**CS 323E** # Midterm **Oct 24, 2000**

Show all your work in solving the following problems. Otherwise points may be deducted.

1. Let $f(x) = \sin(2\pi\, x) + \cos(2\pi\, x)$.

   a) **(4 points)** Show that $|f^{(n)}(x)| \le 2(2\pi)^n$ for all $x$. Note that $f^{(n)}(x)$ denotes the $n$-th derivative of $f$ at $x$.

   b) **(5 points)** Consider the interval $[\frac{1}{10}, \frac{1}{5}]$. Let $p(x)$ be the polynomial interpolating $f$ at $n$ equally spaced points $x_1, x_2, \ldots, x_n$ in this interval ($x_1 = \frac{1}{10}, x_n = \frac{1}{5}$). Find $n$ such that

   $$error(x) = |f(x) - p(x)| \le 10^{-3}$$

   HINT: Use $f(x) = p(x) + \frac{f^{(n)}(\xi)}{n!}(x - x_1)(x - x_2)\ldots(x - x_n)$ and $|(x - x_1)(x - x_2)\ldots(x - x_n)| \le (b - a)^n$.

2. Given the table:

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| $x$ | -1 | 0 | 1/2 | 1 |
| $f(x)$ | 0 | 1 | 0.650068 | 0 |
| $f'(x)$ | 1.16395 | 0 | -1.23204 | -1.16395 |

   a) **(3 points)** Compute the Newton polynomial $p(x)$ passing through $(x_1, f(x_1))$, $(x_2, f(x_2))$, $(x_3, f(x_3))$ and $(x_4, f(x_4))$, i.e., $p(x_i) = f(x_i)$ for $i = 1, 2, 3, 4$.

   b) **(3 points)** Compute the cubic Hermite polynomial $q(x)$ through $(x_1, f(x_1), f'(x_1))$ and $(x_3, f(x_3), f'(x_3))$, i.e., $q(x_1) = f(x_1)$, $q(x_3) = f(x_3)$, $q'(x_1) = f'(x_1)$ and $q'(x_3) = f'(x_3)$.
   HINT: Use $q(x) = a_1 + a_2(x - x_1) + a_3(x - x_1)^2 + a_4(x - x_1)^2(x - x_3)$.

   c) **(4 points)** Find the interpolating polynomial $r(x)$ (of least possible degree) through $(x_1, f(x_1))$, $(x_2, f(x_2), f'(x_2))$ and $(x_3, f(x_3))$, i.e., $r(x_1) = f(x_1)$, $r(x_2) = f(x_2)$, $r'(x_2) = f'(x_2)$ and $r(x_3) = f(x_3)$.

   d) **(1 point)** The tabular data given above is from the function

   $$f(x) = \frac{1 - e^{1-x^2}}{1 - e}.$$

   Among the computed polynomials, which gives the best approximation to $f(-0.1)$?

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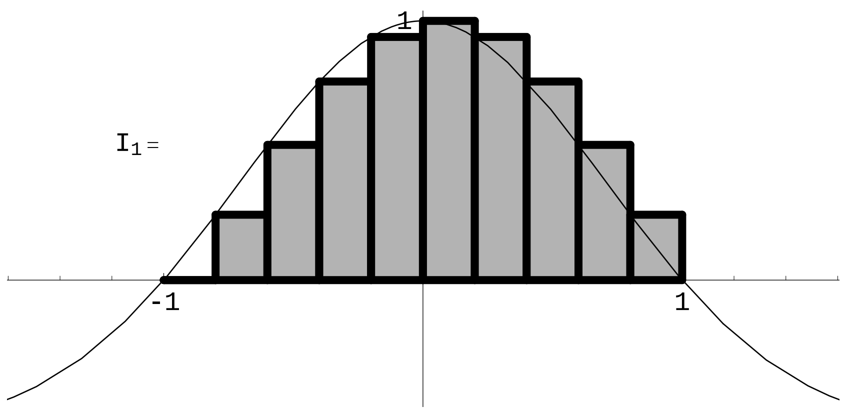
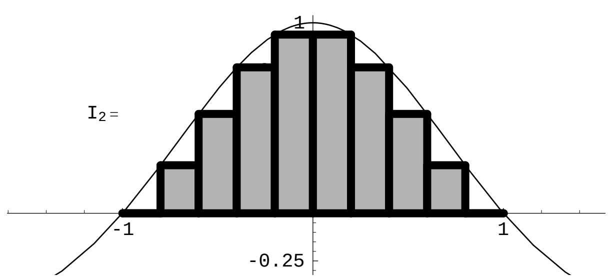
3. The problem is to compute

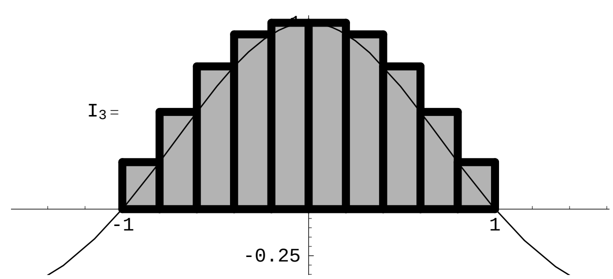
$$I = \int_{-1}^{1} \frac{1 - e^{1-x^2}}{1 - e}\, dx$$

a) **(3 points)** Write a MATLAB function `I1 = left_point_rule(a,b,n)` to compute the above integral by dividing the interval into $n$ equal subintervals and then using the left-point rule; i.e., compute $I_1$ as the shaded area below:




b) **(4 points)** Use or modify the above function to compute $I_2$ and $I_3$, shown below pictorially:




c) **(1 point)** Produce a table listing $I_1$, $I_2$, $I_3$, for $n = 10, 50, 250$,

d) **(2 points)** Use the above table to give error bounds $|I - I_2|$ for $n = 10, 50, 250$.