

## Homework Assignment 5

### Due Tuesday, March 26, 2019

1. (10 points) For each of the following claims, state whether it is true or false. Give a clear and concise explanation to justify your answer.
  - (a) Consider a first order-theory  $T$  with signature  $\Sigma$  and axioms  $A$ . Let  $F_A$  denote the conjunction of every axiom in  $A$ , and let  $\phi$  be a formula over  $\Sigma$ . If  $\phi$  is satisfiable modulo  $T$ , then  $F_A \rightarrow \phi$  is also satisfiable in standard first-order logic.
  - (b) Consider a first order-theory  $T$  with signature  $\Sigma$  and axioms  $A$ . Let  $F_A$  denote the conjunction of every axiom in  $A$ , and let  $\phi$  be a formula over  $\Sigma$ . If  $F_A \rightarrow \phi$  is satisfiable in standard first-order logic, then  $\phi$  is satisfiable modulo  $T$ .
2. (15 points) Consider two theories  $T_1$  and  $T_2$  such that  $\Sigma_1 = \Sigma_2 = \Sigma$  and  $A_1 \supseteq A_2$ . Also, let  $F$  be a formula over signature  $\Sigma$ .
  - (a) If  $F$  is satisfiable modulo  $T_2$ , is it also satisfiable modulo  $T_1$ ? Prove your answer or give a counterexample.
  - (b) If  $F$  is valid modulo  $T_2$ , is it also valid modulo  $T_1$ ? Prove your answer or give a counterexample.
  - (c) If  $T_2$  is complete, what, if anything, can we say about the completeness of  $T_1$ ? Justify your answer.
3. (10 points) Consider the formula  $\forall x.(2 \cdot x - 3 > y)$  in the theory of integers  $T_{\mathbb{Z}}$ . Give a Presburger arithmetic encoding of this formula.
4. (10 points) Decide whether the formula below is valid. If it is valid or unsatisfiable, use the semantic argument method to prove your claim. If the formula is satisfiable but not valid, provide both a falsifying and satisfying structure.

$$a\langle i \triangleleft e \rangle[j] = e \rightarrow i = j \vee a[j] = e$$

5. (10 points) Apply the congruence closure algorithm to decide the satisfiability of the following  $T_{=}$  formula:

$$f(g(x)) = g(f(x)) \wedge f(g(f(y))) = x \wedge f(y) = x \wedge g(f(x)) \neq x$$

Show all the equalities processed by the algorithm and the DAG data structure (with representatives) before and after processing each equality.