CS389L: Problem Set 5

1. Consider an arbitrary CNF formula ϕ in propositional logic.

   (a) Is it possible to convert ϕ to a conjunctive $T_{\mathbb{Q}}$ formula ψ such that ϕ and ψ are equisatisfiable? If so, describe a technique for doing this; if not, explain why this is not possible. Note that ψ is not allowed to contain disjunctions.

   (b) Is it possible to convert χ to a conjunctive $T_{\mathbb{Z}}$ formula ψ such that ϕ and χ are equisatisfiable? If so, describe a technique for doing this; if not, explain why this is not possible. Note that ψ is not allowed to contain disjunctions.

   (c) If you answered "yes" to either part (a) or (b), demonstrate how your conversion works on the following formula:

   $$(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge \neg p$$

   How can you use the translated formula to determine if the original formula is satisfiable?

2. Consider the following inequality system over integers:

   $$\begin{array}{rcc} 4y & \leq & 2x \\ 2y & \leq & -x+3 \\ 4y & \geq & 1 \end{array}$$

   Use the Omega test to determine whether this system is satisfiable or not.

3. Recall that the Nelson-Oppen method requires the theories to be combined to be *stably infinite*. Give an example to demonstrate why this restriction is necessary. Specifically, give two theories T_1, T_2, and a $(T_1 \cup T_2)$- formula F such that F is unsatisfiable but we would conclude otherwise using the Nelson-Oppen method.

4. Consider the following formula F in $T_{=} \cup T_{\mathbb{Z}}$:

   $$g(f(x-2)) = x+2 \wedge g(f(y)) = y-2 \wedge y = x-2$$

   (a) Purify F by writing it as an equisatisfiable formula of the form $F_1 \wedge F_2$ such that F_1 is in $T_{=}$ and F_2 is in $T_{\mathbb{Z}}$.

   (b) Decide the satisfiability of F using the Nelson-Oppen method.

5. Recall that the DPLL(T) framework invokes the solver for theory T to learn conflict clauses and theory propagation lemmas. In this question, we will explore the inference of theory propagation lemmas.

   (a) Let D_T be a decision procedure for a conjunction of Σ_T literals. Describe how one can perform exhaustive theory propagation for T without modifying the decision procedure D_T. (Note: *exhaustive* means that we want to infer all theory propagation lemmas.)

   (b) Suppose T is the theory of equality with interpreted functions. Explain how we can infer theory propagation lemmas for this theory after running the congruence closure algorithm on the current partial assignment.