CS311H: Discrete Mathematics

Intro and Propositional Logic

Instructor: Işıl Dillig

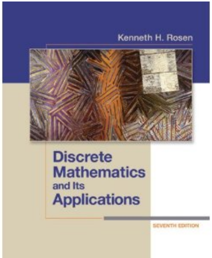
Course Staff

- Instructor: Prof. Işıl Dillig
- TAs: Angela Zhang, Noah Schell, Kush Sharma, Archit Patil
- Course webpage: http://www.cs.utexas.edu/~isil/cs311h
- Contains syllabus, slides from lectures etc.

About this Course

- Give mathematical background you need for computer science
- Topics: Logic, proof techniques, number theory, combinatorics, graph theory, basic complexity theory ...
- These will come up again and again in higher-level CS courses
  - Master CS311H material if you want to do well in future courses!

Textbook



- Textbook (optional): Discrete Mathematics and Its Applications by Kenneth Rosen
- Textbook not a substitute for lectures:
  - Class presentation may not follow book
  - Skip many chapters and cover extra material

Ed Discussion

- We will be using Ed Discussion for all course-related discussions
- Make sure you can access Ed Discussion! (link available through Canvas + webpage)
- Please post class-related questions on Ed Discussion instead of emailing instructor TA's
  - You will get answers quicker, and it will benefit the whole class
- If you have a more personal question, please send private message (also through Ed Discussion)

Discussion Sections and Office Hours

- Discussion sections on Friday 1-2:30 pm, 2-3:30pm
- Please attend the section you were officially assigned to.
- Discussion sections are required – you will be given weekly quizzes
- In addition, TAs will answer questions, solve new problems, and go over quizzes/exams
- Lots of office hours – times and location will be posted on Ed Discussion!

Requirements

- Exams + quizzes+ class attendance/participation
- Three exams scheduled for Sep 23, Oct 28, Dec 4 (in person, closed-book + closed-notes)
- Weekly quizzes during discussion section; lowest quiz grade will be dropped
- There will also be weekly problem sets but they will not be graded.

Grading

- Exam: collectively 50% of final grade
- Quizzes: 45% of final grade
- Attendance/participation: 5% of final grade
- Final grades will be curved

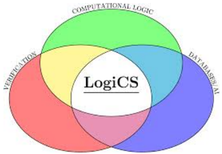
Class Participation

- Everyone expected to attend lectures and participate
- 5% of course grade for participation (attendance, asking/answering questions, being active on Ed Discussion)
- Please ask questions!
  - Will make class more fun for everyone
  - Others also benefit from your questions

Let's get started!

Logic

- Logic: study of valid reasoning; fundamental to CS
- Allows us to represent knowledge in a formal/mathematical way and automate some types of reasoning
- **Many applications in CS:** AI, programming languages, databases, computer architecture, automated testing and program analysis, ...



Propositional Logic

- Simplest logic is propositional logic
- Building blocks of propositional logic are propositions
- A proposition is a statement that is either true or false
- Examples:
  - "CS311 is a course in discrete mathematics": True
  - "Austin is located in California": False
  - "Pay attention": Not a proposition
  - "$x+1=2$": Not a proposition

Propositional Variables, Truth Value

- Truth value of a proposition identifies whether a proposition is true (written T) or false (written F)
- What is truth value of "Today is Friday"?
- Variables that represent propositions are called propositional variables
- Denote propositional variables using lower-case letters, such as $p, p_1, p_2, q, r, s, \ldots$
- Truth value of a propositional variable is either T or F.

Compound Propositions

- More complex propositions formed using logical connectives (also called boolean connectives)
- Three basic logical connectives:
  1. $\wedge$: conjunction (read "and"),
  2. $\vee$: disjunction (read "or")
  3. $\neg$: negation (read "not")
- Propositions formed using these logical connectives called compound propositions; otherwise atomic propositions
- A propositional formula is either an atomic or compound proposition

Negation

- Negation of a proposition p, written $\neg p$, represents the statement "It is not the case that p".
- If p is T, $\neg p$ is F and vice versa.
- In simple English, what is $\neg p$ if p stands for ...
  - "Less than 80 students are enrolled in CS311"?

Conjunction

- Conjunction of two propositions p and q, written $p \wedge q$, is the proposition "p and q"
- $p \wedge q$ is T if both p is true and q is true, and F otherwise.
- What is the conjunction and the truth value of $p \wedge q$ for ...
  - $p =$ "It is Thursday", $q =$ "It is morning" ?

Disjunction

- Disjunction of two propositions p and q, written $p \vee q$, is the proposition "p or q"
- $p \vee q$ is T if either p is true or q is true, and F otherwise.
- What is the disjunction and the truth value of $p \vee q$ for ...
  - $p =$ "It is spring semester", $q =$ "Today is Thursday"?

Propositional Formulas and Truth Tables

- Truth table for propositional formula F shows truth value of F for every possible value of its constituent atomic propositions

- Example: Truth table for $\neg p$

p	$\neg p$
T	F
F	T

- Example: Truth table for $p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Constructing Truth Tables

Useful strategy for constructing truth tables for a formula F:

1. Identify F's constituent atomic propositions
2. Identify F's compound propositions in increasing order of complexity, including F itself
3. Construct a table enumerating all combinations of truth values for atomic propositions
4. Fill in values of compound propositions for each row

Examples

Construct truth tables for the following formulas:

1. $(p \lor q) \land \neg p$
2. $(p \land q) \lor (\neg p \land \neg q)$
3. $(p \lor q \lor \neg r) \land r$

More Logical Connectives

- $\wedge, \vee, \neg$ most common boolean connectives, but there are other boolean connectives as well
- Other connectives: exclusive or $\oplus$, implication $\rightarrow$, biconditional $\leftrightarrow$
- Exclusive or: $p \oplus q$ is true when exactly one of p and q is true, and false otherwise
- Truth table:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Implication (Conditional)

- An implication (or conditional) $p \rightarrow q$ is read "if p then q" or "p implies q"
- It is false if p is true and q is false, and true otherwise
- Exercise: Draw truth table for $p \rightarrow q$
- In an implication $p \rightarrow q$, p is called antecedent and q is called consequent

Converting English into Logic

Let $p =$" I major in CS" and $q =$ "I will find a good job". How do we translate following English sentences into logical formulas?

- "If I major in CS, then I will find a good job":
- "I will not find a good job unless I major in CS":
- "It is sufficient for me to major in CS to find a good job":
- "It is necessary for me to major in CS to find a good job":

More English - Logic Conversions

Let $p =$" I major in CS", $q =$ "I will find a good job", $r =$ "I can program". How do we translate following English sentences into logical formulas?

- " I will not find a good job unless I major in CS or I can program":
- " I will not find a good job unless I major in CS and I can program":
- "A prerequisite for finding a good job is that I can program":
- "If I major in CS, then I will be able to program and I can find a good job":

Converse of a Implication

- The converse of an implication $p \to q$ is $q \to p$.
- What is the converse of "If I am a CS major, then I can program"?
- Note: It is possible for a implication to be true, but its converse to be false, e.g., $F \to T$ is true, but converse false

Inverse of an Implication

- The inverse of an implication $p \to q$ is $\neg p \to \neg q$.
- What is the inverse of "If I get an A in CS311, then I am smart"?
- Note: It is possible for a implication to be true, but its inverse to be false. $F \to T$ is true, but inverse is false

Contrapositive of Implication

- The contrapositive of an implication $p \to q$ is $\neg q \to \neg p$.
- What is the contrapositive of "If I am a CS major, then I can program"?
- Question: Is it possible for an implication to be true, but its contrapositive to be false?

Question

- Given $p \to q$, is it possible that its converse is true, but inverse is false?

Biconditionals

- A biconditional $p \leftrightarrow q$ is the proposition "p if and only if q".
- The biconditional $p \leftrightarrow q$ is true if p and q have same truth value, and false otherwise.
- Exercise: Construct a truth table for $p \leftrightarrow q$
- Question: How can we express $p \leftrightarrow q$ using the other boolean connectives?

Operator Precedence

- Given a formula $p \wedge q \vee r$, do we parse this as $(p \wedge q) \vee r$ or $p \wedge (q \vee r)$?
- Without settling on a convention, formulas without explicit paranthesization are ambiguous.
- To avoid ambiguity, we will specify precedence for logical connectives.

Operator Precedence, cont.

- Negation ($\neg$) has higher precedence than all other connectives.
- Question: Does $\neg p \wedge q$ mean (i) $\neg(p \wedge q)$ or (ii) $(\neg p) \wedge q$?
- Conjunction ($\wedge$) has next highest predence.
- Question: Does $p \wedge q \vee q$ mean (i) $(p \wedge q) \vee r$ or (ii) $p \wedge (q \vee r)$?
- Disjunction ($\vee$) has third highest precedence.
- Next highest is precedence is $\rightarrow$, and lowest precedence is $\leftrightarrow$

Operator Precedence Example

- Which is the correct interpretation of the formula

$$p \lor q \land r \leftrightarrow q \to \neg r$$

(A) $((p \lor (q \land r)) \leftrightarrow q) \to (\neg r)$

(B) $((p \lor q) \land r) \leftrightarrow q) \to (\neg r)$

(C) $(p \lor (q \land r)) \leftrightarrow (q \to (\neg r))$

(D) $(p \lor ((q \land r) \leftrightarrow q)) \to (\neg r)$