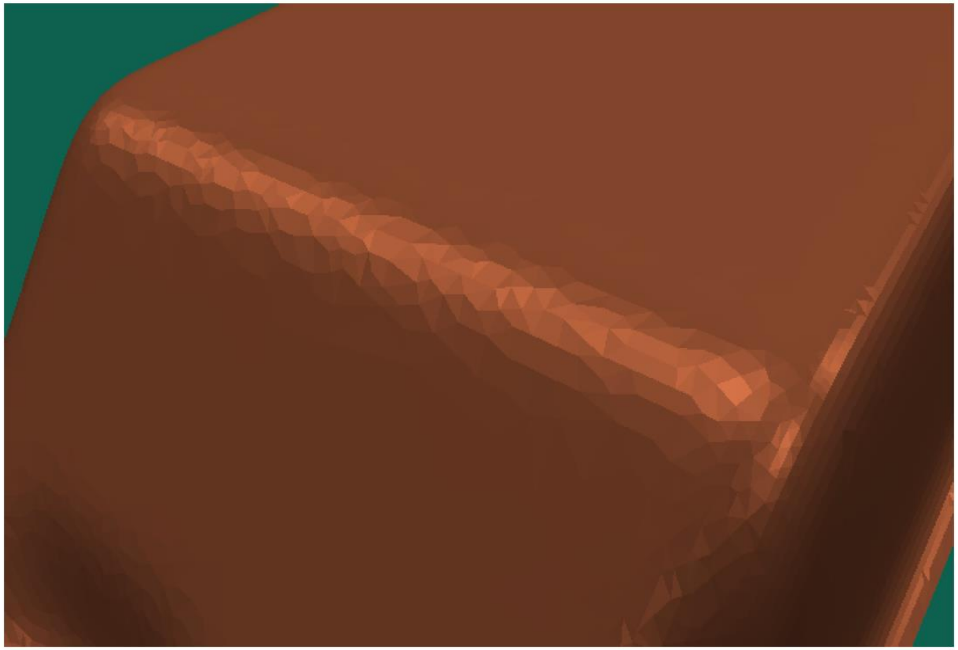

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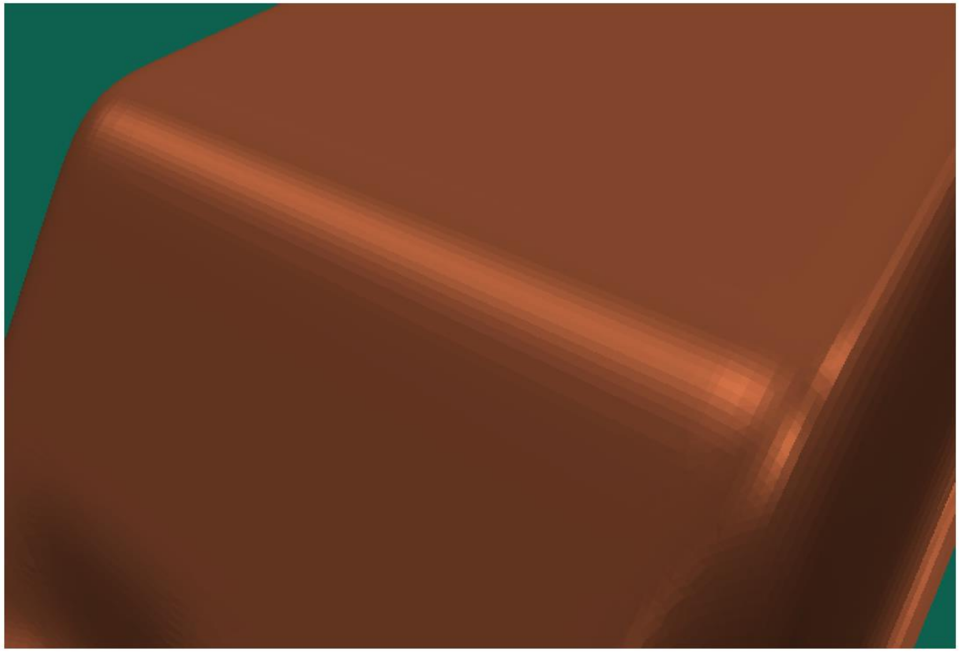

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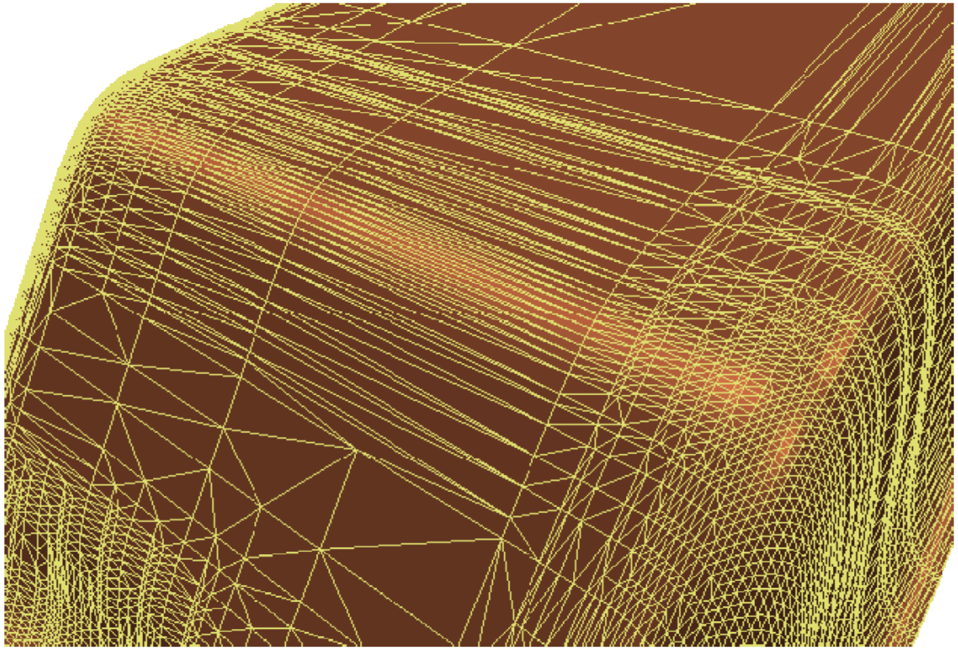

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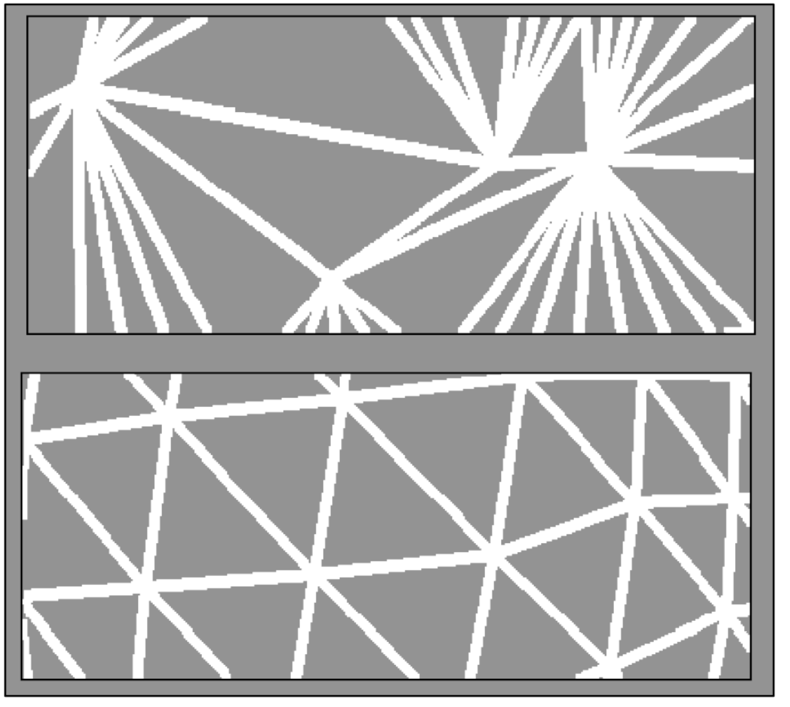


Comparison

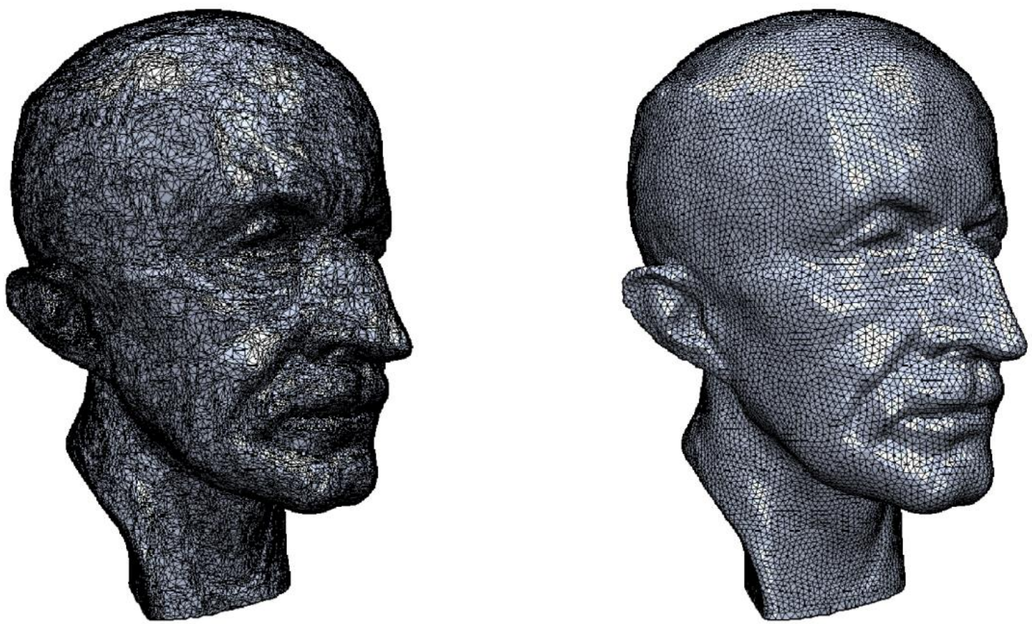
- Vertex clustering
  - fast, but difficult to control simplified mesh
  - Topology changes, non-manifold meshes
  - Global error bound, but often not close to optimum

- Incremental decimation with quadratic error metrics
  - good trade-off between mesh quality and speed
  - explicit control over mesh topology
  - restricting normal deviation improves mesh quality

Remeshing

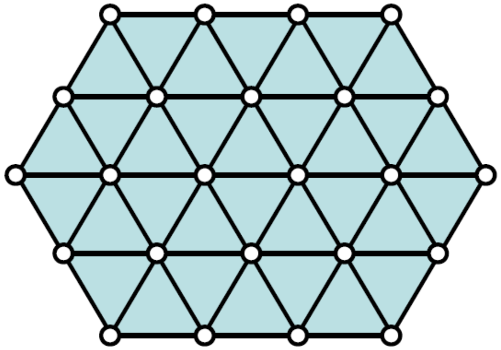
Remeshing

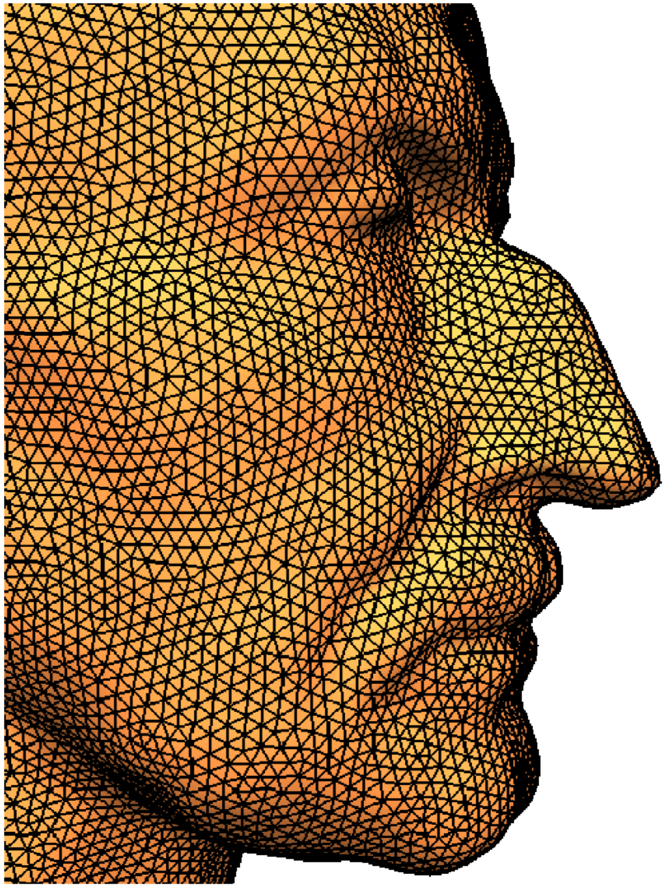
Given a 3D mesh, find a “better” discrete representation of the underlying surface



What is a good mesh?

- Equal edge lengths
- Equilateral triangles
- Valence close to 6





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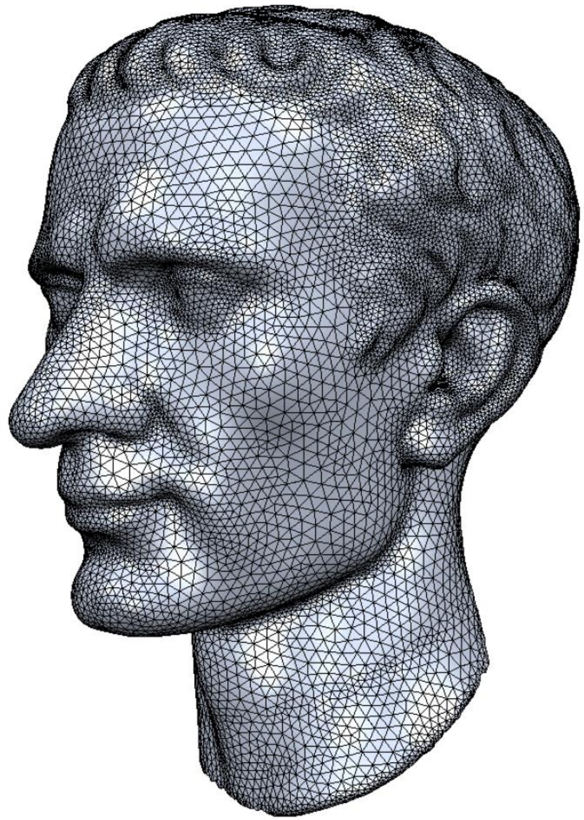
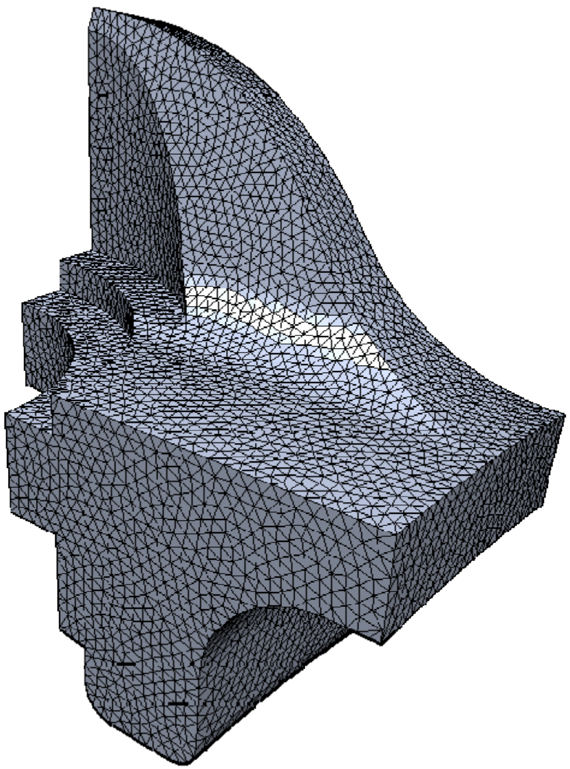
- Uniform vs. adaptive sampling



What is a good mesh?

- Equal edge lengths
- Equilateral triangles
- Valence close to 6
- Uniform vs. adaptive sampling
- Feature preservation



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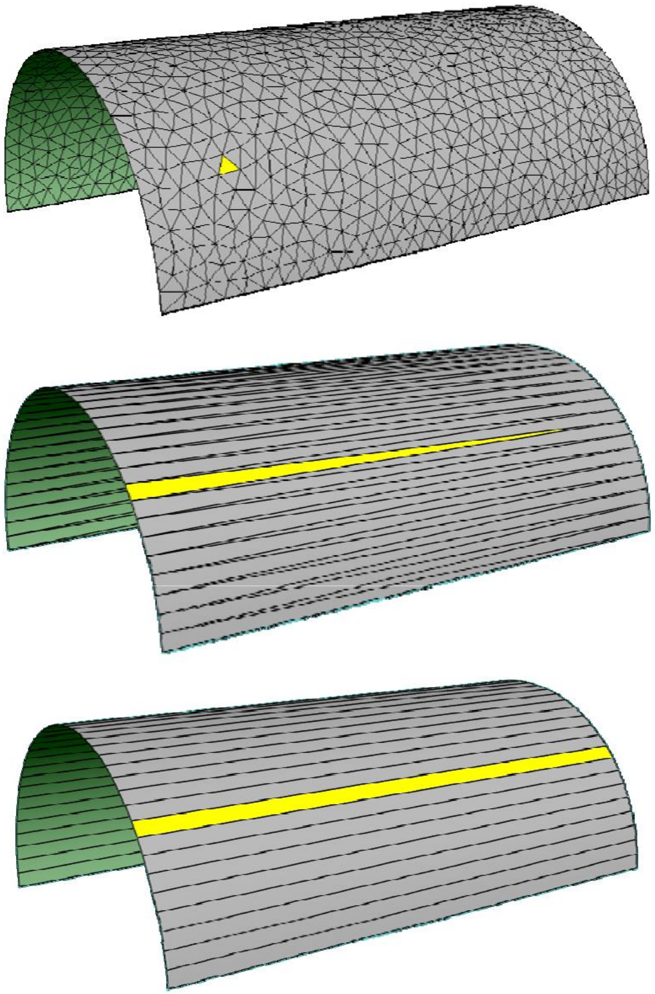
- Feature preservation
- Alignment to curvature lines
- Isotropic vs. anisotropic



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