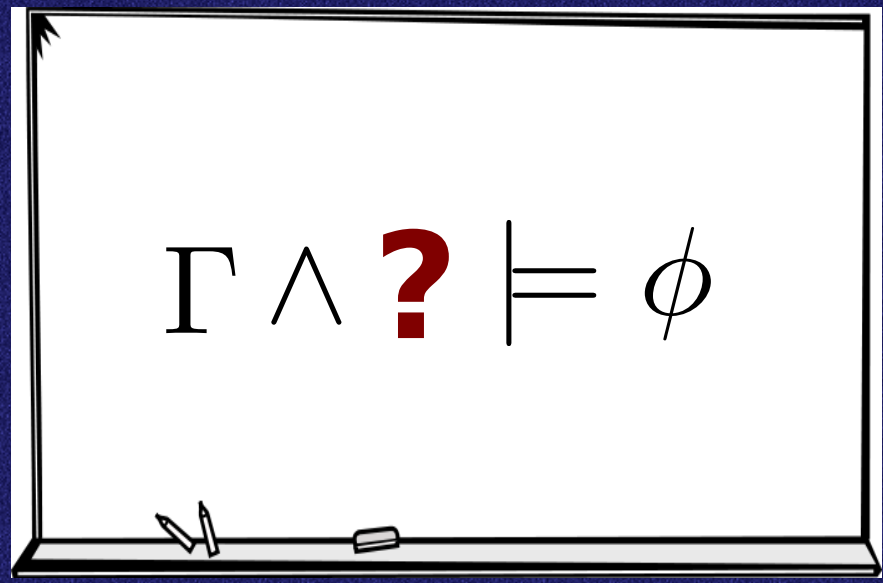
Logical Abduction and its Applications in Program Verification



Isil Dillig
MSR Cambridge

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- Deduction: Infers valid conclusion from premises
- Abduction: Infers missing premise to explain a given conclusion
- Given known facts Γ and desired outcome ϕ, abductive inference finds "simple" explanatory hypothesis ψ such that

$$\Gamma \wedge \psi \models \phi \quad \text{and} \quad \mathrm{SAT}(\Gamma \wedge \psi)$$

Simple Example

- Facts: “If it rains, then it is wet and cloudy”, “If it is wet, then it is slippery”: $R \Rightarrow W \wedge C \ \wedge W \Rightarrow S$

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- Abductive explanation: R, i.e., “It is rainy”

Arithmetic Example

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int x = 0;
int y = 0;

while(x < n)
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   x = x+1;
   y = y+2;
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assert( x + y >= 3*n);
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- Abductive explanation: $y \geq 2x$
  - corresponds to missing loop invariant

Abduction vs. Interpolation

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- Abduction + Interpolation: Yin-and-Yang of logical inference

Road Map

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3. Applications of abduction in program analysis/verification
4. Thoughts on abduction vs. interpolation in verification

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- So, what kind of solutions do want to compute?

Which Abductive Explanations Are Good?

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- This simplicity criterion makes sense in verification because we want proof subgoals to be local and refer to few variables

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- First talk about how to compute solutions with fewest variables
- Then talk about how to obtain most general solution containing these variables

Minimum Satisfying Assignments

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Minimum satisfying assignment (MSA):

- ✓ assigns values to a subset of variables in formula
- ✓ sufficient to make formula true
- ✓ Among all other partial satisfying assignments, contains fewest variables

Example

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- Note: Algorithm for computing MSAs given in our CAV'12 paper, "Minimum Satisfying Assignments for SMT" by Dillig & McMillan

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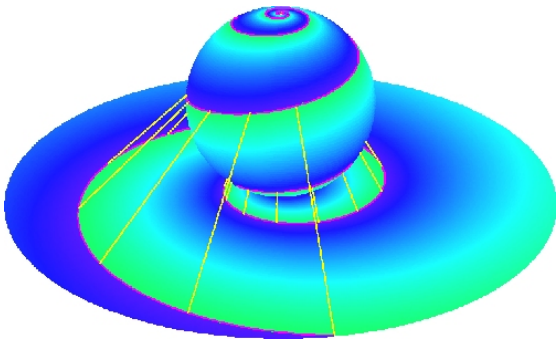
- Furthermore, it makes assumptions about as few variables as possible
- But it is not the most general solution

Finding Most General Solutions



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Quantifier elimination

- To find most general solution containing variables in the MSA, universally quantify all other variables $\overline{V}$ and apply quantifier elimination to $\forall \overline{V}.\ \Gamma \Rightarrow \phi$

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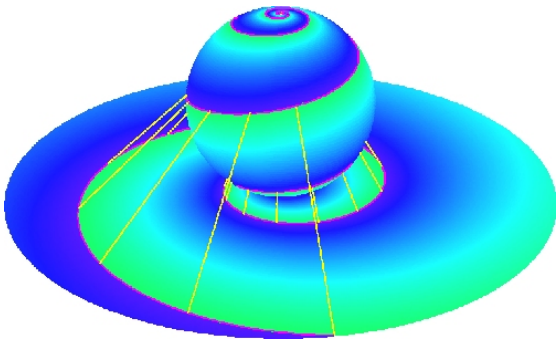

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- The resulting formula captures all satisfying assignments containing only MSA variables

Abduction Algorithm

- abduce yields formula E such that

$$I \wedge E \models \phi$$

and E is consistent with I

$\text{abduce}(I, \phi)$ {

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  - uses algorithm from SAS'10 paper "Small Formulas for Large Programs: ..."

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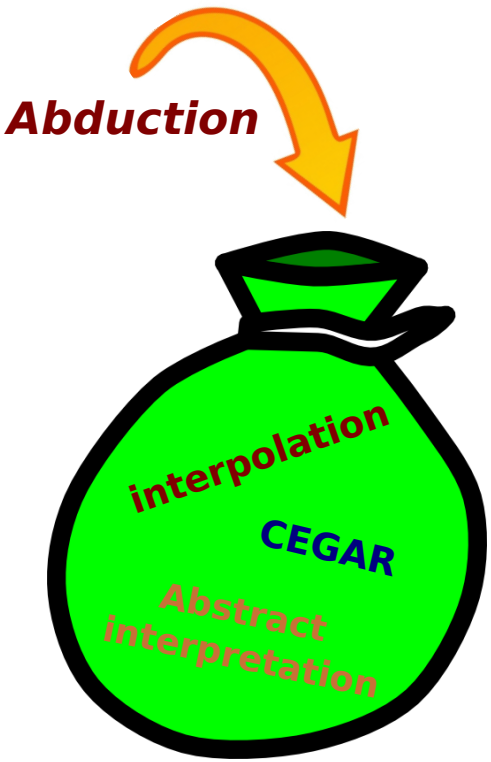
Abduction in Program Analysis

Useful technique to add to our bag of tricks; lots of applications!



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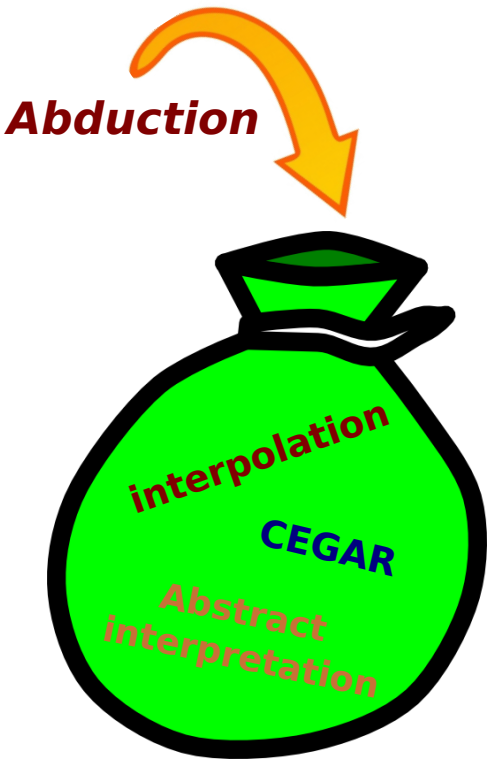

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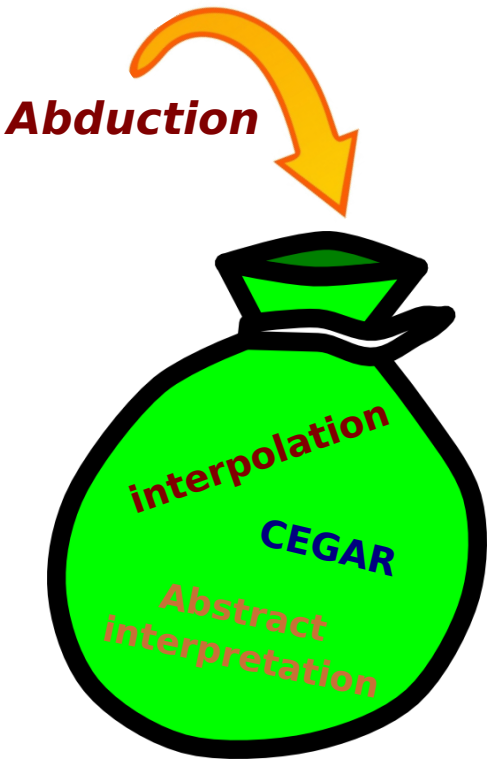

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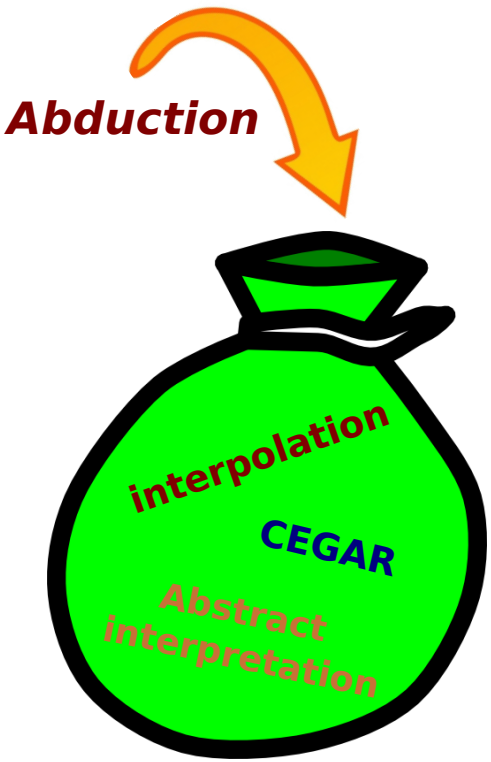

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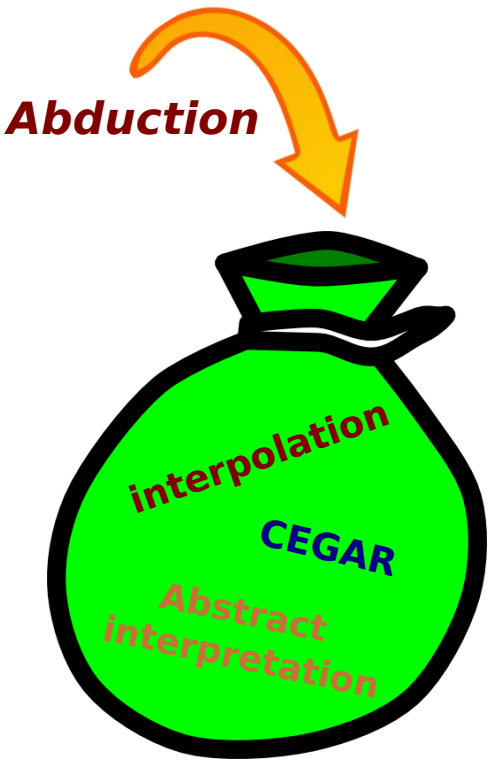

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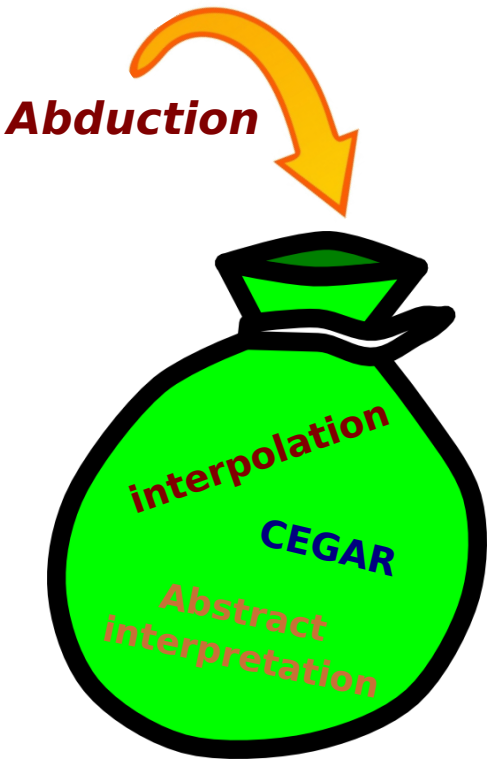

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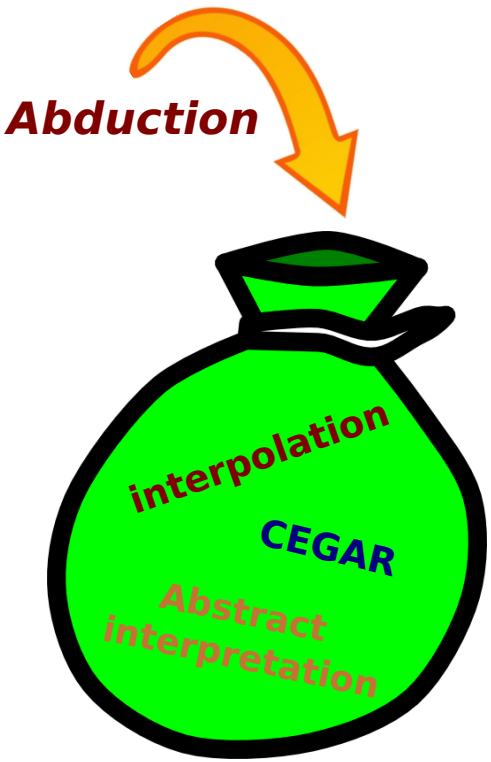

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- Otherwise, strengten LHS using abduction

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- In either case, strengthen invariant to $I \wedge \psi$ and try to prove correctness

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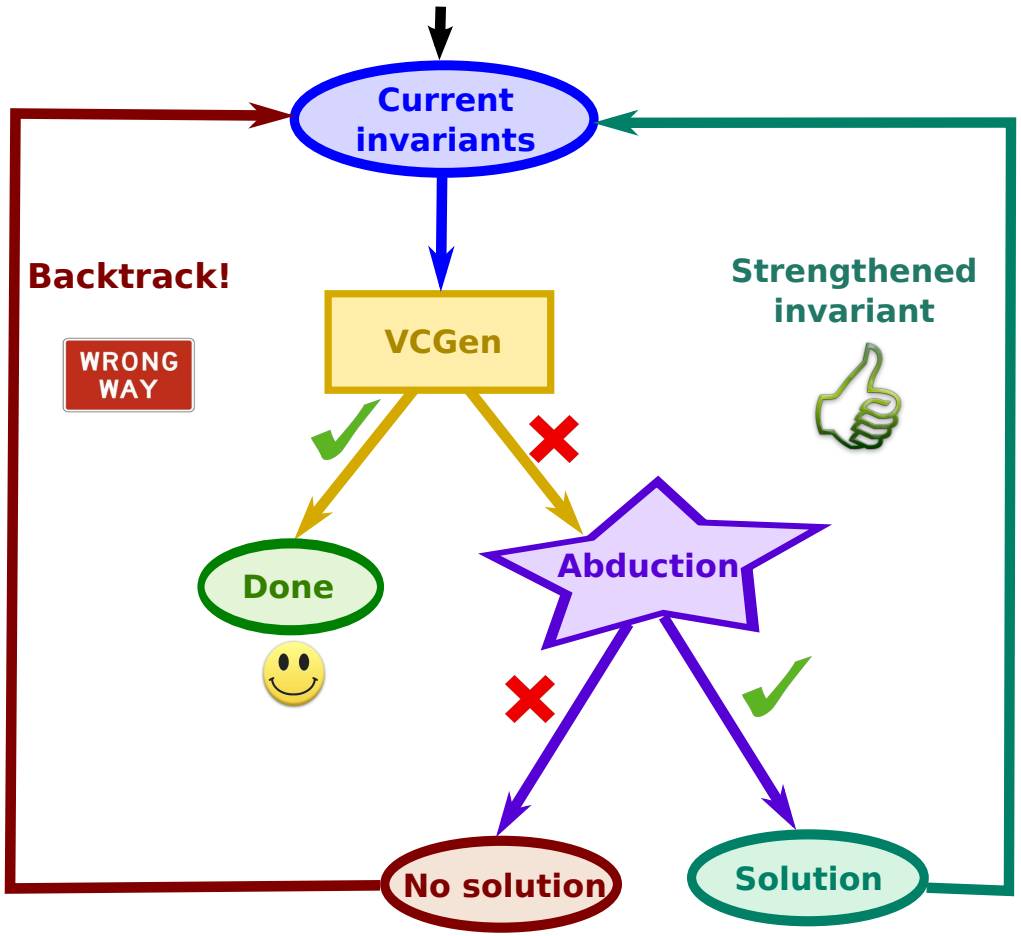

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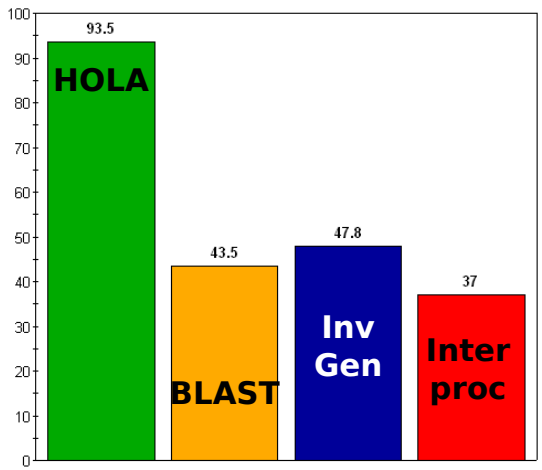

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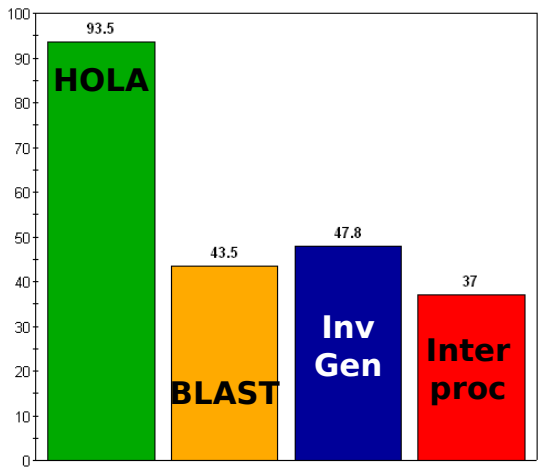

- But not strictly better: cannot prove two benchmarks at least one tool can show

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- Dual concepts, so their combination could be synergistic
  - combine model checking with Hoare-style reasoning?

Questions?