# Automated Specification Analysis Using an Interactive Theorem Prover

#### Harsh Raju Chamarthi and Pete Manolios

Northeastern University

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 Teaching freshmen how to reason about programs using ACL2s



- Teaching freshmen how to reason about programs using ACL2s
- Success of QuickCheck



Counterexamples!!

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- Combining testing and theorem-proving (ACL2 2011 Workshop)





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Introduction		Experimental Evaluation	Discussion and Conclusions
Overview			
Goal Analyse spec	ifications - Find co	ounterexamples!	

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			Constraint Solver

#### Overview

#### Goal

Analyse specifications - Find counterexamples!

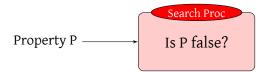
#### The problem

What to do when the *Search Procedure* doesnt return an answer?

#### The main idea

Reduce the search space and guide the procedure towards a counterexample

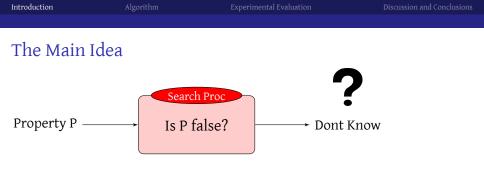
!	QuickCheck	
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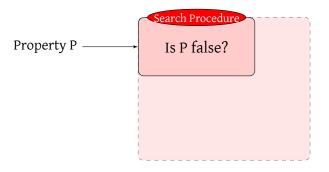


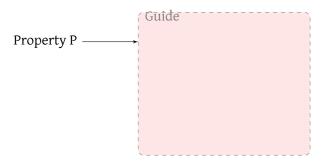
	Experimental Evaluation	
a		
Search Proc		
$\rightarrow$ Is P false	? → Yes	
	Search Proc	Search Proc

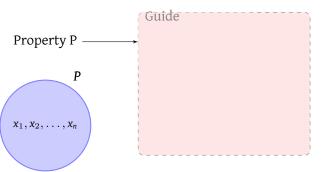
Introduction		Experimental Evaluation	
The Main Ide	a		
Property P ———	$\rightarrow \qquad \qquad$		Yes

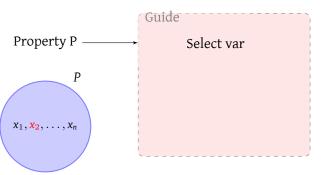
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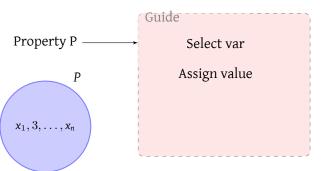


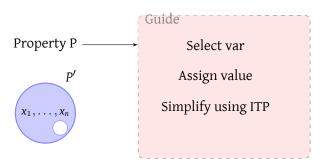


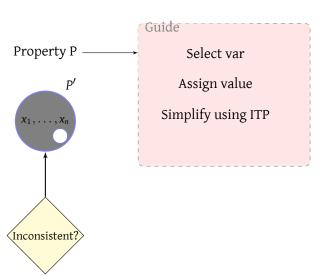


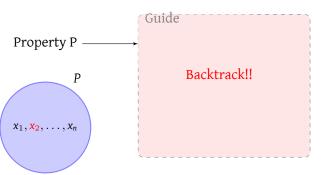


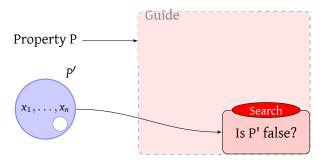




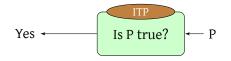


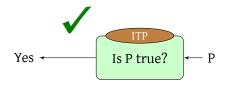


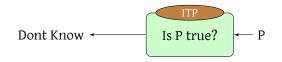


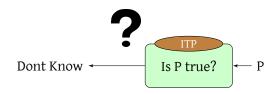


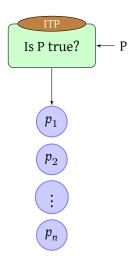


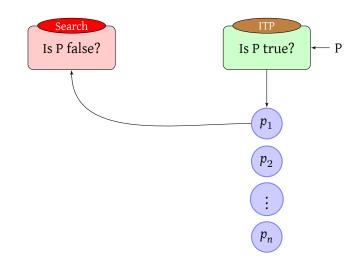












# Assumptions

- Specification language L
  - Multi-sorted first-order logic
  - Extensible
  - Executable

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  - Multi-sorted first-order logic
  - Extensible -- can introduce new function and predicate symbols using well-founded recursive definitions
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- ► An Interactive Theorem Prover (ITP) that can reason about specifications in *L*.
  - Smash takes as input a *goal*, a formula written in *L*, and returns a list of equi-valid *subgoals*.
  - ▶ Simplify takes as input an *L*-formula, *c*, and a list of formulas, *H*, and returns a *simplified* formula that is equivalent to *c* assuming *H* are true.

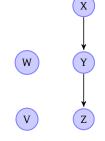




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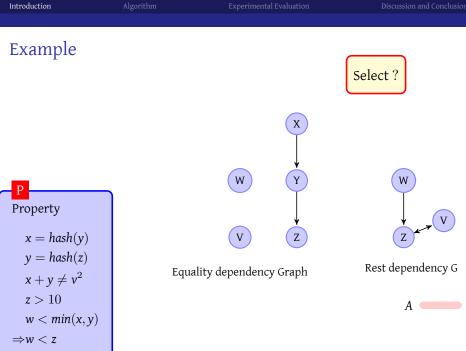


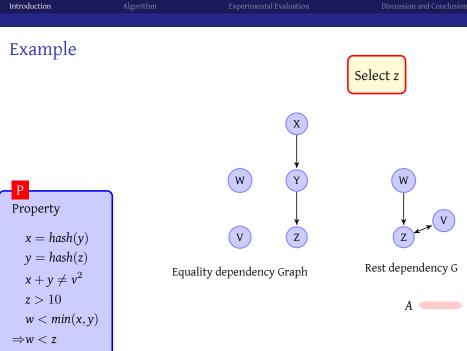


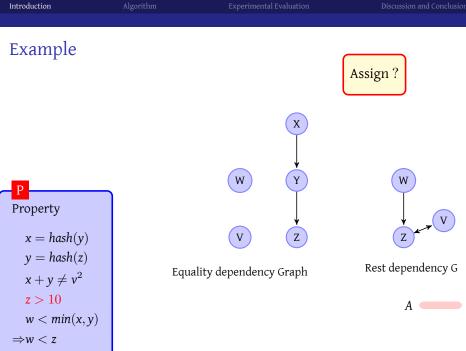


Equality dependency Graph

A







Assign 
$$z = 34$$
 (decision)

P  
Property  

$$x = hash(y)$$
  
 $y = hash(34)$   
 $x + y \neq v^2$   
 $w < min(x, y)$   
 $\Rightarrow w < 34$ 

A

Propagate with Simplify



A

Propagate with Simplify

#### P' Property

x = 3623878690

$$y = 268959709$$

$$x + 268959709 \neq v^2$$

w < x

$$\Rightarrow w < 34$$

А



#### P' Property

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# Update Assignment

#### P' Property

$$x = 3623878690$$

$$y = 268959709$$

$$x + 268959709 \neq v^2$$

w < x

 $\Rightarrow w < 34$ 

A 
$$z = 34$$

Input: Property P, Summary *summary* Output: Status, Summary of the analysis of P if P is closed then return AnalyzeConst(P)

initialize

Search till ...

if StopCond(*summary*) then return (done, *summary*)

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n, status := 0, not-done

Search till ...

if StopCond(*summary*) then return (done, *summary*)

Input: Property *P*, Summary *summary* Output: Status, Summary of the analysis of *P* if *P* is closed then return AnalyzeConst(*P*)

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```
Input: Property P, Summary summary
Output: Status, Summary of the analysis of P
if P is closed then return AnalyzeConst(P)
initialize
while \negStopCond(summary) \land n \leq slimit do
A, n := Search(P), n + 1
summary := updateA(summary, P, A)
if StopCond(summary) then return (done, summary)
Decompose P into subgoals using ITP
```

If progress then Recurse on each subgoal

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S := Smash(P)
```

```
summary := updateS(summary, P, S)
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Input: Property P, Summary summary
Output: Status, Summary of the analysis of P
  if P is closed then return AnalyzeConst(P)
  initialize
  Search till ...
  if StopCond(summary) then return (done, summary)
  Decompose P into subgoals ...
  if S \neq \{P\} then
      for all p \in S do
         status, summary := Analyze(p, summary)
         if status = done then return (done, summary)
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Input: Property *P*, Summary *summary* Output: Status, Summary of the analysis of *P* if *P* is closed then return AnalyzeConst(*P*)

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Search till ...

if StopCond(*summary*) then return (done, *summary*)

Decompose P into subgoals ...

If progress then Recurse on each subgoal ...

Input: Property *P* with at least one free variable Output: A counterexample (assignment) or *fail* **local Stack** *A* **of (var, val, # assigns, type, property)** Initialize and Select first variable Iteratively construct counterexample or *fail* 

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Input: Property *P* with at least one free variable Output: A counterexample (assignment) or *fail* local Stack *A* of (var, val, # assigns, type, property) A, i, x := [], 0,Select(*P*) Iteratively construct counterexample or *fail* 

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Input: Property P with at least one free variable
Output: A counterexample (assignment) or fail
  local Stack A of (var, val, # assigns, type, property)
  A, i, x := [], 0, \operatorname{Select}(P)
  while true do
       v, t := Assign(x, P)
      P' := \operatorname{Propagate}(x, v, P)
       if \neginconsistent(P') then
           Extend A, continue search if not done
       else if A \neq [] then
           backtrack
       fail if ...
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Input: Property P with at least one free variable
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  local Stack A of (var, val, # assigns, type, property)
  A, i, x := [], 0, Select(P)
  while true do
       v, t := Assign(x, P)
       P' := \operatorname{Propagate}(x, v, P)
       if \neginconsistent(P') then
           if t = ``decision" then i := i + 1
           A := push((x, v, i, t, P), A)
           if A is complete then return A
           i, P, x := 0, P', \operatorname{Select}(P')
       else if A \neq [] then
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       P' := \operatorname{Propagate}(x, v, P)
       if \neginconsistent(P') then
           Extend A, continue search if not done
       else if A \neq [] then
           repeat
               (x, ..., i, t, P) := head(A)
               A := pop(A)
           until (t = ``decision" \land i < blimit) \lor A = []
       fail if ...
```

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      else if A \neq [] then
           backtrack
       if A = [] \land (t = ``implied'' \lor i > blimit) then
           return fail
```

Input: Property *P* with at least one free variable Output: A free variable in *P* if  $\exists h \in hyps(P)$  of form x = c then return x

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 $G_{=} := EqualityDependencyGraph(P, vars(P))$ 

1. Case: x = y. Add  $x \leftrightarrow y$ .

```
2. Case: x = fterm

y \in freeVars(fterm) and

x \notin freeVars(fterm)

add x \rightarrow y
```

Input: Property *P* with at least one free variable Output: A free variable in *P* if  $\exists h \in hyps(P)$  of form x = c then return x $G_{=} :=$  EqualityDependencyGraph(P, vars(P))Do SCC on  $G_{=}$ , collect the leaf components in *L leaves*\_= := pick *x* from each  $l \in L$ 

Input: Property *P* with at least one free variable Output: A free variable in *P* if  $\exists h \in hyps(P)$  of form x = c then return x $G_{=} := EqualityDependencyGraph(P, vars(P))$ SCC on  $G_{=}$  $G_{bal} := RestDependencyGraph(P, leaves_{=})$ 

- 1.  $x \bowtie y$  where  $\bowtie \in \{<, \le, >, \ge\}$ : No edge
- 2.  $x \bowtie fterm$  such that  $\bowtie$  is a binary relation,  $y \in freeVars(fterm)$  and  $x \notin freeVars(fterm)$ : Add  $x \rightarrow y$
- 3.  $R(term_1, term_2, ..., term_n)$ , such that  $x \in freeVars(term_i), y \in freeVars(term_j), i \neq j$ ,  $n \geq 2$  and R is an arbitrary n-ary relation: Add  $x \leftrightarrow y$ .

Input: Property *P* with at least one free variable Output: A free variable in *P* 

if  $\exists h \in hyps(P)$  of form x = c then return x

 $G_{=} := EqualityDependencyGraph(P, vars(P))$ 

SCC on  $G_{=}$ 

 $G_{\bowtie} := \mathsf{RestDependencyGraph}(P, \mathit{leaves}_{=})$ 

Do SCC on  $G_{\bowtie}$  to get dag  $D_{\bowtie}$ 

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 $G_{\bowtie} := \mathsf{RestDependencyGraph}(P, \mathit{leaves}_{=})$ 

Do SCC on  $G_{\bowtie}$  to get dag  $D_{\bowtie}$ 

X := the leaf in  $D_{\bowtie}$  with maximum  $i_{=}$  value return X

 $i_{=}(x)$  denotes number of nodes that can reach it in  $G_{=}$  $i_{=}(X)$  denotes max value of  $i_{=}$  among nodes  $\in X$ 

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- 1. Fetch
- 2. Read
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#### Can we find design errors that lead to hazards?

- 1. Assuming designer has modelled both ISA and MA
- 2. Formalize above correctness condition
- 3. Analyze it using our method (demo)

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#### Correctness

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#### Observations

- 1. No assertions were written
- 2. No lemmas were specified
- 3. No manual tests or test driver given.

Alloy

- Alloy is a declarative modeling language based on sets and relations (relational logic with transitive closure)
- Used for describing and analyzing high-level specifications and designs.

#### Automatic Analysis

Given a bound on # model elements, called *scope*, Alloy models (and its specifications) translated into Boolean formulas and shipped to off-the-shelf SAT solvers.

	Alloy Analyzer			Our method	
Property	Scope	Time	Result	Time	Result
delUndoesAdd	25	26.41		0.07	QED
addIdempotent	25	37.76		0.19	QED
addLocal	3	0.08	CE	1.35	CE
lookupYields	3	0.05	CE	0.83	CE
writeRead	34	99.69		0.02	QED
writeIdempotent	33	44.13		0.01	QED
hidePreservesInv	61	24.91		0.26	QED
cutPaste	3	0.20	CE	0.49	CE
pasteCut	3	0.20	CE	1.38	CE
pasteAffectsHidden	27	117.63		0.42	QED
markSweepSound	8	47.34		0.28	QED
markSweepComplete	7	58.12		0.34	QED

Table: Comparison with Alloy Analyzer (AA)

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 Table: Comparison with Alloy Analyzer (AA)

<sup>&</sup>lt;sup>1</sup>Ghazi and Taghdiri. Relational Reasoning by SMT Solving. In FM 2011

#### Methodology

Modeled above examples in ACL2, mimicking original formulation in Alloy.

Used *set* types and *map* types *i.e.*, binary relations, provided by our data definition framework.

#### Observations

- 1. The ordered sets and records library in ACL2 distribution, powerful enough to prove all the properties that Alloy posits are true
- 2. No intermediate lemmas provided, no hint or guidance offered to the theorem prover
- 3. Highlights effectiveness of powerful libraries by the tool-writer put to use by the choice of right abstractions by the programmer

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# Conclusions

- Automatically analyze properties, interleaving ITP and testing in a fine-grained fashion
- Search algorithm guides testing when it is stuck (Decision Procedures can also benefit)
- Select algorithm can be used as a starting point by concolic testing
- Combining automated methods with ITP technology results in a more powerful, yet automated method.
- Better interactive theorem proving experience

#### The End

# Thank you