Problem Set 1
Due Thursday, January 31 at 3:30 pm

The answers to the homework assignment should be your own individual work. Please hand in a hard copy of your solutions in class on the due date.

1. (15 points) Decide whether each propositional logic formula below is valid or not. If the formula is valid, prove that it is valid using the semantic argument method. Otherwise, prove it is not valid by providing a falsifying interpretation and in addition state whether the formula is contingent or unsatisfiable.

   (a) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

   (b) $(p \rightarrow (q \rightarrow r)) \rightarrow (\neg r \rightarrow (\neg q \rightarrow \neg p))$

2. (10 points) Convert the following formula to equivalent formulas in NNF, CNF, and DNF:

$$\neg(\neg(p \land q) \rightarrow \neg r)$$

3. (10 points) Convert the same formula as in Problem 3 to an equi-satisfiable CNF formula using Tseitin's transformation.

4. (15 points) Let ϕ be a propositional formula in NNF and let I be an interpretation of ϕ. Let the positive set of I with respect to ϕ, denoted $pos(I, \phi)$, be the literals of ϕ that are satisfied by I. As an example, for the formula $\phi = (\neg x \land y) \lor z$ and the interpretation $I = [x \mapsto \bot, y \mapsto \top, z \mapsto \bot]$, we have $pos(I, \phi) = \{\neg x, y\}$. Prove the following theorem about the monotonicity of NNF:

   Monotonity of NNF: For every interpretation I and I' such that $pos(I, \phi) \subseteq pos(I', \phi)$, if $I \models \phi$, then $I' \models \phi$.

   (**Hint:** Use structural induction.)

5. (25 points) In this problem, we will prove the correctness of Tseitin's encoding. Let F be a propositional logic formula and let F' be a CNF formula obtained using Tseitin's transformation from formula F.

   (a) Let I be an interpretation of formula F, and let the Tseitin extension $TS(I)$ be an interpretation that is the same as I except that it also assigns values to the new variables in F'. Specifically, if variable p_G in F' represents subformula G of F, then I' assigns to p_G whatever G evaluates to under I. Prove (using induction) that, for any interpretation I, an extension I' of I is a satisfying interpretation of

$$\bigwedge_{(G = G_1 \circ G_2) \in S_F} p_G \leftrightarrow (p_{G_1} \circ p_{G_2})$$

   iff $I' = TS(I)$.

   (b) Now, use the lemma from part (a) to prove the equisatisfiability of F and F' (its Tseitin encoding).