CS 323E **Homework 9** **Due: 5pm, Wed Nov 22, 2000**

1. (5 points)
   Let MyPLU(A) be the function which computes the LU factorization with partial pivoting of the matrix A:

```
function [P,L,U]=MyPLU(A)
n=length(A);
P=eye(n,n);
for k=1:n-1,
(*)   [max,ind]=max(abs(A(k:n,k)));  p=k+ind-1;
      P([k p],:)=P([p k],:);    % modifies the permutation matrix
      A([k p],:)=A([p k],:);    % modifies the input matrix A
      for i=k+1:n,
        A(i,k)=A(i,k)/A(k,k);
        for j=k+1:n,
          A(i,j)=A(i,j)-A(i,k)*A(k,j);
        end
      end
end
...
```

   Can the line (*) be substituted by `[max,ind]=max(abs(A(:,k)));`? i.e.: at the k-th step, can the scan for the maximum element involve the rows formerly computed? Give reasons for your answer.

2. The MATLAB command `hilb(n)` generates an $n \times n$ Hilbert matrix, which we denote by H_n. Try $n = 3, 10, 20$ in the following problems:

   1) (4 points)
      Solve:

      $$H_n x_n = b_n$$

      for x_n, where $b_n = H_n * \texttt{ones}(\texttt{n}, 1)$.
      Use the MATLAB command "\" to solve the above system. (See `help mldivide`).

   b) (2 points)
      How close is x_n to the exact solution? Comment.

   c) (4 points)
      Explain the accuracy of x_n. Use the command `cond` to get the condition number of H_n.

   d) (5 points)
      Does the MATLAB command "\" do pivoting? Give reason for your answer.

3. (Use pen & paper). Let

$$A = \begin{bmatrix} 10^{-16} & 10^{-17} \\ -10^{-16} & 10^{-17} \end{bmatrix}$$

a) (2 points)
Compute the determinant of A.

b) (5 points)
Compute $\kappa_1(A) = \|A\|_1 \cdot \|A^{-1}\|_1$.

c) (2 points)
Is A nearly singular? Comment.

d) (1 point)
Does the "smallness" of the determinant imply that A is nearly singular?

4. The MATLAB command `pascal(n)` generates an $n \times n$ Pascal matrix, which we denote by P_n. Try $n = 16$ in the following.

a) (1 point)
Using MATLAB, find the determinant of P_n.

b) (1 point)
Using MATLAB, find the condition number of P_n.

c) (3 points)
Is P_n close to singularity? Comment.