

1. The MATLAB command `hilb(n)` generates an $n \times n$ Hilbert matrix, which we denote by H_n . Try $n = 3, 10, 20$ in the following problems:

a) (4 points)

Solve:

$$H_n x_n = b_n$$

for x_n , where $b_n = H_n * \mathbf{ones}(n, 1)$.

Use the MATLAB command “\” to solve the above system. (See `help mldivide`).

b) (2 points)

How close is x_n to the exact solution? Comment.

c) (4 points)

Explain the accuracy of x_n . Use the command `cond` to get the condition number of H_n .

2. (5 points)

Does the MATLAB command “\” do pivoting? Give an example to justify your answer.

3. (Use pen & paper). Let

$$A = \begin{bmatrix} 10^{-16} & 10^{-17} \\ -10^{-16} & 10^{-17} \end{bmatrix}$$

a) (2 points)

Compute the determinant of A .

b) (5 points)

Compute $\kappa_1(A) = \|A\|_1 \cdot \|A^{-1}\|_1$.

c) (2 points)

Is A nearly singular? Comment.

d) (1 point)

Does the small magnitude of the determinant imply that A is nearly singular?

4. The MATLAB command `pascal(n)` generates an $n \times n$ Pascal matrix, which we denote by P_n . Try $n = 16$ in the following.

a) (1 point)

Using MATLAB, find the determinant of P_n .

b) (1 point)

Using MATLAB, find the condition number of P_n .

c) (3 points)

Is P_n close to singularity? Comment.