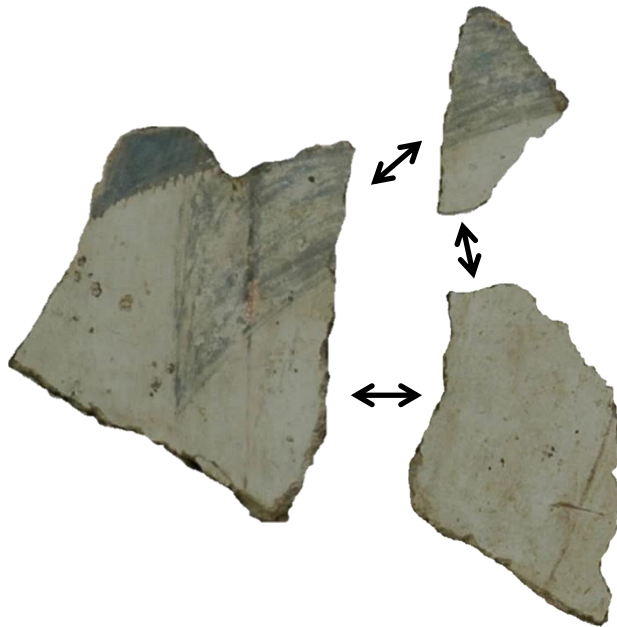
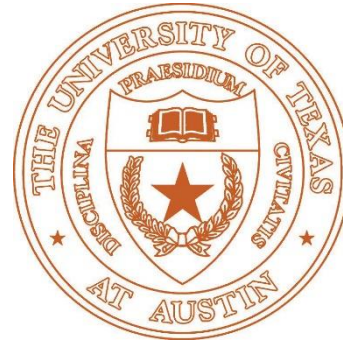


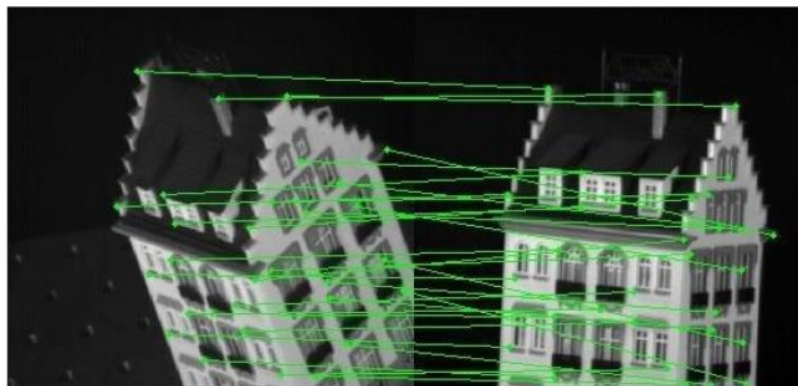
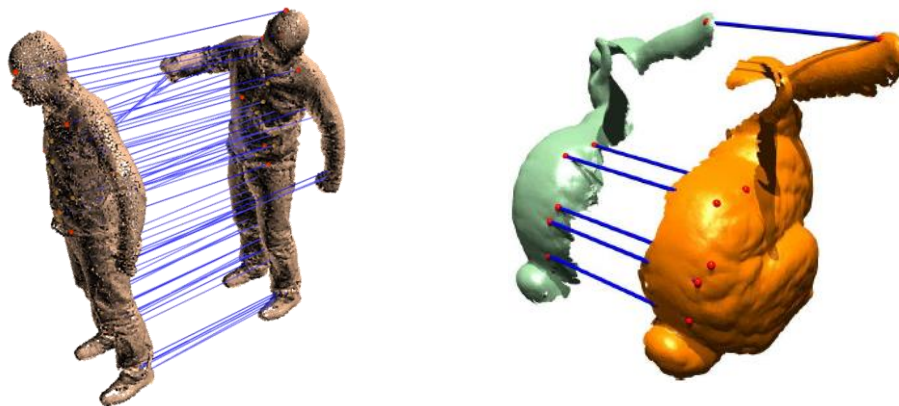
# Data-Driven Shape Correspondence



Qixing Huang  
December 5<sup>th</sup> 2016

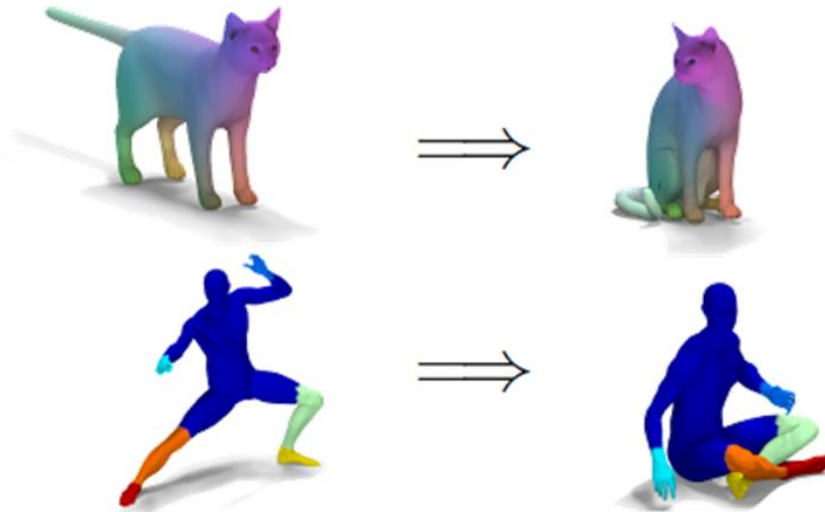


# Maps between objects

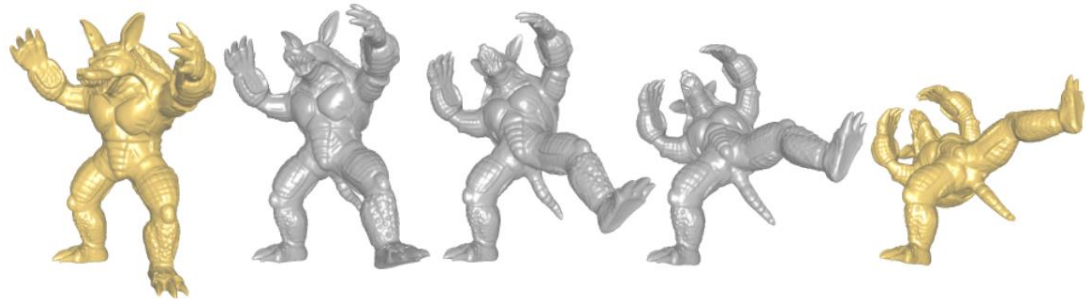


# Maps for propagation and interpolation

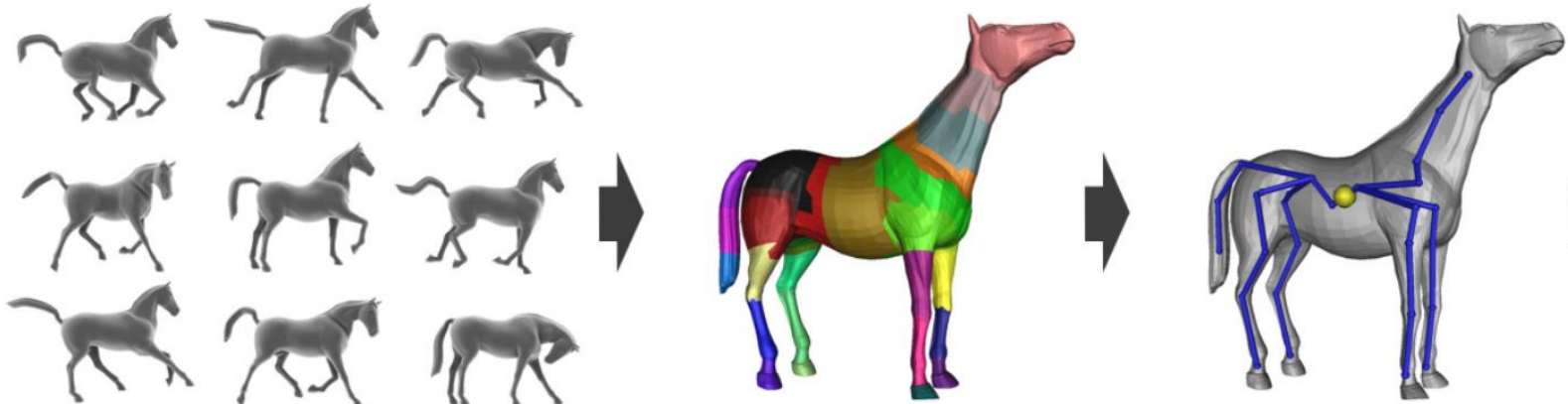
Propagation:



Interpolation:

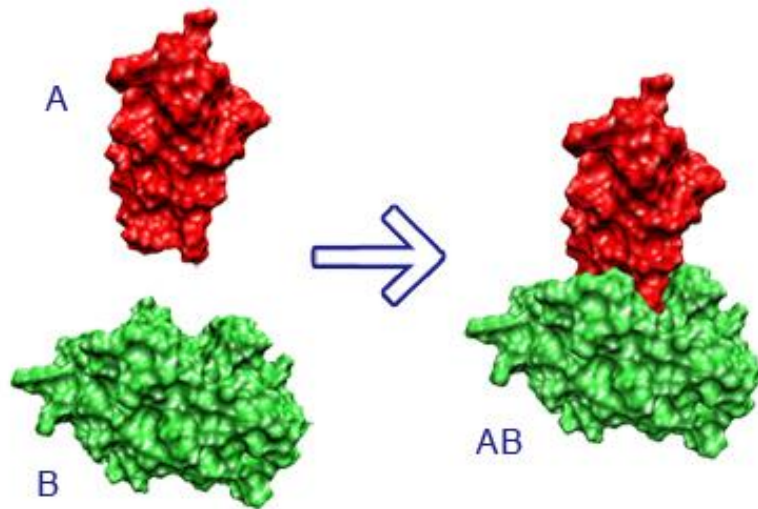


# Aggregating information



Segmentation and Skeleton Extraction

# Matching in other domains



Protein docking

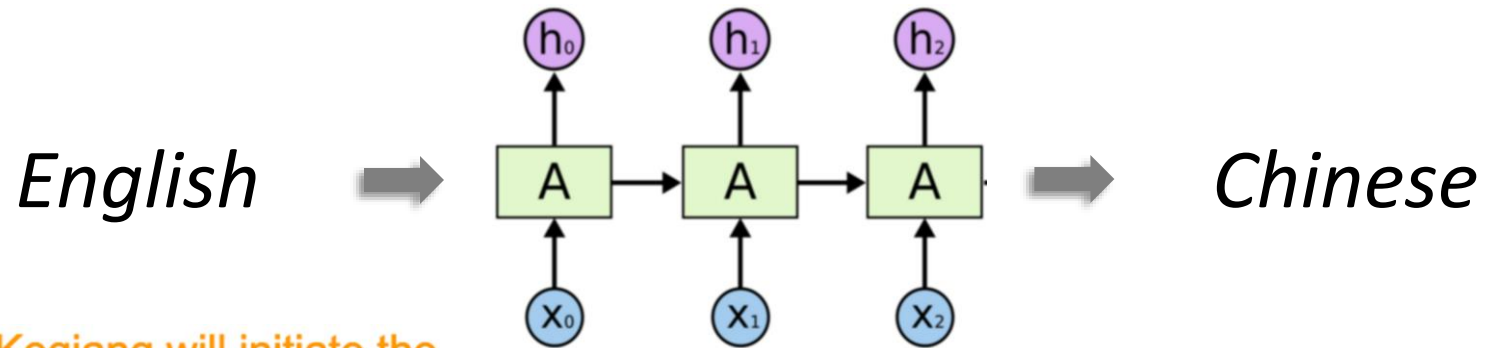


Brain matching

<http://zbi-www.bioinf.uni-sb.de/en/about-bioinformatics/docking.html>

<http://step.polymtl.ca/~rv101/images/research-brains.png>

# Mapping between different domains



Li Keqiang will initiate the annual dialogue mechanism between premiers of China and Canada during this visit, and hold the first annual dialogue with Premier Trudeau of Canada.

李克強此行將啟動中加總理年度對話機制，與加拿大總理杜魯多舉行兩國總理首次年度對話。

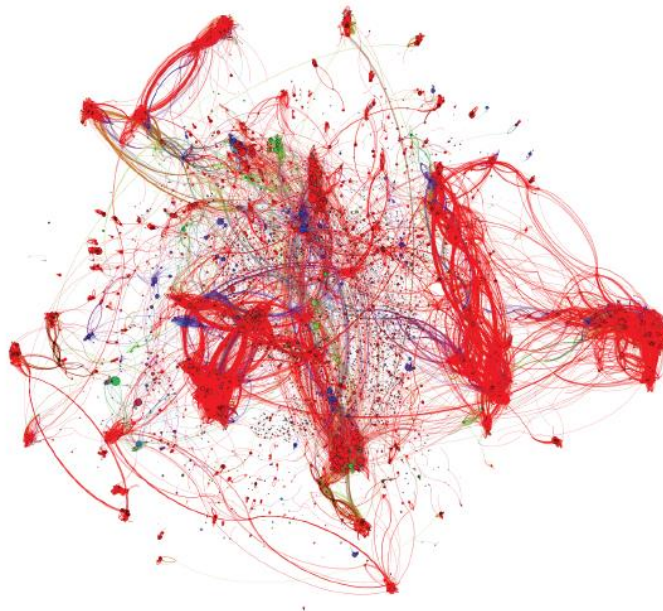
# Matching is ubiquitous

- Introduction to machine learning (Coursera)
  - 40K code submissions for linear SVM



# Matching is ubiquitous

- Grading using maps between AST trees
  - Graders mark a very small portion of them
  - Propagate labels to other programs

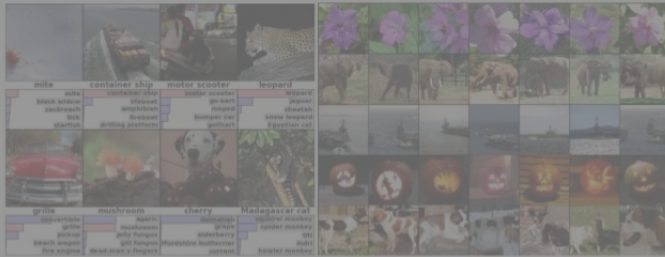


Credit: Jonathan Huang



Matching is Hard!

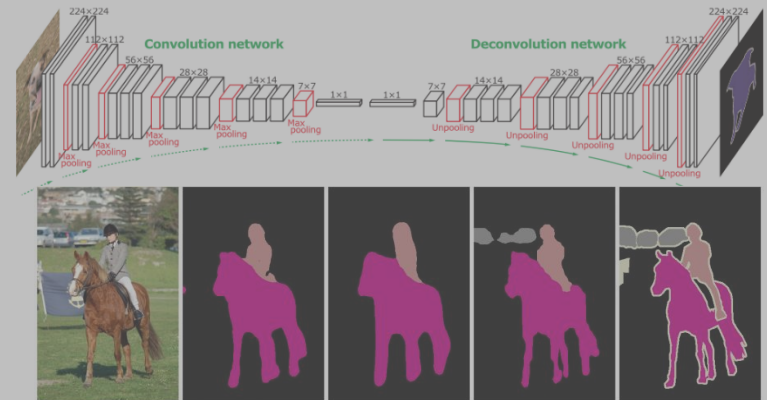
IMAGENET



1000 classes

[He et al. 16]

Classification

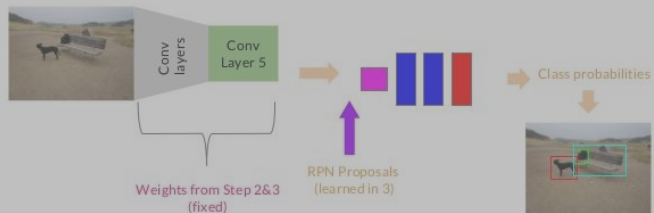


[Zheng et al. 16]

Segmentation

Faster R-CNN: 4-step training

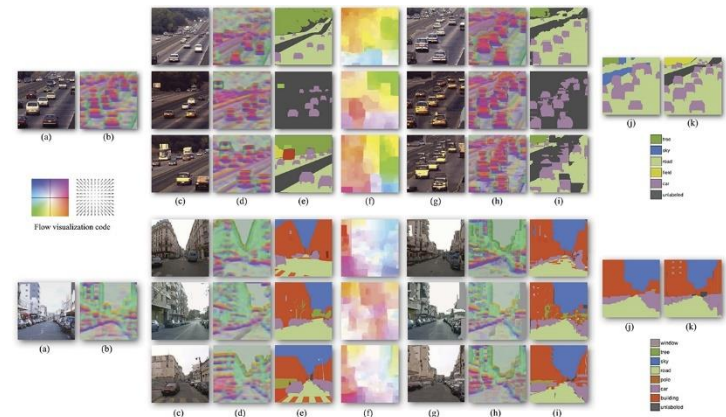
Step 4: Fine tune FC layers of Fast R-CNN using same shared convolutional layers as in 3.



22

[Ren et al. 16]

Recognition

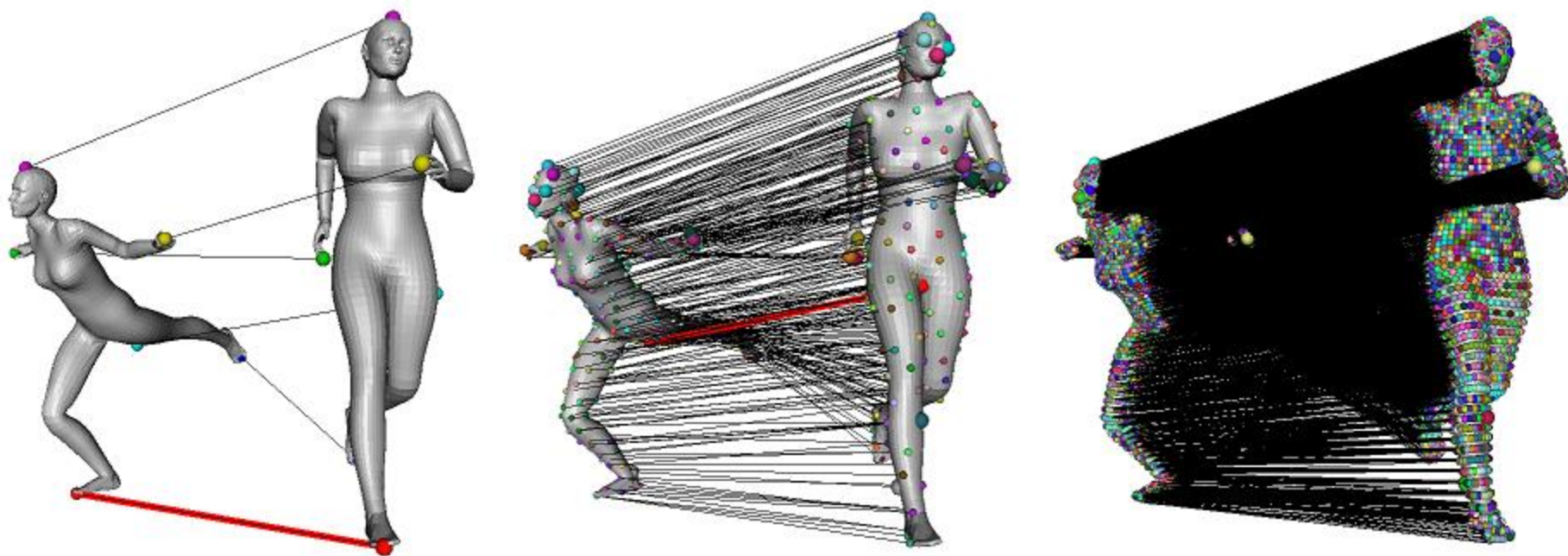


[Liu et al. 08]

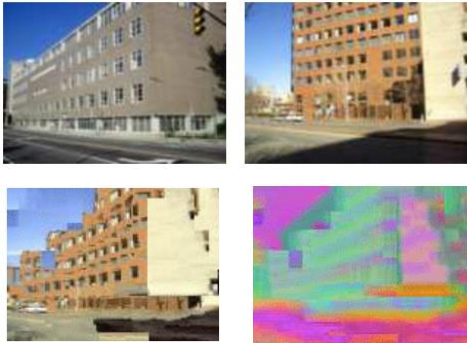
Correspondence

Fundamental Challenge:  
Lack of Training Data

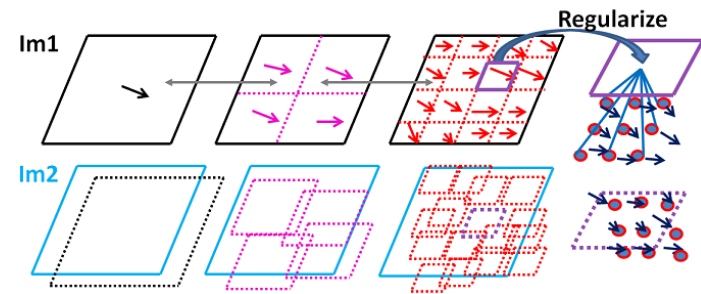
# Hard to label dense correspondences



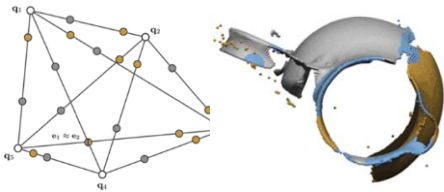
# State-of-the-art pair-wise methods



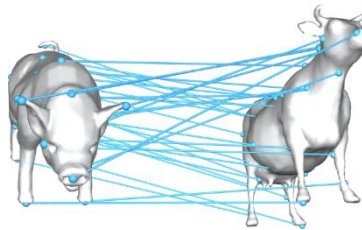
SIFTFlow [Liu et al. 08]



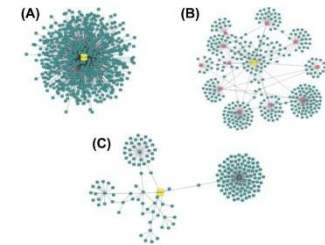
DSP [Kim et al. 13]



4-points voting  
[Aigor et al. 08]

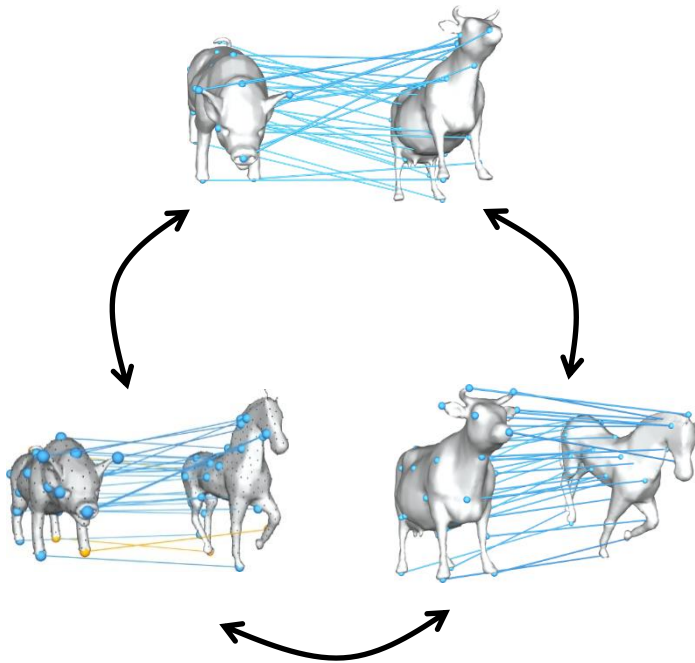


Blended intrinsic maps  
[Kim et al. 11]

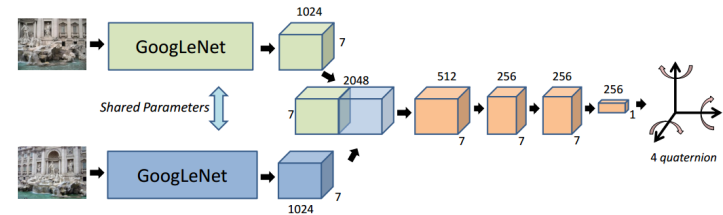


HubAlign  
[Hashemifar et al. 14]

# Outline



Map synchronization



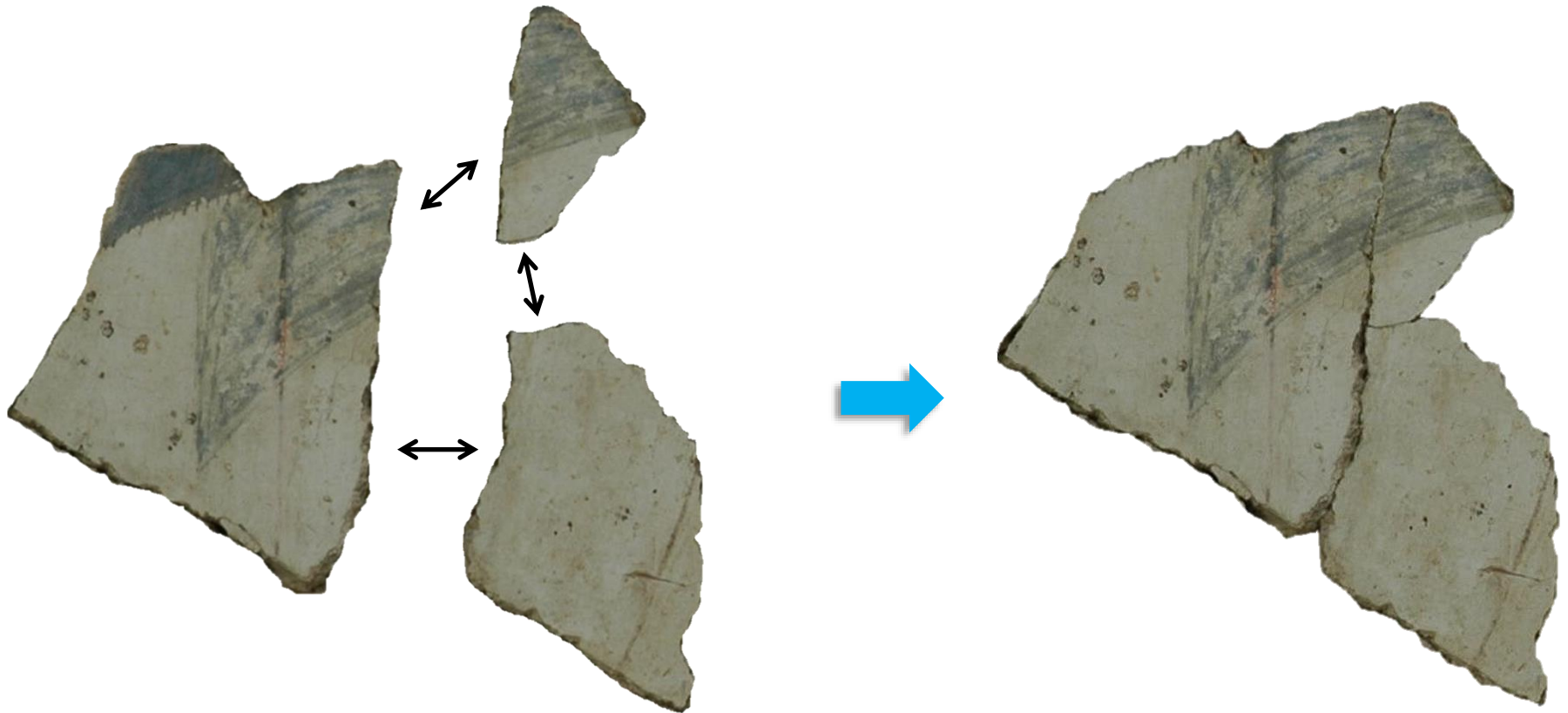
Supervising  
neural networks

# Ambiguities in assembling pieces



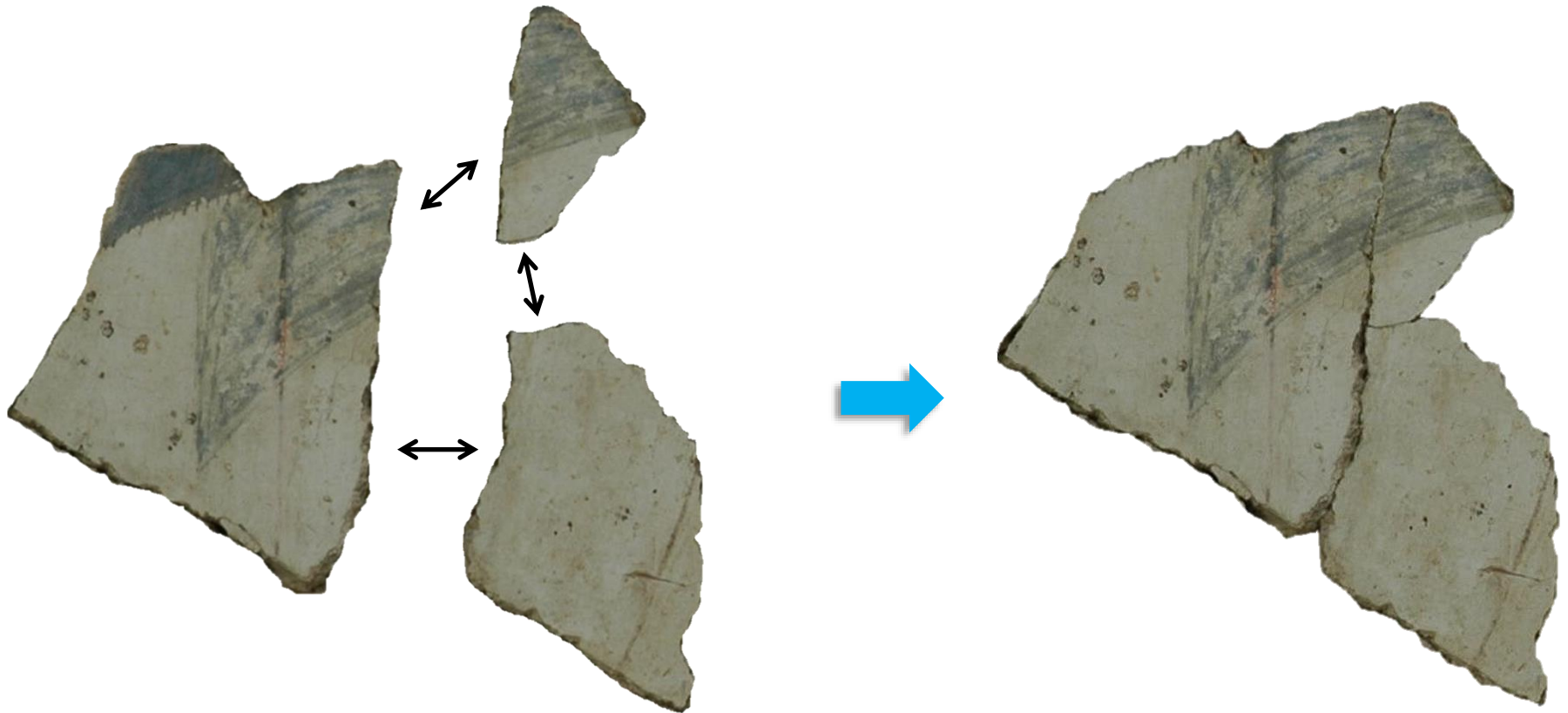


# Resolving ambiguities by looking at additional pieces



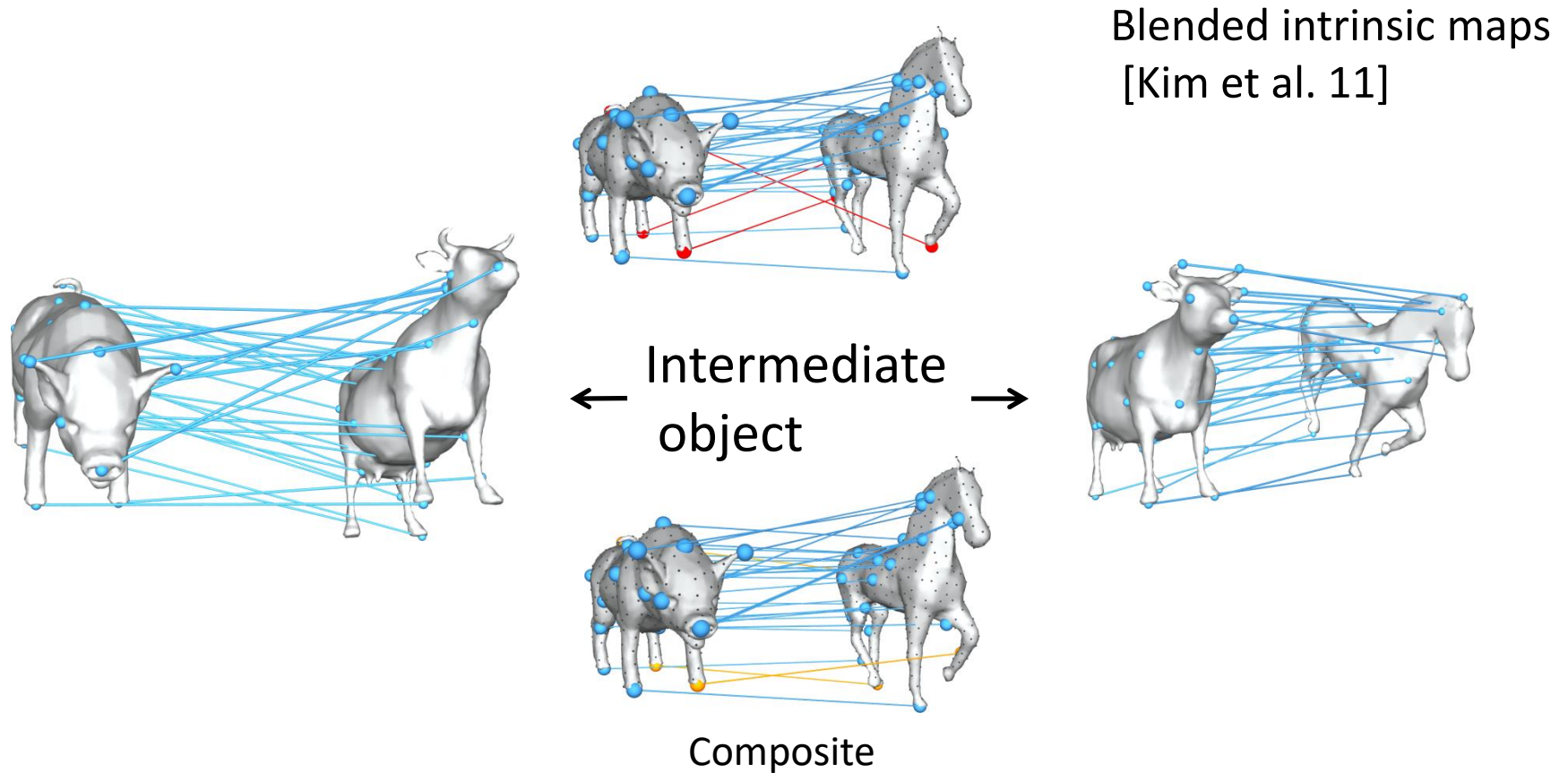


# Resolving ambiguities by looking at additional pieces

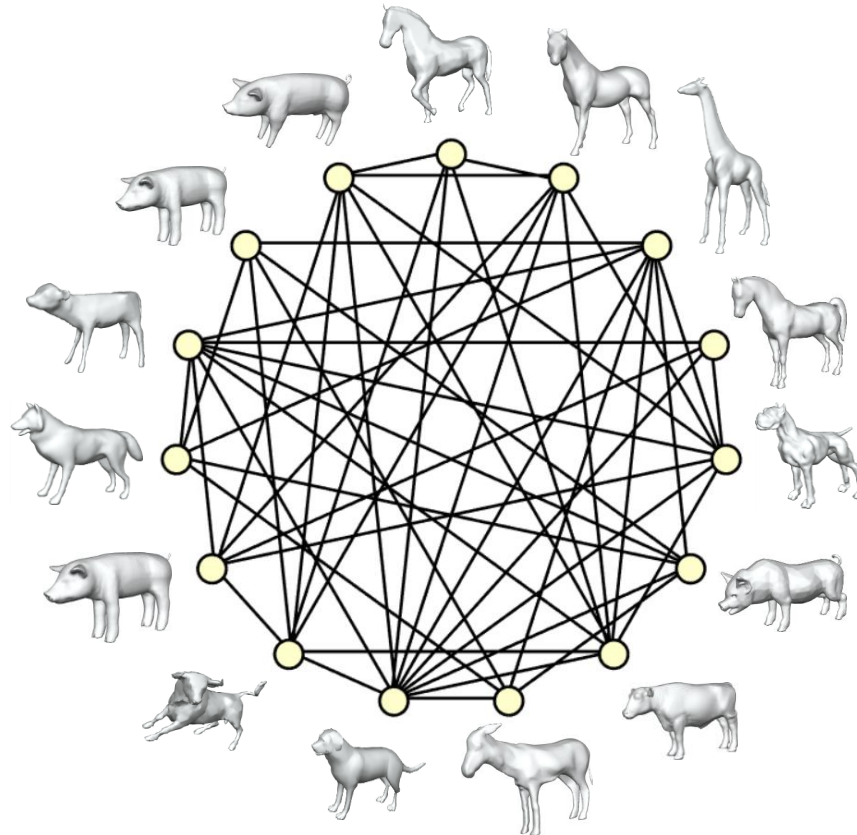


# Matching through intermediate objects

## --- map propagation

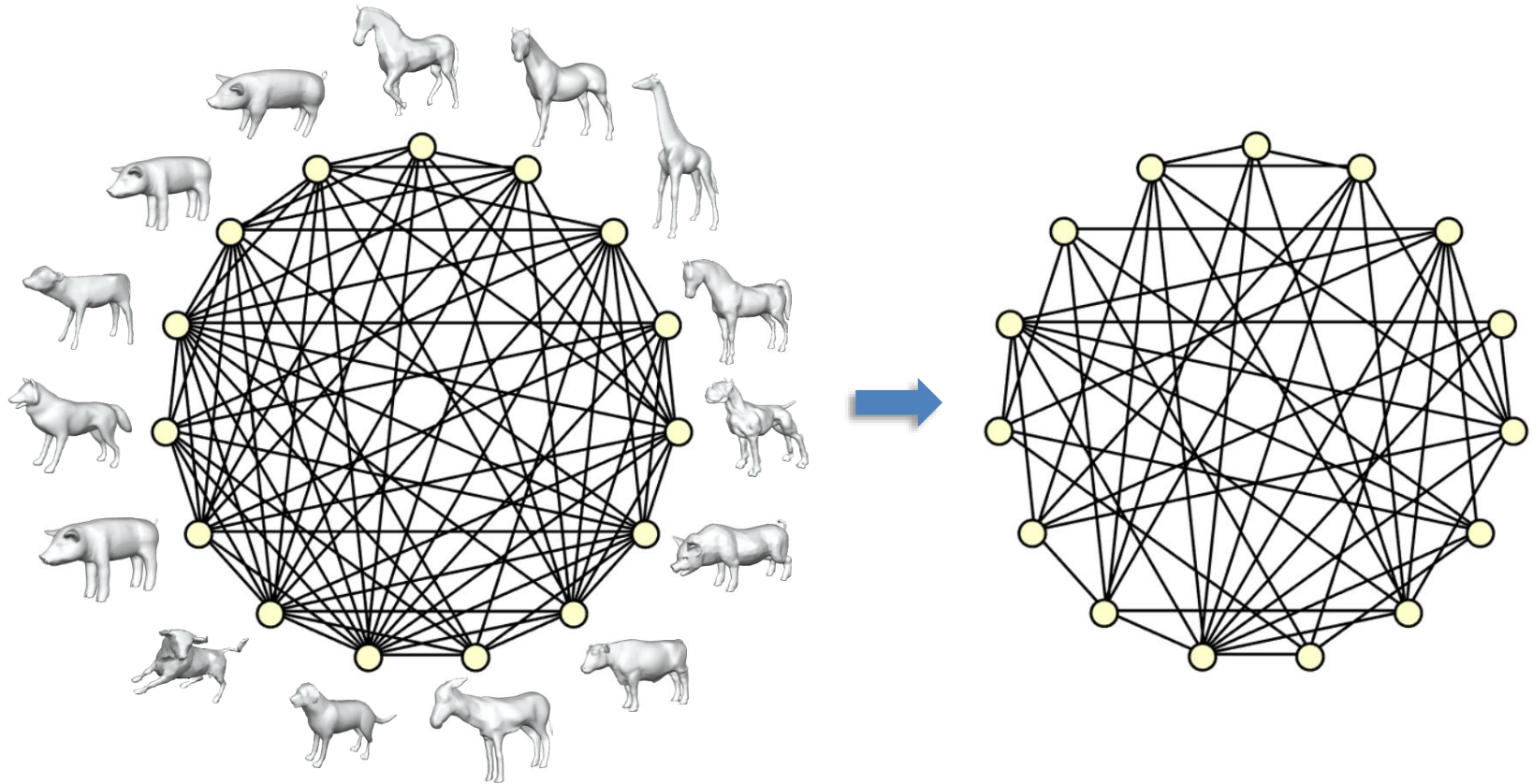


# Pair-wise maps usually contain sufficient information



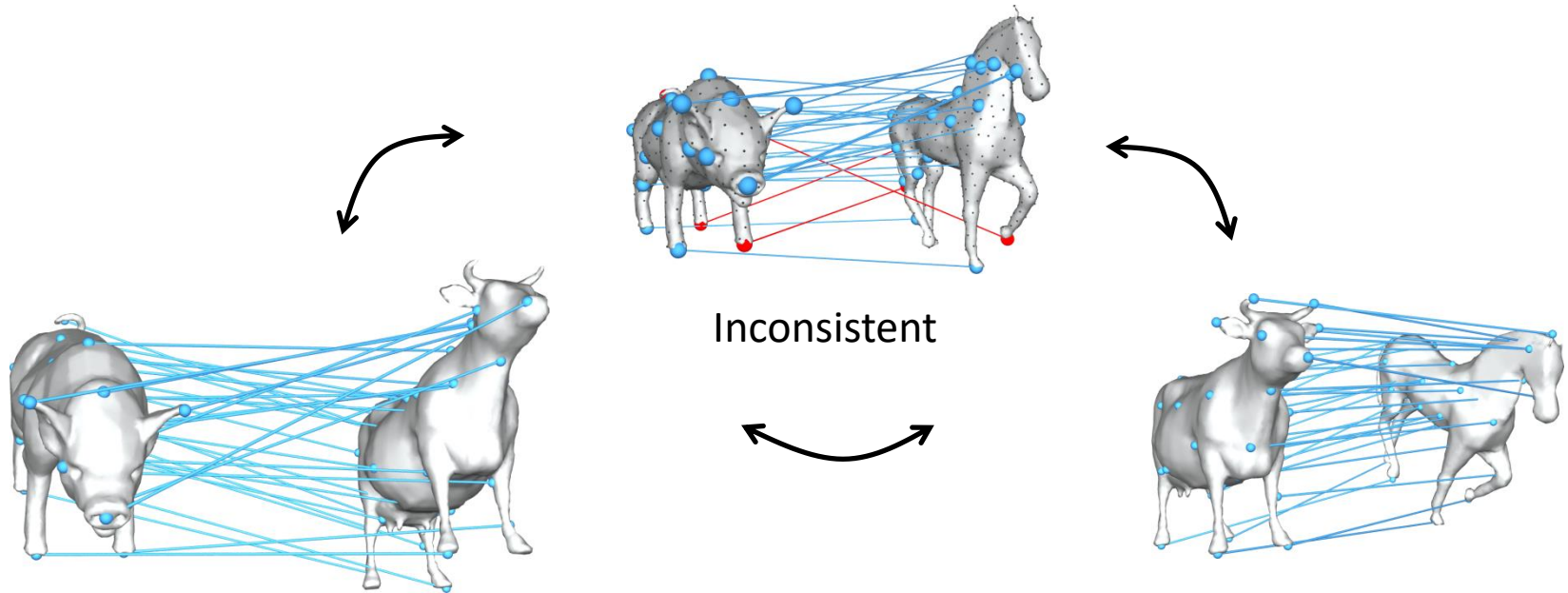
Network of approximately correct blended intrinsic maps

# Map synchronization problem



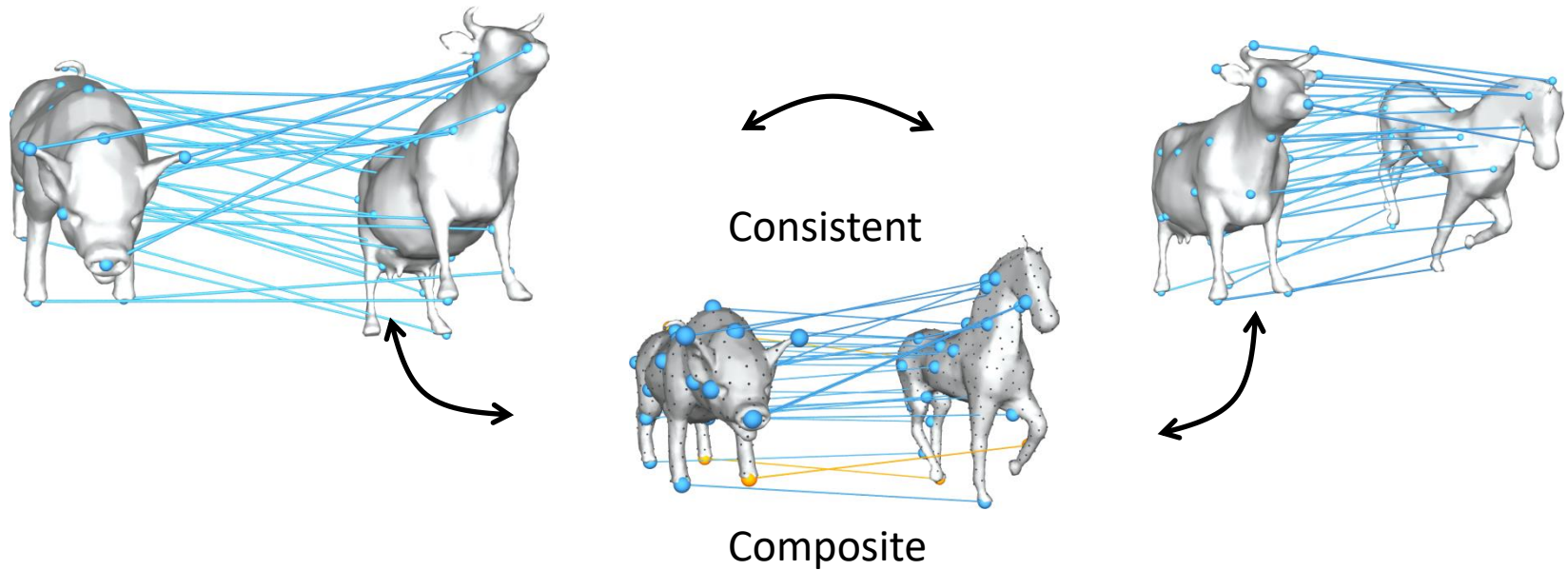
Identify correct maps among a (sparse) network of maps

# A natural constraint on maps is that they should be consistent along cycles





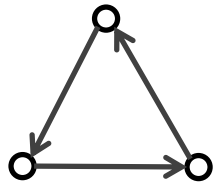
# A natural constraint on maps is that they should be consistent along cycles



# Literature on utilizing the cycle-consistency constraint

- Spanning tree optimization [Huber et al. 01, Huang et al. 06, Cho et al. 08, Crandell et al. 11, Huang et al. 12]
- Sampling inconsistent cycles [Zach et al. 10, Nyugen et al. 11, Zhou et al. 15]

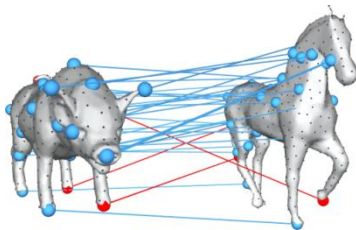
# Compressive sensing view of map synchronization



Cycle-consistency



Compressible



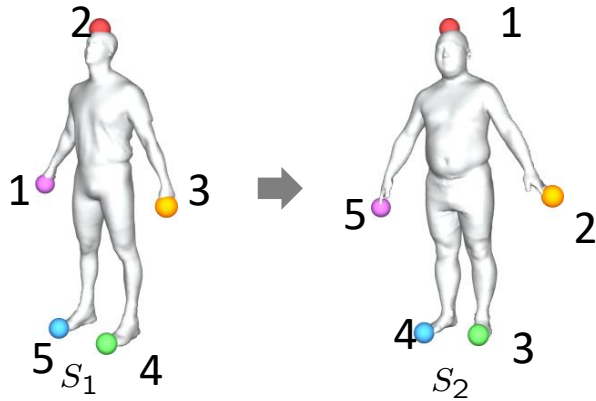
Input maps



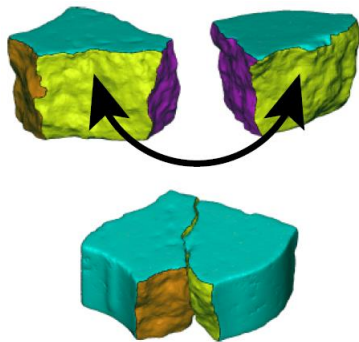
Noisy observations



# Matrix representation of maps



$$X_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



$$T_{12} = \begin{bmatrix} R_{12} & t_{12} \\ 0 & 1 \end{bmatrix}$$

# Map synchronization as constrained matrix recovery

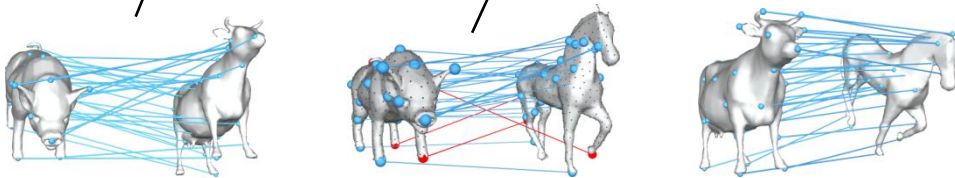
$$\begin{aligned}
 X &= \begin{bmatrix} I_m & X_{12} & \cdots & X_{1n} \\ X_{21} & I_m & \cdots & \vdots \\ \vdots & \vdots & \ddots & X_{n-1,n} \\ X_{n1} & \cdots & X_{n,n-1} & I_m \end{bmatrix} & X_{ij} = X_{j1} X_{i1}^T \\
 &= \begin{bmatrix} I_m \\ \vdots \\ X_{n1} \end{bmatrix} \begin{bmatrix} I_m & \cdots & X_{n1}^T \end{bmatrix}
 \end{aligned}$$

# Map synchronization as constrained matrix recovery

$$X = \begin{bmatrix} I_m & X_{12} & \cdots & X_{1n} \\ X_{21} & I_m & \cdots & \vdots \\ \vdots & \vdots & \ddots & X_{n-1,n} \\ X_{n1} & \cdots & X_{n,n-1} & I_m \end{bmatrix}$$

**Parameter-free**

**Theoretical guarantees**



Noisy measurements of matrix blocks

Q. H and L. Guibas, *Consistent Shape Maps via Semidefinite Programming*, Sym. on Geometry Processing'13

Y. Chen, L. Guibas, Q. H, *Near-Optimal Joint Object Matching via Convex Relaxation*, ICML'14

Q. H, F. Wang, L. Guibas, *Functional Map Networks for Analyzing and Exploring Large Shape Collections*, SIGGRAPH' 14

S. Shen, Q.H., N. Srebro, S. Sunghavi, *Normalized Spectral Map Synchronization*, NIPS' 16

# Permutation synchronization

Objective function:

$$\text{minimize } \sum_{(i,j) \in \mathcal{G}} \|X_{ij}^{\text{input}} - X_{ij}\|_1$$

Observation graph

Constraints:

$$X \succeq 0 \quad \leftarrow \text{cycle-consistency}$$

**Semidefinite  
Program**

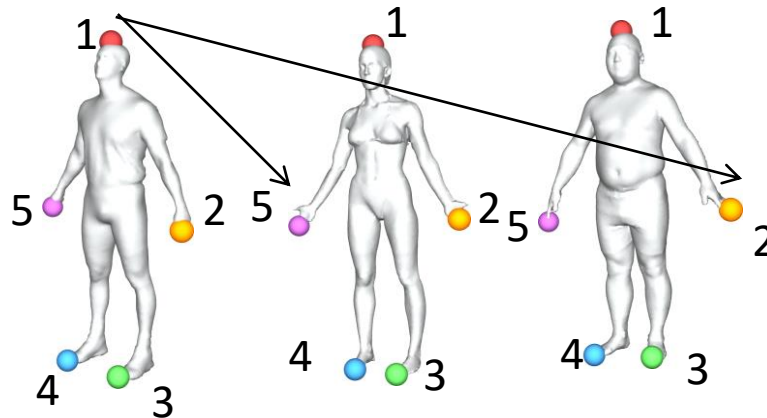
$$\begin{aligned} X_{ii} &= I_m, \quad 1 \leq i \leq n \\ X_{ij} \mathbf{1} &= \mathbf{1}, X_{ij}^T \mathbf{1} = \mathbf{1}, \quad 1 \leq i < j \leq n \\ 0 &\leq X \leq 1 \end{aligned}$$

mapping constraint

# Deterministic guarantee

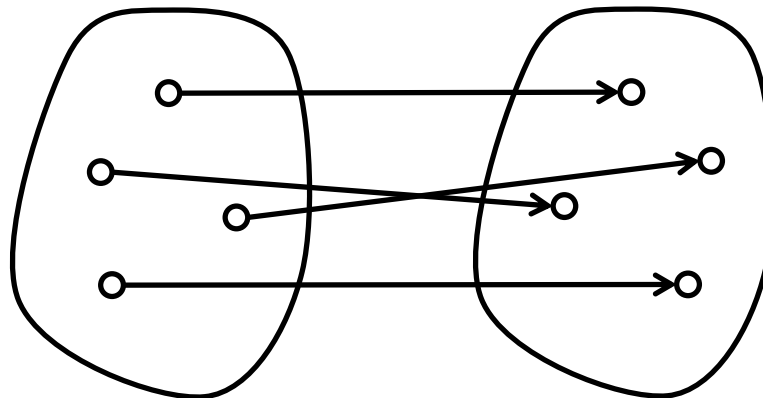
- Theorem: Given noisy input maps, permutation synchronization recovers the underlying maps if*

$$\text{\#incorrect corres. of each point} < \frac{\lambda_2(G)}{4}$$



# Optimality when the object graph $G$ is a clique

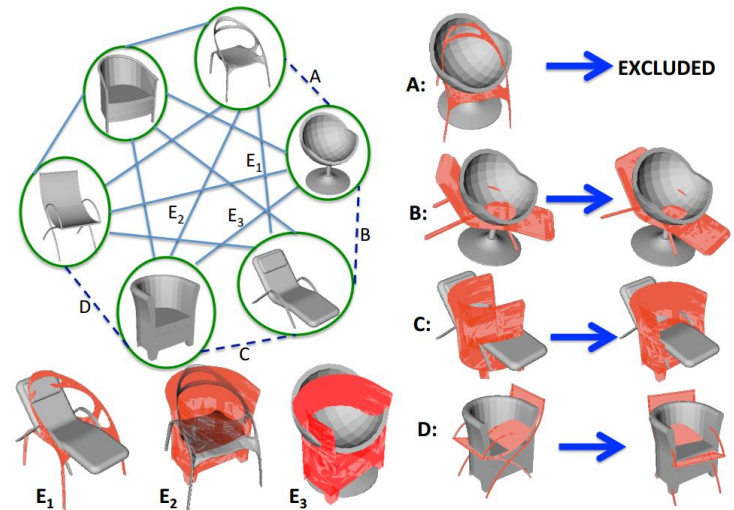
- 25% incorrect correspondences
- Worst-case scenario
  - Two clusters of objects of equal size
  - Wrong correspondences between objects of different clusters only (50%)



# Justification of maximizing $\lambda_2(G)$ for map graph construction

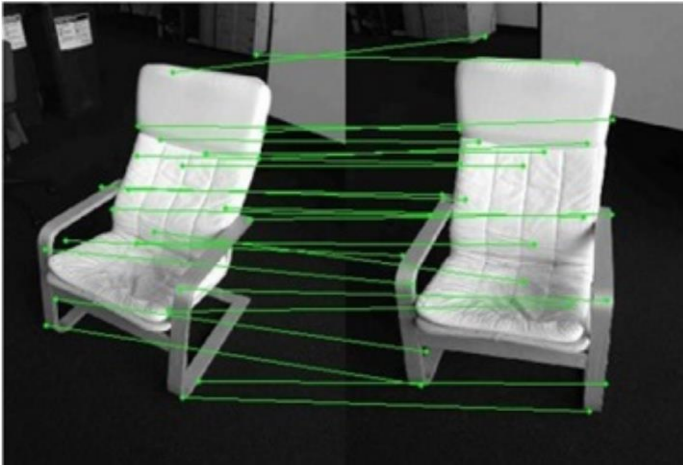


Imageweb [Heath et al 10]



Fuzzy correspondences  
on shapes [Kim et al 12]

# Variants



Partial maps [CGH'14]  
Spectral Sync. [SHSS'16]

Near-optimal!



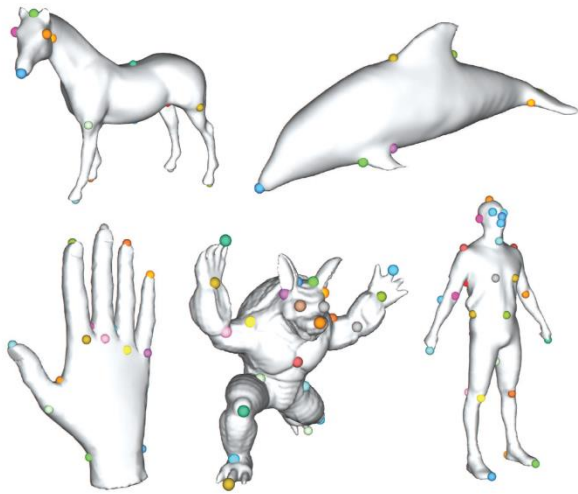
Rotation Sync.  
[Wang and Singer'14,...]

Near-optimal?

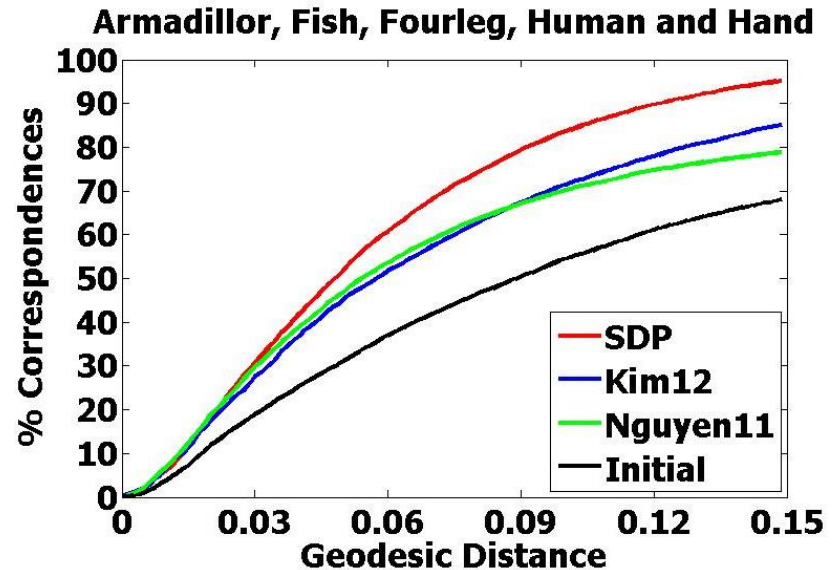


# Experimental Results

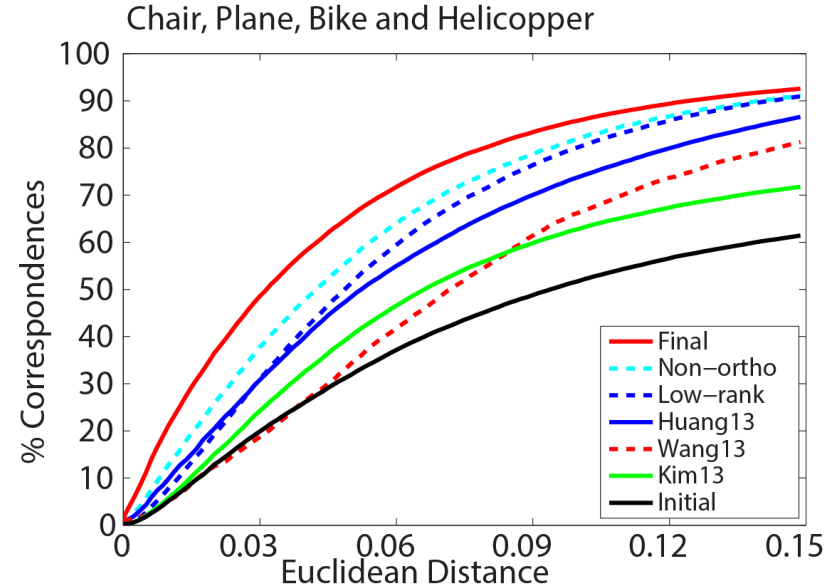
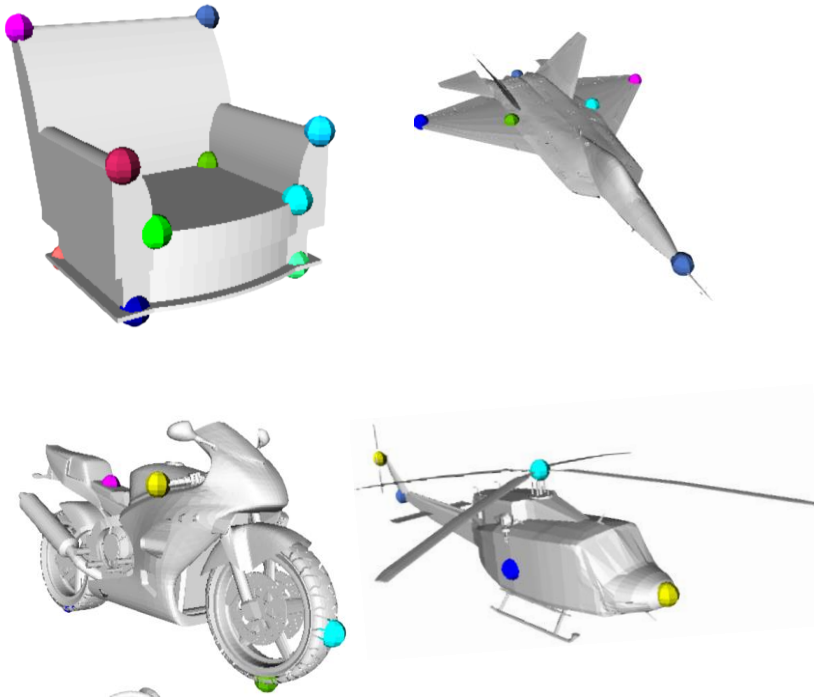
# Constrained matrix recovery achieves state-of-the-art performance



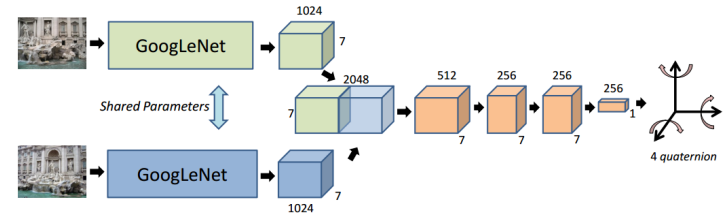
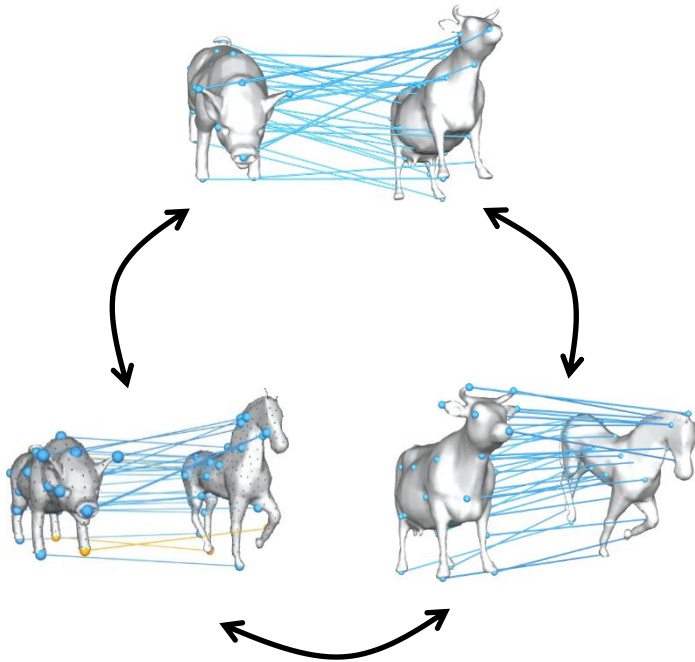
SHREC07-UnSym



# Constrained matrix recovery achieves state-of-the-art performance



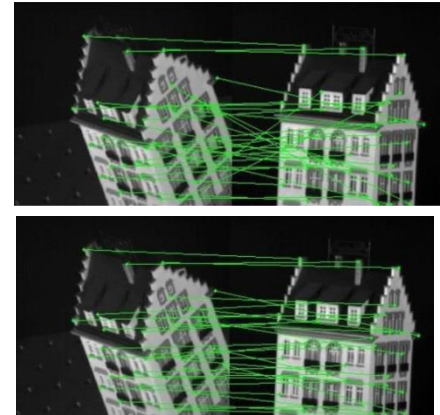
# Outline



Supervising  
neural networks

Matrix recovery perspective

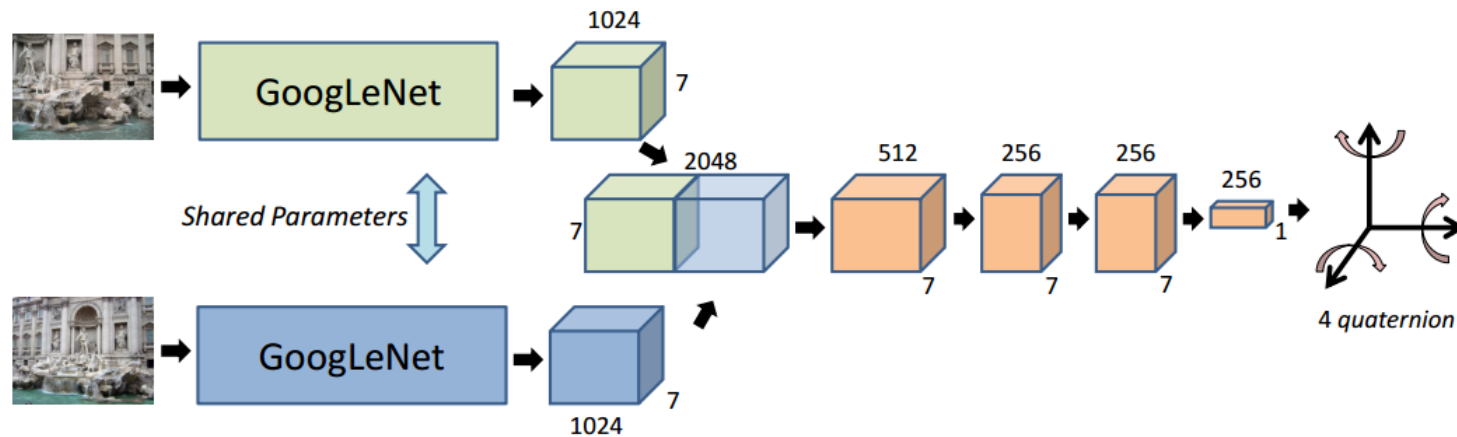
# Map synchronization versus learning pair-wise matching



CMU Hotel dataset

Pair-wise (RANSAC)	Joint Matching (from RANSAC)	Pairwise (Learning) Leordeanu et al. 12
64.1%	97.4%	94.6%
Joint Matching (from Learning)	Pairwise (Learning) Leordeanu et al. 12	Joint Matching (from Learning)
100%	95.1%	100%

# Unsupervised Learning of Relative Camera Poses Using Neural Networks



# Alternating minimization converges



SydneyHouse [Chu et al.16]

125K Training, 30K Testing



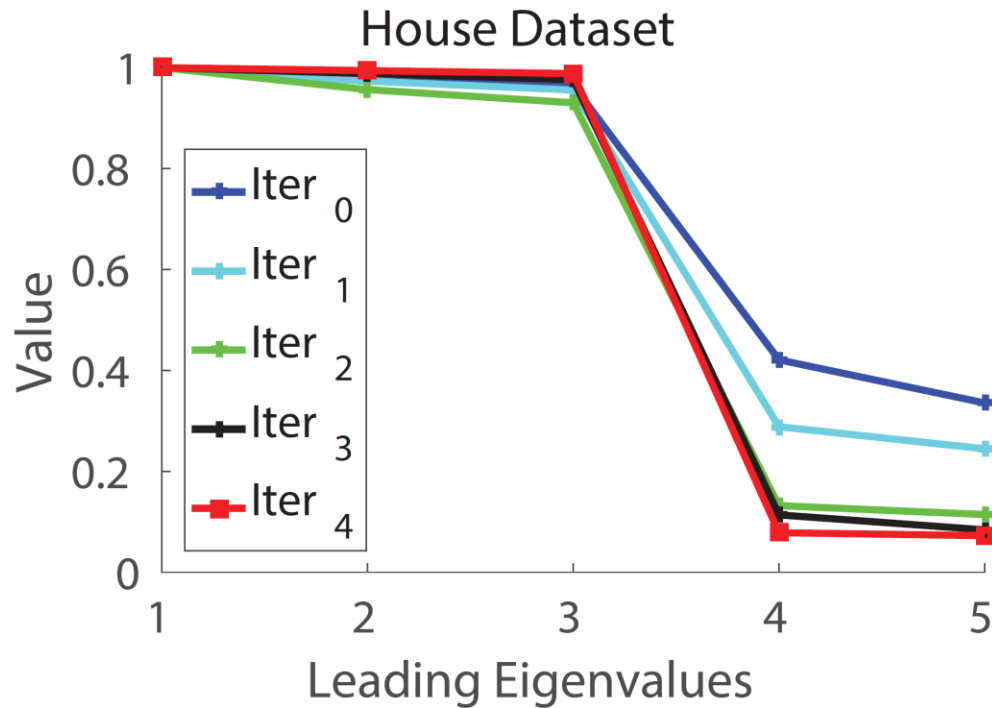
PoseNet [Kendall et al.15]

390K Training, 91K Testing

Network	Iter. 0	Iter. 1	Iter. 2	Iter. 3	Iter. 4
PoseNet	13.35	10.14	9.23	8.94	8.92
SydneyHouse	6.13	5.02	4.95	4.89	4.87



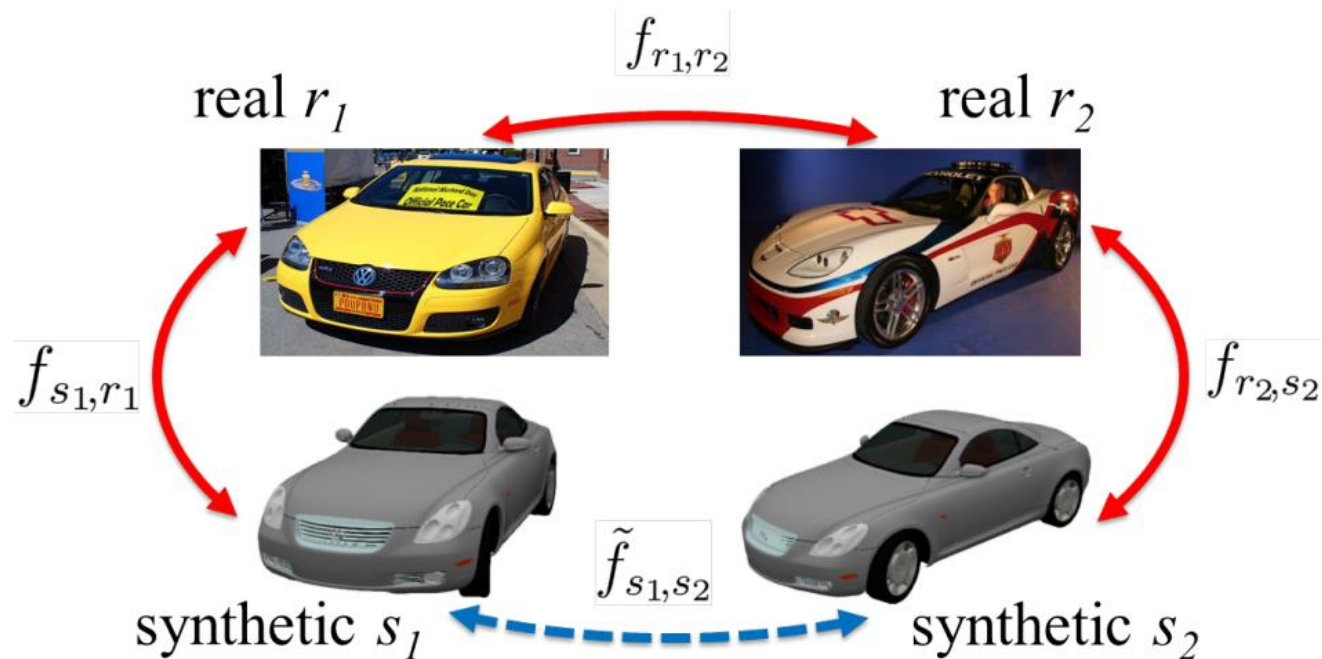
# Predicted maps become more consistent



# Cycle-consistency perspective

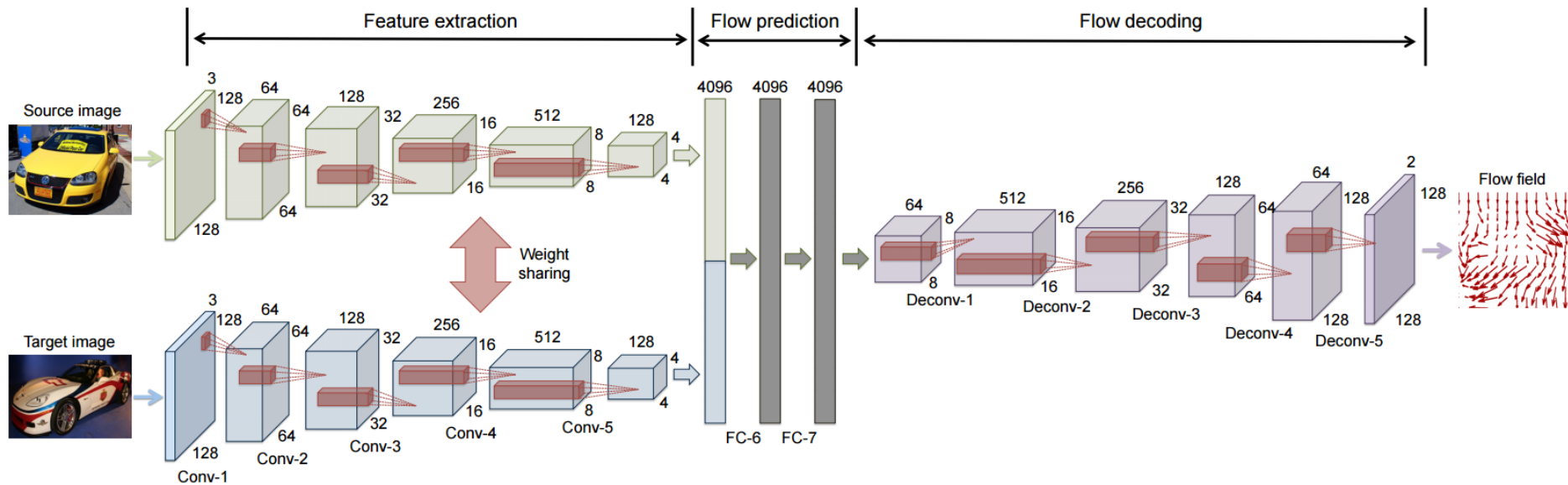
[Zhou-Krähenbühl-Abruy-Huang-Efros, CVPR' 16]

# Connecting real images through synthetic images



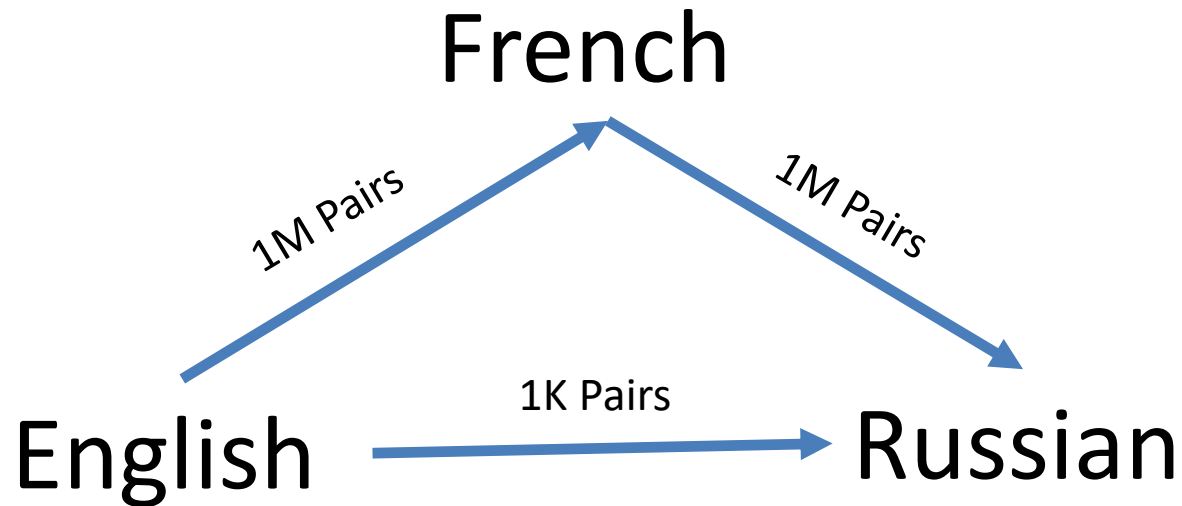
$$\tilde{f}_{s_1, s_2} = f_{s_1, r_1} \circ f_{r_1, r_2} \circ f_{r_2, s_2}$$

# Flow architecture



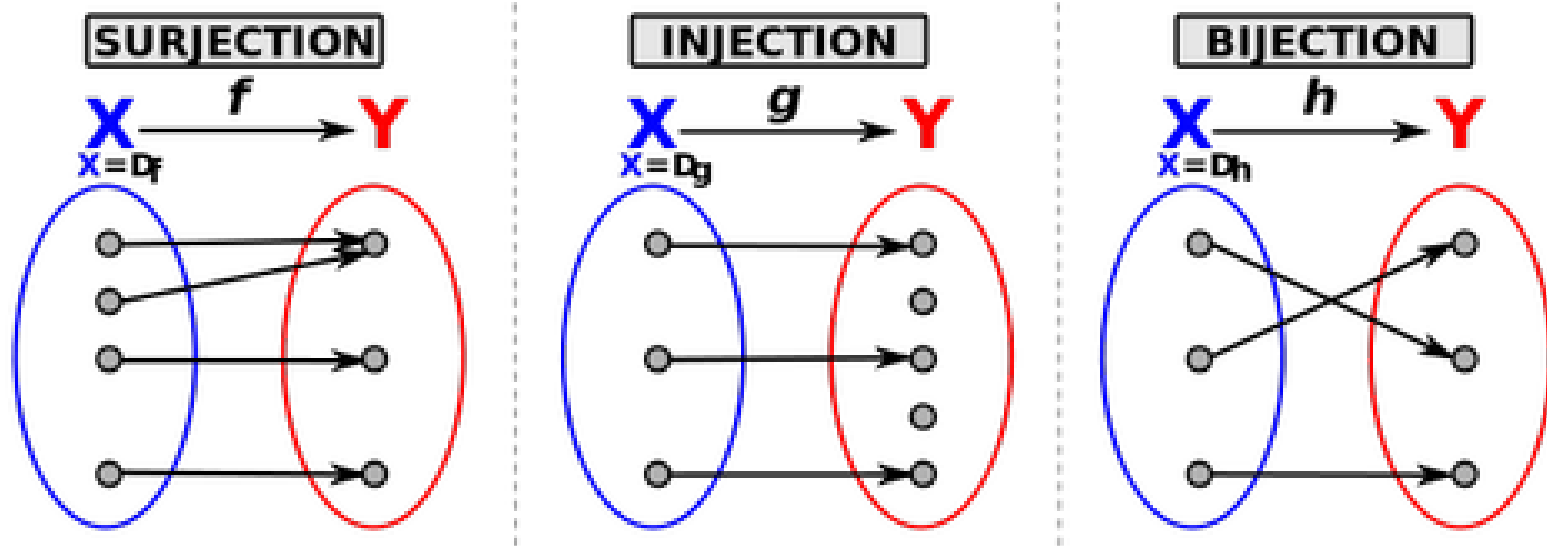
*FlowNet* [Fischer et al 15]

# Multi-lingual Translation [Cho et al. ]

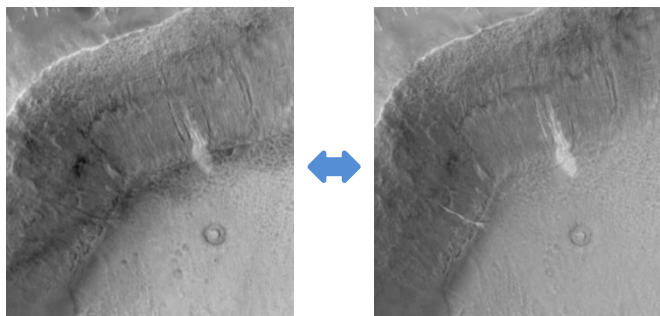


Looking Ahead

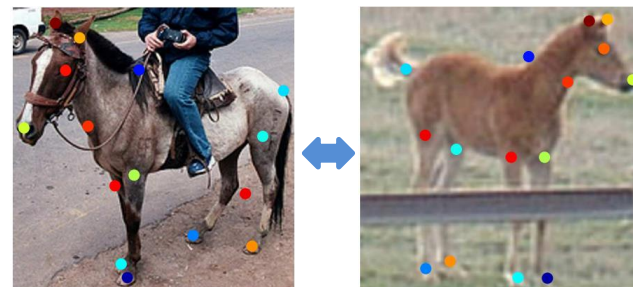
# Maps as functions between sets



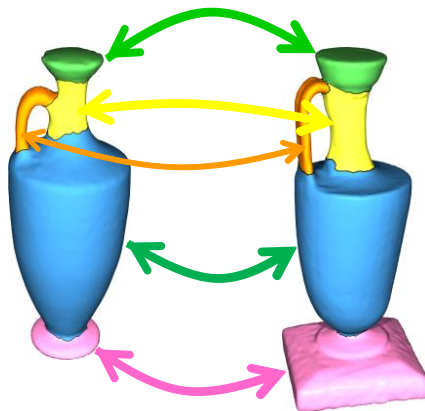
# How to define sets on objects?



Pixels



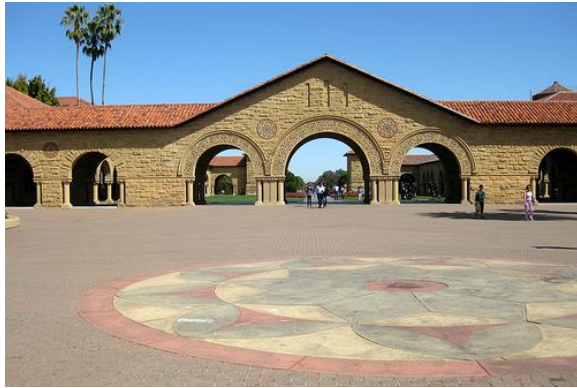
Key points



Parts



# Sets (or representations) and maps should be optimized together



Pixel-wise  
correspondences



Segment  
correspondences

Questions?