

## CS311H: Discrete Mathematics

### Intro and Propositional Logic

Instructor: Işıl Dillig

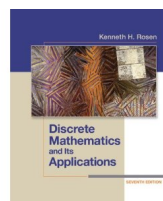
## Course Staff

- ▶ **Instructor:** Prof. Işıl Dillig
- ▶ **TAs:** Zeki Gurbuz, Letizia Fazzini, Sneha Ballabh, Noah Schell
- ▶ **Course webpage:** <http://www.cs.utexas.edu/~isil/cs311h>
- ▶ Contains syllabus, important information about HW policy, slides from lectures etc.

## About this Course

- ▶ Give mathematical background you need for computer science
- ▶ **Topics:** Logic, proof techniques, number theory, combinatorics, graph theory, basic complexity theory . . .
- ▶ These will come up again and again in higher-level CS courses
  - ▶ Master CS311H material if you want to do well in future courses!

## Textbook



- ▶ Textbook (optional): Discrete Mathematics and Its Applications by Kenneth Rosen
- ▶ Textbook not a substitute for lectures:
  - ▶ Class presentation may not follow book
  - ▶ Skip many chapters and cover extra material

## Ed Discussion

- ▶ We will be using Ed Discussion for all course-related discussions
- ▶ Make sure you can access Ed Discussion! (link available through Canvas + webpage)
- ▶ Please post class-related questions on Ed Discussion instead of emailing instructor TA's
  - ▶ You will get answers quicker, and it will benefit the whole class
- ▶ Please use common sense when posting questions: Hints/ideas ok, but cannot post full solutions!!
- ▶ If you have a more personal question, please send private message (also through Ed Discussion)

## Discussion Sections and Office Hours

- ▶ Discussion sections on Friday 12-1pm and Fri 1-2 pm
- ▶ Please attend the section you were officially assigned to.
- ▶ Discussion section will answer questions, solve new problems, and go over previous homework
- ▶ Lots of office hours – times and location will be posted on Ed Discussion!

## Requirements

- ▶ Exams + problem sets + class attendance/participation
- ▶ Three exams scheduled for Sep 26, Oct 31, Dec 5 (in person, closed-book + closed-notes)
- ▶ Weekly problem sets

## Grading

- ▶ **Exam:** collectively 60% of final grade
- ▶ **Homework:** 35% of final grade
- ▶ **Attendance/participation:** 5% of final grade
- ▶ Final grades will be curved

## Homework Policy

- ▶ Homework must be submitted by **1 pm** on the due date
- ▶ Late submissions **not** allowed, lowest homework score dropped when calculating grades
- ▶ Homework must be done **on your own**, but allowed to ask conceptual (high-level) questions on Ed Discussion and during office hours
  - ▶ **Not allowed** to do problem sets in groups
  - ▶ **Not allowed** to check solutions with each other
  - ▶ Collaboration with other students on HW is considered cheating and will get you in very serious trouble

## Homework Policy, cont.

- ▶ You may not use AI Assistants like GPT in solving homework problems
- ▶ You may also not do online search for solutions to similar homework problems
- ▶ You may not discuss homework problems with each other through channels (e.g., Slack, Facebook, WhatsApp etc) that the course staff does not have access to.
- ▶ Homework solutions must be typeset using Latex and submitted through Gradescope

## Honor Code

- ▶ Failing to adhere to the homework policy is a violation of the UT **honor code**
- ▶ We take the honor code extremely seriously: people have failed the class in the past for violating the HW policy
- ▶ In addition to failing the class, your case will be sent to the Dean of Students and placed on your file
- ▶ Please don't risk ruining your career for a slightly better grade on a problem set or exam...

## More on Homework

- ▶ Problem sets in this class will be much harder than what you are used to from high school!!
  - ▶ Normal to spend >30mins on a single HW question
  - ▶ Do not seek help from us unless you've spent at least one hour on each problem
- ▶ Expect each problem set to take > 6 hours

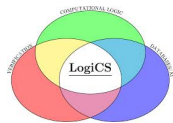
## Class Participation

- ▶ Everyone expected to attend lectures and participate
- ▶ 5% of course grade for participation (attendance, asking/answering questions, being active on Ed Discussion)
- ▶ Please ask questions!
  - ▶ Will make class more fun for everyone
  - ▶ Others also benefit from your questions

## Let's get started!

## Logic

- ▶ Logic: study of valid reasoning; fundamental to CS
- ▶ Allows us to represent knowledge in a formal/mathematical way and automate some types of reasoning
- ▶ **Many applications in CS:**  
AI, programming languages, databases, computer architecture, automated testing and program analysis, ...



## Propositional Logic

- ▶ Simplest logic is **propositional logic**
- ▶ Building blocks of propositional logic are **propositions**
- ▶ A **proposition** is a statement that is either true or false
- ▶ Examples:
  - ▶ "CS311 is a course in discrete mathematics": **True**
  - ▶ "Austin is located in California": **False**
  - ▶ "Pay attention": **Not a proposition**
  - ▶ " $x+1=2$ ": **Not a proposition**

## Propositional Variables, Truth Value

- ▶ **Truth value** of a proposition identifies whether a proposition is true (written **T**) or false (written **F**)
- ▶ What is truth value of "Today is Friday"?
- ▶ Variables that represent propositions are called **propositional variables**
- ▶ Denote propositional variables using lower-case letters, such as  $p, p_1, p_2, q, r, s, \dots$
- ▶ Truth value of a propositional variable is either T or F.

## Compound Propositions

- ▶ More complex propositions formed using **logical connectives** (also called **boolean connectives**)
- ▶ Three basic logical connectives:
  1.  $\wedge$ : **conjunction** (read "and"),
  2.  $\vee$ : **disjunction** (read "or")
  3.  $\neg$ : **negation** (read "not")
- ▶ Propositions formed using these logical connectives called **compound propositions**; otherwise **atomic propositions**
- ▶ A **propositional formula** is either an atomic or compound proposition

## Negation

- ▶ Negation of a proposition  $p$ , written  $\neg p$ , represents the statement "It is not the case that  $p$ ".
- ▶ If  $p$  is  $T$ ,  $\neg p$  is  $F$  and vice versa.
- ▶ In simple English, what is  $\neg p$  if  $p$  stands for ...
  - ▶ "Less than 80 students are enrolled in CS311"?

## Conjunction

- ▶ **Conjunction** of two propositions  $p$  and  $q$ , written  $p \wedge q$ , is the proposition " $p$  and  $q$ ".
- ▶  $p \wedge q$  is  $T$  if **both**  $p$  is true **and**  $q$  is true, and  $F$  otherwise.
- ▶ What is the conjunction and the truth value of  $p \wedge q$  for ...
  - ▶  $p$  = "It is Thursday",  $q$  = "It is morning"?

## Disjunction

- ▶ **Disjunction** of two propositions  $p$  and  $q$ , written  $p \vee q$ , is the proposition " $p$  or  $q$ ".
- ▶  $p \vee q$  is  $T$  if **either**  $p$  is true **or**  $q$  is true, and  $F$  otherwise.
- ▶ What is the disjunction and the truth value of  $p \vee q$  for ...
  - ▶  $p$  = "It is spring semester",  $q$  = "Today is Thursday"?

## Propositional Formulas and Truth Tables

- ▶ **Truth table** for propositional formula  $F$  shows truth value of  $F$  for every possible value of its constituent atomic propositions

- ▶ **Example:** Truth table for  $\neg p$

$p$	$\neg p$
T	F
F	T

- ▶ **Example:** Truth table for  $p \vee q$

$p$	$q$	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

## Constructing Truth Tables

Useful strategy for constructing truth tables for a formula  $F$ :

1. Identify  $F$ 's constituent atomic propositions
2. Identify  $F$ 's compound propositions in increasing order of complexity, including  $F$  itself
3. Construct a table enumerating all combinations of truth values for atomic propositions
4. Fill in values of compound propositions for each row

## Examples

Construct truth tables for the following formulas:

1.  $(p \vee q) \wedge \neg p$
2.  $(p \wedge q) \vee (\neg p \wedge \neg q)$
3.  $(p \vee q \vee \neg r) \wedge r$

## More Logical Connectives

- ▶  $\wedge, \vee, \neg$  most common boolean connectives, but there are other boolean connectives as well
- ▶ Other connectives: **exclusive or**  $\oplus$ , **implication**  $\rightarrow$ , **biconditional**  $\leftrightarrow$
- ▶ **Exclusive or**:  $p \oplus q$  is true when exactly one of  $p$  and  $q$  is true, and false otherwise

$p$	$q$	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

- ▶ Truth table:

## Implication (Conditional)

- ▶ An **implication** (or conditional)  $p \rightarrow q$  is read "if  $p$  then  $q$ " or " $p$  implies  $q$ "
- ▶ It is false if  $p$  is true and  $q$  is false, and true otherwise
- ▶ **Exercise**: Draw truth table for  $p \rightarrow q$
- ▶ In an implication  $p \rightarrow q$ ,  $p$  is called **antecedent** and  $q$  is called **consequent**

## Converting English into Logic

Let  $p$  = "I major in CS" and  $q$  = "I will find a good job". How do we translate following English sentences into logical formulas?

- ▶ "If I major in CS, then I will find a good job":  $p \rightarrow q$
- ▶ "I will not find a good job unless I major in CS":  $\neg q \rightarrow p$
- ▶ "It is sufficient for me to major in CS to find a good job":  $p \rightarrow q$
- ▶ "It is necessary for me to major in CS to find a good job":  $\neg q \rightarrow p$

## More English - Logic Conversions

Let  $p$  = "I major in CS",  $q$  = "I will find a good job",  $r$  = "I can program". How do we translate following English sentences into logical formulas?

- ▶ "I will not find a good job unless I major in CS or I can program":  $\neg q \rightarrow p \vee r$
- ▶ "I will not find a good job unless I major in CS and I can program":  $\neg q \rightarrow p \wedge r$
- ▶ "A prerequisite for finding a good job is that I can program":  $q \rightarrow r$
- ▶ "If I major in CS, then I will be able to program and I can find a good job":  $p \rightarrow (r \wedge q)$

## Converse of a Implication

- ▶ The **converse** of an implication  $p \rightarrow q$  is  $q \rightarrow p$ .
- ▶ What is the converse of "If I am a CS major, then I can program"?
- ▶ **Note**: It is possible for a implication to be true, but its converse to be false, e.g.,  $F \rightarrow T$  is true, but converse false

## Inverse of an Implication

- ▶ The **inverse** of an implication  $p \rightarrow q$  is  $\neg p \rightarrow \neg q$ .
- ▶ What is the inverse of "If I get an A in CS311, then I am smart"?
- ▶ **Note**: It is possible for a implication to be true, but its inverse to be false.  $F \rightarrow T$  is true, but inverse is false

## Contrapositive of Implication

- ▶ The **contrapositive** of an implication  $p \rightarrow q$  is  $\neg q \rightarrow \neg p$ .
- ▶ What is the contrapositive of "If I am a CS major, then I can program"?
- ▶ **Question:** Is it possible for an implication to be true, but its contrapositive to be false?

## Question

- ▶ Given  $p \rightarrow q$ , is it possible that its converse is true, but inverse is false?

## Biconditionals

- ▶ A **biconditional**  $p \leftrightarrow q$  is the proposition "p if and only if q".
- ▶ The biconditional  $p \leftrightarrow q$  is true if  $p$  and  $q$  have same truth value, and false otherwise.
- ▶ **Exercise:** Construct a truth table for  $p \leftrightarrow q$
- ▶ **Question:** How can we express  $p \leftrightarrow q$  using the other boolean connectives?

## Operator Precedence

- ▶ Given a formula  $p \wedge q \vee r$ , do we parse this as  $(p \wedge q) \vee r$  or  $p \wedge (q \vee r)$ ?
- ▶ Without settling on a convention, formulas without explicit paranthesization are ambiguous.
- ▶ To avoid ambiguity, we will specify **precedence** for logical connectives.

## Operator Precedence, cont.

- ▶ Negation ( $\neg$ ) has **higher precedence** than all other connectives.
- ▶ **Question:** Does  $\neg p \wedge q$  mean (i)  $\neg(p \wedge q)$  or (ii)  $(\neg p) \wedge q$ ?
- ▶ Conjunction ( $\wedge$ ) has next highest precedence.
- ▶ **Question:** Does  $p \wedge q \vee r$  mean (i)  $(p \wedge q) \vee r$  or (ii)  $p \wedge (q \vee r)$ ?
- ▶ Disjunction ( $\vee$ ) has third highest precedence.
- ▶ Next highest is precedence is  $\rightarrow$ , and lowest precedence is  $\leftrightarrow$

## Operator Precedence Example

- ▶ Which is the correct interpretation of the formula

$$p \vee q \wedge r \leftrightarrow q \rightarrow \neg r$$

- (A)  $((p \vee (q \wedge r)) \leftrightarrow q) \rightarrow (\neg r)$
- (B)  $((p \vee q) \wedge r) \leftrightarrow q \rightarrow (\neg r)$
- (C)  $(p \vee (q \wedge r)) \leftrightarrow (q \rightarrow (\neg r))$
- (D)  $(p \vee ((q \wedge r) \leftrightarrow q)) \rightarrow (\neg r)$