EXPLAIN: A Tool for Performing Abductive Inference

Isil Dillig
MSR Cambridge

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- Deduction: Infers valid conclusion from premises
- Abduction: Infers missing premise to explain a given conclusion
- Given known facts Γ and desired outcome ϕ, abductive inference finds "simple" explanatory hypothesis ψ such that

$$\Gamma \wedge \psi \models \phi \quad \text{and} \quad \mathrm{SAT}(\Gamma \wedge \psi)$$

Simple Example

- Facts: “If it rains, then it is wet and cloudy”, “If it is wet, then it is slippery”:
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- Conclusion: "It is cloudy and slippery", i.e., $C \land S$
- Abductive explanation: R, i.e., "It is rainy"

Arithmetic Example

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int x = 0;
int y = 0;

while(x < n)
{
   x = x+1;
   y = y+2;
}

assert( x + y >= 3*n);
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- Suppose we know $x \geq n$
  - e.g., from loop termination condition
- Desired conclusion $x + y \geq 3n$
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- Abductive explanation: $y \geq 2x$
  - corresponds to missing loop invariant

Properties of Desired Solutions

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- Trivial solution: ϕ, but not useful because does not take into account what we know
- So, what kind of solutions do want to compute?

Which Abductive Explanations Are Good?

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Occam's Razor

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- Generality: If explanation A is logically weaker than explanation B, always prefer A
- Simplicity: Not clear-cut, but we use number of variables
- This simplicity criterion makes sense in verification because we want proof subgoals to be local and refer to few variables

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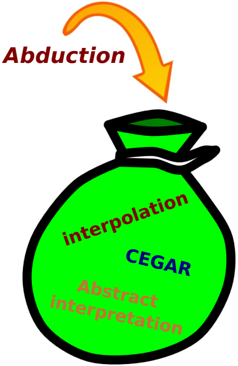
```
abduce(I, φ) {
  V = msa(I ⇒ φ, I)
  ψ = QE(∀V̄.(I ⇒ φ))
  ψ' = simplify(ψ, I)
  return ψ'
}
```

Abduction in Program Analysis

Useful technique to add to our bag of tricks; lots of applications!



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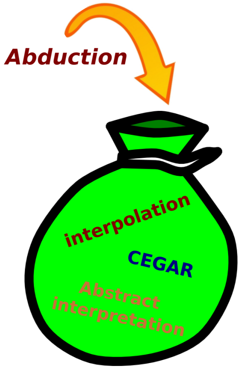

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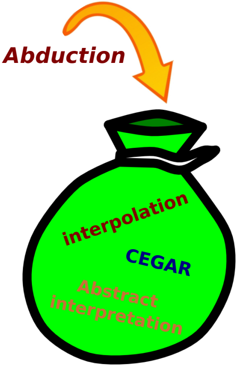

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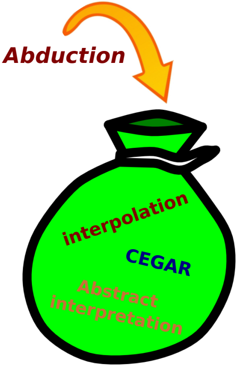

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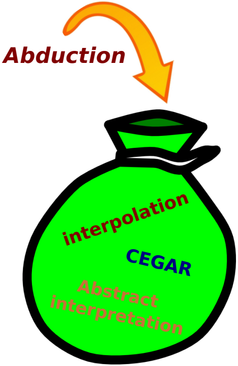

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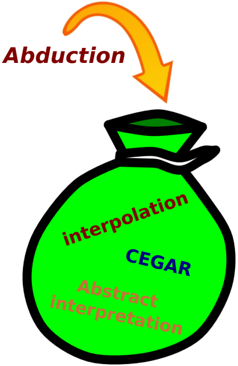

- Loop invariant generation
- Synthesis of compositional program proofs
- Inference of missing library specifications
- Explaining static analysis warnings to programmers
- Modular analysis using separation logic

EXPLAIN

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  http://www.cs.wm.edu/˜tdillig/mistral

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- Try it out!

Questions?