CS311H: Discrete Mathematics

Intro and Propositional Logic

Instructor: Işıl Dillig

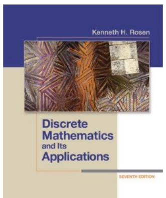
Course Staff

- Instructor: Prof. Işıl Dillig
- Proctors: Ana Brendel, Shray Vats, Arnav Mohan
- Course webpage: http://www.cs.utexas.edu/~isil/cs311h-csb/
- Contains contact info, office hours, syllabus, slides from lectures etc.

About this Course

- Give mathematical background you need for computer science
- Topics: Logic, proof techniques, number theory, combinatorics, graph theory, basic complexity theory ...
- These will come up again and again in higher-level CS courses
  - Master CS311H material if you want to do well in future courses!

Textbook



- Textbook (optional): Discrete Mathematics and Its Applications by Kenneth Rosen
- Textbook not a substitute for lectures:
  - Class presentation may not follow book
  - Skip many chapters and cover extra material

Piazza

- Piazza page: https://piazza.com/utexas/fall2021/cs311hcsb
- Homework #0: Make sure you can access Piazza page!
- Please post class-related questions on Piazza instead of emailing instructor TA's
  - If something is not clear to you, it won't be to others either
  - You'll get answers a lot quicker
- Please use common sense when posting questions on Piazza
  - Hints/ideas ok, but cannot post full solutions!!
- If you have a more personal question, ok to use email (cs311h-staff@cs.utexas.edu)

Discussion Sections and Office Hours

- Discussion sections on Fridays 2-3pm in JGB 2.202
- Discussion section will answer questions, solve new problems, and go over previous homework
- Lots of office hours – check webpage for info!

Requirements

- Exams + problem sets + class attendance/participation
- Three exams scheduled for Sep 30, Oct 28, Dec 1
- 9 or 10 problem sets (about once every week)

Grading

- Exam: collectively 45% of final grade
- Homework: 50% of final grade
- Attendance/participation: 5% of final grade
- Final grades will be curved

Homework Policy

- Homework must be submitted by **noon** on the due date
- Late submissions **not** allowed, lowest homework score dropped when calculating grades
- Homework must be done **on your own**, but allowed to ask questions on Piazza and during office hours
  - **Not allowed** to do problem sets in groups
  - **Not allowed** to check solutions with each other
- Homework solutions must be typeset using Latex and submitted through Gradescope
- A few of the homework assignments will involve programming, rest are problem sets

More on Homework

- Problem sets in this class will be much harder than what you are used to from high school!!
  - Normal to spend >30mins on a single HW question
  - Do not seek help from us unless you've spent at least one hour on each problem
- Expect each problem set to take > 6 hours

Exam Policy

- Exams will be on-line and cumulative
- Can only consult lecture notes and textbook but nothing else
- Cannot talk with others about exam questions
- Honor code taken very seriously at UT (both for exams + ps)
  - May be expelled for violating honor code!
  - Please read departmental guidelines (link from course webpage)

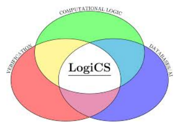
Class Participation

- Everyone expected to attend lectures and participate
- 5% of course grade for participation (attendance, asking/answering questions, being active on Piazza)
- Please ask questions!
  - Will make class more fun for everyone
  - Others also benefit from your questions

Let's get started!

Logic

- Logic: study of valid reasoning; fundamental to CS
- Allows us to represent knowledge in a formal/mathematical way and automate some types of reasoning
- **Many applications in CS:** AI, programming languages, databases, computer architecture, automated testing and program analysis, ...



Propositional Logic

- Simplest logic is propositional logic
- Building blocks of propositional logic are propositions
- A proposition is a statement that is either true or false
- Examples:
  - "CS311 is a course in discrete mathematics": True
  - "Austin is located in California": False
  - "Pay attention": Not a proposition
  - "x+1 =2": Not a proposition

Propositional Variables, Truth Value

- Truth value of a proposition identifies whether a proposition is true (written T) or false (written F)
- What is truth value of "Today is Friday"?
- Variables that represent propositions are called propositional variables
- Denote propositional variables using lower-case letters, such as $p, p_1, p_2, q, r, s, \ldots$
- Truth value of a propositional variable is either T or F.

Compound Propositions

- More complex propositions formed using logical connectives (also called boolean connectives)
- Three basic logical connectives:
  1. $\land$: conjunction (read "and"),
  2. $\lor$: disjunction (read "or")
  3. $\neg$: negation (read "not")
- Propositions formed using these logical connectives called compound propositions; otherwise atomic propositions
- A propositional formula is either an atomic or compound proposition

Negation

- Negation of a proposition p, written $\neg p$, represents the statement "It is not the case that p".
- If p is T, $\neg p$ is F and vice versa.
- In simple English, what is $\neg p$ if p stands for ...
  - "Less than 80 students are enrolled in CS311"?

Conjunction

- Conjunction of two propositions p and q, written $p \wedge q$, is the proposition "p and q"
- $p \wedge q$ is T if both p is true and q is true, and F otherwise.
- What is the conjunction and the truth value of $p \wedge q$ for ...
  - $p =$ "It is Thursday", $q =$ "It is morning" ?

Disjunction

- Disjunction of two propositions p and q, written $p \vee q$, is the proposition "p or q"
- $p \vee q$ is T if either p is true or q is true, and F otherwise.
- What is the disjunction and the truth value of $p \vee q$ for ...
  - $p =$ "It is spring semester", $q =$ "Today is Thursday"?

Propositional Formulas and Truth Tables

- Truth table for propositional formula F shows truth value of F for every possible value of its constituent atomic propositions
- Example: Truth table for $\neg p$

p	$\neg p$
T	F
F	T

- Example: Truth table for $p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Constructing Truth Tables

Useful strategy for constructing truth tables for a formula F:

1. Identify F's constituent atomic propositions
2. Identify F's compound propositions in increasing order of complexity, including F itself
3. Construct a table enumerating all combinations of truth values for atomic propositions
4. Fill in values of compound propositions for each row

Examples

Construct truth tables for the following formulas:

1. $(p \vee q) \wedge \neg p$
2. $(p \wedge q) \vee (\neg p \wedge \neg q)$
3. $(p \vee q \vee \neg r) \wedge r$

More Logical Connectives

- $\wedge, \vee, \neg$ most common boolean connectives, but there are other boolean connectives as well
- Other connectives: exclusive or $\oplus$, implication $\rightarrow$, biconditional $\leftrightarrow$
- Exclusive or: $p \oplus q$ is true when exactly one of p and q is true, and false otherwise
- Truth table:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Implication (Conditional)

- An implication (or conditional) $p \rightarrow q$ is read "if p then q" or "p implies q"
- It is false if p is true and q is false, and true otherwise
- Exercise: Draw truth table for $p \rightarrow q$
- In an implication $p \rightarrow q$, p is called antecedent and q is called consequent

Converting English into Logic

Let $p =$" I major in CS" and $q =$ "I will find a good job". How do we translate following English sentences into logical formulas?

- "If I major in CS, then I will find a good job": $p \rightarrow q$
- "I will not find a good job unless I major in CS": $\neg p \rightarrow \neg q$
- "It is sufficient for me to major in CS to find a good job": $p \rightarrow q$
- "It is necessary for me to major in CS to find a good job": $\neg p \rightarrow \neg q$

More English - Logic Conversions

Let $p =$" I major in CS", $q =$ "I will find a good job", $r =$ "I can program". How do we translate following English sentences into logical formulas?

- " I will not find a good job unless I major in CS or I can program": $(\neg p \wedge \neg r) \rightarrow \neg q$
- " I will not find a good job unless I major in CS and I can program": $(\neg p \vee \neg r) \rightarrow \neg q$
- "A prerequisite for finding a good job is that I can program": $\neg r \rightarrow \neg q$
- "If I major in CS, then I will be able to program and I can find a good job": $p \rightarrow (r \wedge q)$

Converse of a Implication

- The converse of an implication $p \rightarrow q$ is $q \rightarrow p$.
- What is the converse of "If I am a CS major, then I can program"?
- Note: It is possible for a implication to be true, but its converse to be false, e.g., $F \rightarrow T$ is true, but converse false

Inverse of an Implication

- The inverse of an implication $p \rightarrow q$ is $\neg p \rightarrow \neg q$.
- What is the inverse of "If I get an A in CS311, then I am smart"?
- Note: It is possible for a implication to be true, but its inverse to be false. $F \rightarrow T$ is true, but inverse is false

Contrapositive of Implication

- The contrapositive of an implication $p \rightarrow q$ is $\neg q \rightarrow \neg p$.
- What is the contrapositive of "If I am a CS major, then I can program"?
- Question: Is it possible for an implication to be true, but its contrapositive to be false?

Question

- Given $p \rightarrow q$, is it possible that its converse is true, but inverse is false?
- Converse: $q \rightarrow p$
- Inverse: $\neg p \rightarrow \neg q$
- Inverse is contrapositive of converse, hence they always have same truth value

Biconditionals

- A biconditional $p \leftrightarrow q$ is the proposition "p if and only if q".
- The biconditional $p \leftrightarrow q$ is true if p and q have same truth value, and false otherwise.
- Exercise: Construct a truth table for $p \leftrightarrow q$
- Question: How can we express $p \leftrightarrow q$ using the other boolean connectives?

Operator Precedence

- Given a formula $p \wedge q \vee r$, do we parse this as $(p \wedge q) \vee r$ or $p \wedge (q \vee r)$?
- Without settling on a convention, formulas without explicit paranthesization are ambiguous.
- To avoid ambiguity, we will specify precedence for logical connectives.

Operator Precedence, cont.

- Negation ($\neg$) has higher precedence than all other connectives.
- Question: Does $\neg p \wedge q$ mean (i) $\neg(p \wedge q)$ or (ii) $(\neg p) \wedge q$?
- Conjunction ($\wedge$) has next highest predence.
- Question: Does $p \wedge q \vee q$ mean (i) $(p \wedge q) \vee r$ or (ii) $p \wedge (q \vee r)$?
- Disjunction ($\vee$) has third highest precedence.
- Next highest is precedence is $\rightarrow$, and lowest precedence is $\leftrightarrow$

Operator Precedence Example

- Which is the correct interpretation of the formula

$$p \vee q \wedge r \leftrightarrow q \rightarrow \neg r$$

(A) $((p \vee (q \wedge r)) \leftrightarrow q) \rightarrow (\neg r)$

(B) $((p \vee q) \wedge r) \leftrightarrow q) \rightarrow (\neg r)$

(C) $(p \vee (q \wedge r)) \leftrightarrow (q \rightarrow (\neg r))$

(D) $(p \vee ((q \wedge r) \leftrightarrow q)) \rightarrow (\neg r)$