CS311H: Discrete Mathematics

Intro and Propositional Logic

Instructor: Isıl Dillig

Course Staff

- Instructor: Prof. Isıl Dillig
- Proctors: Ana Brendel, Shray Vats, Arnav Mohan
- Course webpage: http://www.cs.utexas.edu/~isil/cs311h-csb/
  - Contains contact info, office hours, syllabus, slides from lectures etc.

About this Course

- Give mathematical background you need for computer science
- Topics: Logic, proof techniques, number theory, combinatorics, graph theory, basic complexity theory ...
- These will come up again and again in higher-level CS courses
  - Master CS311H material if you want to do well in future courses!

Textbook

- Textbook (optional): Discrete Mathematics and Its Applications by Kenneth Rosen
- Textbook not a substitute for lectures:
  - Class presentation may not follow book
  - Skip many chapters and cover extra material

Piazza

- Piazza page: https://piazza.com/utexas/fall2021/cs311hcsb
- Homework #0: Make sure you can access Piazza page!
- Please post class-related questions on Piazza instead of emailing instructor TA’s
  - If something is not clear to you, it won’t be to others either
  - You’ll get answers a lot quicker
- Please use common sense when posting questions on Piazza
  - Hints/ideas ok, but cannot post full solutions!!
- If you have a more personal question, ok to use email (cs311h-staff@cs.utexas.edu)

Discussion Sections and Office Hours

- Discussion sections on Fridays 2-3pm in JGB 2.202
- Discussion section will answer questions, solve new problems, and go over previous homework
- Lots of office hours – check webpage for info!
Requirements

- Exams + problem sets + class attendance/participation
- Three exams scheduled for Sep 30, Oct 28, Dec 1
- 9 or 10 problem sets (about once every week)

Grading

- Exam: collectively 45% of final grade
- Homework: 50% of final grade
- Attendance/participation: 5% of final grade
- Final grades will be curved

Homework Policy

- Homework must be submitted by noon on the due date
- Late submissions not allowed, lowest homework score dropped when calculating grades
- Homework must be done on your own, but allowed to ask questions on Piazza and during office hours
  - Not allowed to do problem sets in groups
  - Not allowed to check solutions with each other
- Homework solutions must be typeset using Latex and submitted through Gradescope
- A few of the homework assignments will involve programming, rest are problem sets

More on Homework

- Problem sets in this class will be much harder than what you are used to from high school!!
  - Normal to spend >30mins on a single HW question
  - Do not seek help from us unless you’ve spent at least one hour on each problem
- Expect each problem set to take > 6 hours

Exam Policy

- Exams will be on-line and cumulative
- Can only consult lecture notes and textbook but nothing else
- Cannot talk with others about exam questions
- Honor code taken very seriously at UT (both for exams + ps)
  - May be expelled for violating honor code!
  - Please read departmental guidelines (link from course webpage)

Class Participation

- Everyone expected to attend lectures and participate
- 5% of course grade for participation (attendance, asking/answering questions, being active on Piazza)
- Please ask questions!
  - Will make class more fun for everyone
  - Others also benefit from your questions
Let's get started!

Logic

- Logic: study of valid reasoning; fundamental to CS
- Allows us to represent knowledge in a formal/mathematical way and automate some types of reasoning
- Many applications in CS: AI, programming languages, databases, computer architecture, automated testing and program analysis, . . .

Propositional Logic

- Simplest logic is propositional logic
- Building blocks of propositional logic are propositions
- A proposition is a statement that is either true or false
- Examples:
  - "CS311 is a course in discrete mathematics": True
  - "Austin is located in California": False
  - "Pay attention": Not a proposition
  - "x+1 =2": Not a proposition

Propositional Variables, Truth Value

- Truth value of a proposition identifies whether a proposition is true (written T) or false (written F)
- What is truth value of "Today is Friday"?
- Variables that represent propositions are called propositional variables
- Denote propositional variables using lower-case letters, such as \( p, p_1, p_2, q, r, t, s, . . . \)
- Truth value of a propositional variable is either T or F.

Compound Propositions

- More complex propositions formed using logical connectives (also called boolean connectives)
- Three basic logical connectives:
  1. \( \wedge \): conjunction (read "and")
  2. \( \vee \): disjunction (read "or")
  3. \( \neg \): negation (read "not")
- Propositions formed using these logical connectives called compound propositions; otherwise atomic propositions
- A propositional formula is either an atomic or compound proposition

Negation

- Negation of a proposition \( p \), written \( \neg p \), represents the statement "It is not the case that \( p \)"
- If \( p \) is \( T \), \( \neg p \) is \( F \) and vice versa.
- In simple English, what is \( \neg p \) if \( p \) stands for . . .
  - "Less than 80 students are enrolled in CS311"?
Conjunction

- **Conjunction** of two propositions $p$ and $q$, written $p \land q$, is the proposition "$p$ and $q$"
- $p \land q$ is $T$ if both $p$ is true and $q$ is true, and $F$ otherwise.
- What is the conjunction and the truth value of $p \land q$ for . . .
  - $p = \text{"It is Thursday"}$, $q = \text{"It is morning"}$?

Disjunction

- **Disjunction** of two propositions $p$ and $q$, written $p \lor q$, is the proposition "$p$ or $q$"
- $p \lor q$ is $T$ if either $p$ is true or $q$ is true, and $F$ otherwise.
- What is the disjunction and the truth value of $p \lor q$ for . . .
  - $p = \text{"It is spring semester"}$, $q = \text{"Today is Thursday"}$?

Propositional Formulas and Truth Tables

- Truth table for propositional formula $F$ shows truth value of $F$ for every possible value of its constituent atomic propositions

<table>
<thead>
<tr>
<th>$p$</th>
<th>$\neg p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$F$</td>
</tr>
<tr>
<td>$F$</td>
<td>$T$</td>
</tr>
</tbody>
</table>

- Example: Truth table for $p \lor q$

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \lor q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$T$</td>
<td>$F$</td>
<td>$T$</td>
</tr>
<tr>
<td>$F$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$F$</td>
<td>$F$</td>
<td>$F$</td>
</tr>
</tbody>
</table>

Constructing Truth Tables

Useful strategy for constructing truth tables for a formula $F$:

1. Identify $F$’s constituent atomic propositions
2. Identify $F$’s compound propositions in increasing order of complexity, including $F$ itself
3. Construct a table enumerating all combinations of truth values for atomic propositions
4. Fill in values of compound propositions for each row

Examples

Construct truth tables for the following formulas:

1. $(p \lor q) \land \neg p$
2. $(p \land q) \lor (\neg p \land \neg q)$
3. $(p \lor q \lor \neg r) \land r$

More Logical Connectives

- $\land, \lor, \neg$ most common boolean connectives, but there are other boolean connectives as well
- Other connectives: exclusive or $\oplus$, implication $\rightarrow$, biconditional $\leftrightarrow$
  - Exclusive or: $p \oplus q$ is true when exactly one of $p$ and $q$ is true, and false otherwise
    
    | $p$ | $q$ | $p \oplus q$ |
    |-----|-----|-------------|
    | $T$ | $T$ | $F$         |
    | $T$ | $F$ | $T$         |
    | $F$ | $T$ | $T$         |
    | $F$ | $F$ | $F$         |
    
    - Truth table:
Implication (Conditional)

- An implication (or conditional) \( p \rightarrow q \) is read “if \( p \) then \( q \)” or “\( p \) implies \( q \)”.
- It is false if \( p \) is true and \( q \) is false, and true otherwise.
- Exercise: Draw truth table for \( p \rightarrow q \).
- In an implication \( p \rightarrow q \), \( p \) is called antecedent and \( q \) is called consequent.

Converting English into Logic

Let \( p = \) “I major in CS” and \( q = \) “I will find a good job”. How do we translate following English sentences into logical formulas?

- “If I major in CS, then I will find a good job”: \( p \rightarrow q \)
- “I will not find a good job unless I major in CS”: \( \neg p \rightarrow \neg q \)
- “It is sufficient for me to major in CS to find a good job”: \( p \rightarrow q \)
- “It is necessary for me to major in CS to find a good job”: \( \neg p \rightarrow \neg q \)

More English - Logic Conversions

Let \( p = \) “I major in CS”, \( q = \) “I will find a good job”, \( r = \) “I can program”. How do we translate following English sentences into logical formulas?

- “I will not find a good job unless I major in CS or I can program”: \( \neg p \land \neg r \rightarrow \neg q \)
- “I will not find a good job unless I major in CS and I can program”: \( \neg p \lor \neg r \rightarrow \neg q \)
- “A prerequisite for finding a good job is that I can program”: \( \neg r \rightarrow \neg q \)
- “If I major in CS, then I will be able to program and I can find a good job”: \( p \rightarrow (r \land q) \)

Inverse of an Implication

- The inverse of an implication \( p \rightarrow q \) is \( \neg p \rightarrow \neg q \).
- What is the inverse of “If I get an A in CS311, then I am smart”? 
- Note: It is possible for a implication to be true, but its inverse to be false. \( F \rightarrow T \) is true, but inverse is false.

Contrapositive of Implication

- The contrapositive of an implication \( p \rightarrow q \) is \( \neg q \rightarrow \neg p \).
- What is the contrapositive of “If I am a CS major, then I can program”? 
- Question: Is it possible for an implication to be true, but its contrapositive to be false?
Question

- Given \( p \rightarrow q \), is it possible that its converse is true, but inverse is false?
- Converse: \( q \rightarrow p \)
- Inverse: \( \neg p \rightarrow \neg q \)
- Inverse is contrapositive of converse, hence they always have same truth value

Biconditionals

- A biconditional \( p \leftrightarrow q \) is the proposition "p if and only if q".
- The biconditional \( p \leftrightarrow q \) is true if \( p \) and \( q \) have same truth value, and false otherwise.
- Exercise: Construct a truth table for \( p \leftrightarrow q \)
- Question: How can we express \( p \leftrightarrow q \) using the other boolean connectives?

Operator Precedence

- Given a formula \( p \land q \lor r \), do we parse this as \( (p \land q) \lor r \) or \( p \land (q \lor r) \)?
- Without settling on a convention, formulas without explicit paranthesization are ambiguous.
- To avoid ambiguity, we will specify precedence for logical connectives.

Operator Precedence, cont.

- Negation (\( \neg \)) has higher precedence than all other connectives.
- Question: Does \( \neg p \land q \) mean (i) \( \neg (p \land q) \) or (ii) \( (\neg p) \land q \)?
- Conjunction (\( \land \)) has next highest precedence.
- Question: Does \( p \land q \lor r \) mean (i) \( (p \land q) \lor r \) or (ii) \( p \land (q \lor r) \)?
- Disjunction (\( \lor \)) has third highest precedence.
- Next highest is precedence is \( \rightarrow \), and lowest precedence is \( \leftrightarrow \)

Operator Precedence Example

- Which is the correct interpretation of the formula

\[
p \lor q \land r \leftrightarrow q \rightarrow \neg r
\]

(A) \( ((p \lor (q \land r)) \leftrightarrow q) \rightarrow (\neg r) \)

(B) \( ((p \lor q) \land r) \leftrightarrow q) \rightarrow (\neg r) \)

(C) \( (p \lor (q \land r)) \leftrightarrow (q \rightarrow (\neg r)) \)

(D) \( (p \lor ((q \land r) \leftrightarrow q)) \rightarrow (\neg r) \)