Constraint-Based Analysis in the Presence of Uncertainty and Imprecision

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Motivation

- When we reason about programs statically, uncertainty and imprecision come up everywhere.

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- When we reason about programs statically, uncertainty and imprecision come up everywhere.
    - Uncertainty: We often do not (or cannot) model every aspect of the environment the program executes in
    - Imprecision: Any analysis is necessarily based on some abstraction of the program

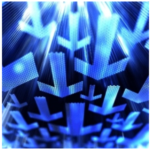
Uncertainty

- User Input

```
if(getUserInput() == 'y') return true;
else return false;
```

Uncertainty

- User Input
- Network data



```
char buf[1024];
recv(socket,buf,1024,0);
struct data* d = (struct data*) buf;
```

Uncertainty

- User Input
- Network data
- System state

```
int* p = malloc(sizeof(int)*num_elems);
if(p == NULL) exit(1);
```

Uncertainty

- User Input
- Network data
- System state
- Many more
  - e.g., calling an unknown function, thread scheduling, . . .

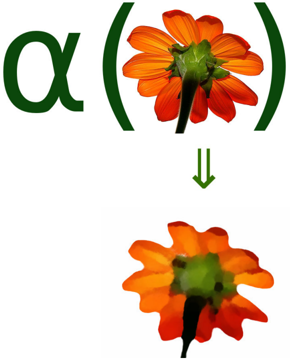
Uncertainty

- User Input
- Network data
- System state
- Many more...

All of these appear as non-deterministic environment choices

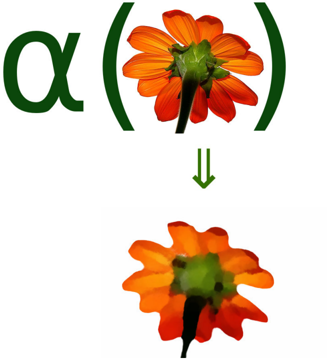
Imprecision

- In constrast to uncertainty, imprecision arises from the abstraction intentionally chosen by the analysis designer



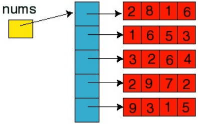
Imprecision

- In constrast to uncertainty, imprecision arises from the abstraction intentionally chosen by the analysis designer
- But imprecision results in similar consequences as uncertainty



Imprecision

- Many program analyis systems do not reason about unbounded data structures or abstract data types




```
int elem = array[i];
assert(elem != -1);
```

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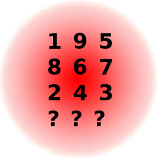

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int elem =  ;
assert(elem != -1);
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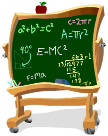
Imprecision

- Many program analyis systems do not reason about unbounded data structures or abstract data types
- Many systems do not track "complicated" arithmetic




```
if(COEF*a*b+MIN_SIZE >= MAX)
     return -1;
```

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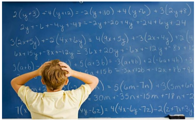
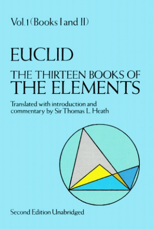


`if(`  `>= MAX)`
`    return -1;`

Imprecision

- Many program analyis systems do not reason about unbounded data structures or abstract data types
- Many systems do not track “complicated” arithmetic
- Many systems cannot infer complicated loop invariants




```
int compute_gcd(int a, int b) {
    while(b!=0) {
     int t = a;
     a = b;
     b = t % b;
   }
   return a;
}
assert(x%compute_gcd(x,y) == 0);
```

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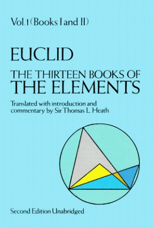


```
int compute_gcd(int a, int b) {
```

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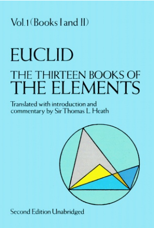


```
int compute_gcd(int a, int b)
{
```

```
}
```

`assert(x%`  `== 0);`

Imprecision

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Sources of imprecision appear as non-deterministic environment choices

Modeling Choice in Constraint Based Analyses

- In constraint-based systems, environment choice is typically modeled using unconstrained variables that we call choice variables.

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Modeling Choice in Constraint Based Analyses

- Unfortunately, the use of choice variables may introduce two problems:
  - Theoretical: It is not clear how to solve recursive constraints containing choice variables.
  - Practical: The number of choice variables is proportional to the size of the analyzed program.

Large formulas $\Rightarrow$ Poor scalability

Recursive Constraint Example

```
bool queryUser(bool featureEnabled) {
   if(!featureEnabled) return false;
   char userInput = getUserInput();
   if(userInput == 'y') return true;
   if(userInput=='n') return false;
   printf("Input must be y or n!  Please try again");
   return queryUser(featureEnabled);
}
```

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```

When does queryUser return true?

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   return queryUser(featureEnabled);
}
```

Given an arbitrary argument α, what is the constraint $\Pi_{\alpha,\text{true}}$ under which queryUser returns true?

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$$\boxed{\Pi_{\alpha,\mathsf{true}} = ?}$$

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$$\Pi_{\alpha,\mathsf{true}} = (\alpha = \mathsf{true}) \wedge ?$$

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$$\Pi_{\alpha,\mathsf{true}} = ((\alpha = \mathsf{true}) \wedge (\beta = \text{'y'} \vee ?))$$

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}
```

$$\Pi_{\alpha,\mathsf{true}} = ((\alpha = \mathsf{true}) \wedge (\beta = \text{'y'} \vee (\beta \neq \text{'n'} \wedge \Pi_{\alpha,\mathsf{true}}[\mathsf{true}/\alpha][\beta'/\beta])))$$

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Recursive Constraint Example

- If we solve this constraint naively using standard fix-point computation, we get:

$$\begin{aligned}\Pi_{\alpha,\mathtt{true}} = \quad & (\alpha = \mathtt{true}) \wedge (\beta = \mathtt{'y'} \vee (\neg(\beta = \mathtt{'n'}) \wedge \\ & \Pi_{\alpha,\mathsf{true}}[\mathsf{true}/\alpha][\beta'/\beta]))\end{aligned}$$

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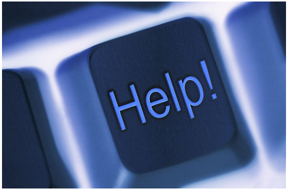
- It is not clear how to solve recursive constraints involving choice variables.

Scalability

Even if we had a way of solving such recursive constraints, choice variables remain a source of scalability problems, even for reasonably sized programs.



Scalability Problem Example

```
Key * key_new_private(int type) {
  Key *k = key_new(type);
  switch (type) {
    case KEY_RSA1:
    case KEY_RSA:
      if ((k->rsa->d = BN_new()) == NULL) fatal("BN_new failed");
      if ((k->rsa->iqmp = BN_new()) == NULL) fatal("BN_new failed");
      if ((k->rsa->q = BN_new()) == NULL) fatal("BN_new failed");
      if ((k->rsa->p = BN_new()) == NULL) fatal("BN_new failed");
      if ((k->rsa->dmq1 = BN_new()) == NULL) fatal("BN_new failed");
      if ((k->rsa->dmp1 = BN_new()) == NULL) fatal("BN_new failed");
      break;
    case KEY_DSA:
      if ((k->dsa->priv_key = BN_new()) == NULL) fatal("BN_new failed");
    default:
      break;  }
  return k;  }
```

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      break;
    case KEY_DSA:
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    default:
      break;  }
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```

Assume KEY_RSA1, KEY_RSA, and KEY_DSA are #define'd as 1, 2 and 3 respectively.

Scalability Problem Example

```
Key * key_new_private(int type) {
  Key *k = key_new(type);
  switch (type) {
    case  1:
    case  2:
      if ((k->rsa->d = BN_new()) == NULL) fatal("BN_new failed");
      if ((k->rsa->iqmp = BN_new()) == NULL) fatal("BN_new failed");
      if ((k->rsa->q = BN_new()) == NULL) fatal("BN_new failed");
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      if ((k->rsa->dmp1 = BN_new()) == NULL) fatal("BN_new failed");
      break;
    case  3:
      if ((k->dsa->priv_key = BN_new()) == NULL) fatal("BN_new failed");
    default:
      break;  }
  return k;  }
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Scalability Problem Example

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Key * key_new_private(int type) {
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      break;
    case  3:
      if ((k->dsa->priv_key = BN_new()) == NULL) fatal("BN_new failed");
    default:
      break;  }
  return k;  }
```

What is the constraint under which `key_new_private` successfully returns a new key?

Scalability Problem Example

- Denoting the argument of `key_new_private` by α, let us slice the relevant part of the function:

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- Denoting the argument of key_new_private by α, let us slice the relevant part of the function:

```
key_new_private(α) {
  if (α == 1 || α == 2 ) {
      if (BN_new() == NULL) /* fail */
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  }
  if (α == 3)
      if (BN_new()) == NULL) /* fail */
  /* success */
}
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  }
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```

- Here, `BN_NEW` is a `malloc` wrapper; hence, its return value should be treated as non-deterministic environment choice

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  /* success */
}
```

- We replace each call to `BN_NEW` with a fresh choice variable β_i.

Scalability Problem Example

```
key_new_private(α) {
  if (α == 1 || α == 2 ) {
    if (β1 == 0 || β2 == 0 || β3 == 0 || β4 == 0 || β5 == 0)
      /* fail */
  }
  if (α == 3)
      if (β6 == 0) /* fail */
  /* success */
}
```

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key_new_private(α) {
  if (α == 1 || α == 2 ) {
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  /* success */
}
```

- The condition under which the function succeeds is:

$$\boxed{\begin{array}{c}(1 \leq \alpha \leq 2 \wedge (\beta_1 \neq 0 \wedge \beta_2 \neq 0 \wedge \beta_3 \neq 0 \wedge \beta_4 \neq 0 \wedge \beta_5 \neq 0)\\ \vee(\alpha = 3 \wedge \beta_6 \neq 0) \vee \alpha \leq 0 \vee \alpha \geq 4)\end{array}}$$

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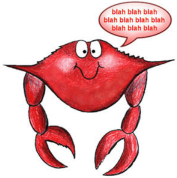

- Very verbose way of stating the success condition!

Scalability Problem Example

- Now consider some calling context of this function:

```
Key* rsa1_key = key_new_private(KEY_RSA1);
Key* rsa_key = key_new_private(KEY_RSA);
Key* dsa_key = key_new_private(KEY_DSA);
/* SUCCESS */
```

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/* SUCCESS */
```

- What is the constraint under which we reach `/*SUCCESS*/`?

$$
\begin{gathered}
(1 \leq 1 \leq 2 \wedge (\beta_1 \neq 0 \wedge \beta_2 \neq 0 \wedge \beta_3 \neq 0 \wedge \beta_4 \neq 0 \wedge \beta_5 \neq 0) \\
\vee (1 = 3 \wedge \beta_6 \neq 0) \vee 1 \leq 0 \vee 1 \geq 4) \quad \wedge \\
(1 \leq 2 \leq 2 \wedge (\beta'_1 \neq 0 \wedge \beta'_2 \neq 0 \wedge \beta'_3 \neq 0 \wedge \beta'_4 \neq 0 \wedge \beta'_5 \neq 0) \\
\vee (2 = 3 \wedge \beta'_6 \neq 0) \vee 2 \leq 0 \vee 2 \geq 4) \quad \wedge \\
(1 \leq 3 \leq 2 \wedge (\beta''_1 \neq 0 \wedge \beta''_2 \neq 0 \wedge \beta''_3 \neq 0 \wedge \beta''_4 \neq 0 \wedge \beta''_5 \neq 0) \\
\vee (3 = 3 \wedge \beta''_6 \neq 0) \vee 3 \leq 0 \vee 3 \geq 4)
\end{gathered}
$$

Scalability Problem Example

- Now consider some calling context of this function:

```
Key* rsa1_key = key_new_private(KEY_RSA1);
Key* rsa_key = key_new_private(KEY_RSA);
Key* dsa_key = key_new_private(KEY_DSA);
/* SUCCESS */
```

- What is the constraint under which we reach `/*SUCCESS*/`?

$$
\begin{array}{c}
(1 \leq 1 \leq 2 \wedge (\beta_1 \neq 0 \wedge \beta_2 \neq 0 \wedge \beta_3 \neq 0 \wedge \beta_4 \neq 0 \wedge \beta_5 \neq 0) \\
\vee (1 = 3 \wedge \beta_6 \neq 0) \vee 1 \leq 0 \vee 1 \geq 4) \quad \wedge \\
(1 \leq 2 \leq 2 \wedge (\beta'_1 \neq 0 \wedge \beta'_2 \neq 0 \wedge \beta'_3 \neq 0 \wedge \beta'_4 \neq 0 \wedge \beta'_5 \neq 0) \\
\vee (2 = 3 \wedge \beta'_6 \neq 0) \vee 2 \leq 0 \vee 2 \geq 4) \quad \wedge \\
(1 \leq 3 \leq 2 \wedge (\beta''_1 \neq 0 \wedge \beta''_2 \neq 0 \wedge \beta''_3 \neq 0 \wedge \beta''_4 \neq 0 \wedge \beta''_5 \neq 0) \\
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\end{array}
$$

Conclusion from the Examples

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- Introducing choice variables causes problems both with scalability and solving recursive constraints
- It is desirable to eliminate these choice variables from the constraints
- Idea: Compute an over-approximation of the constraint not containing any choice variables

Strongest Necessary Conditions

- An over-approximation $\lceil \phi \rceil$ of a constraint ϕ not containing choice variables is implied by the original constraint, i.e. $\lceil \phi \rceil$ is a necessary condition.

$$\phi \Rightarrow \lceil \phi \rceil$$

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$$\forall \phi'.((\phi \Rightarrow \phi') \Rightarrow (\lceil \phi \rceil \Rightarrow \phi'))$$

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$$\phi \Rightarrow \lceil\phi\rceil$$

- But rather than computing any necessary condition, we want to compute the strongest necessary condition:

$$\forall\phi'.((\phi \Rightarrow \phi') \Rightarrow (\lceil\phi\rceil \Rightarrow \phi'))$$

- Because strongest necessary condition $\lceil\phi\rceil$ preserves the satisfiability of ϕ:

$$\mathrm{SAT}(\phi) \Leftrightarrow \mathrm{SAT}(\lceil\phi\rceil)$$

SNC Example 1

- Consider the constraint from `key_new_private`:

$$
\boxed{\begin{array}{c}(1 \leq \alpha \leq 2 \wedge (\beta_1 \neq 0 \wedge \beta_2 \neq 0 \wedge \beta_3 \neq 0 \wedge \beta_4 \neq 0 \wedge \beta_5 \neq 0) \\ \vee(\alpha = 3 \wedge \beta_6 \neq 0) \vee \alpha \leq 0 \vee \alpha \geq 4)\end{array}}
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- The strongest necessary condition for this formula is just true.

SNC Example 1

```
Key * key_new_private(int type) {
  Key *k = key_new(type);
  switch (type) {
    case KEY_RSA1:
    case KEY_RSA:
      if ((k->rsa->d = BN_new()) == NULL) fatal("BN_new failed");
      if ((k->rsa->iqmp = BN_new()) == NULL) fatal("BN_new failed");
      if ((k->rsa->q = BN_new()) == NULL) fatal("BN_new failed");
      if ((k->rsa->p = BN_new()) == NULL) fatal("BN_new failed");
      if ((k->rsa->dmq1 = BN_new()) == NULL) fatal("BN_new failed");
      if ((k->rsa->dmp1 = BN_new()) == NULL) fatal("BN_new failed");
      break;
    case KEY_DSA:
      if ((k->dsa->priv_key = BN_new()) == NULL) fatal("BN_new failed");
    default:
      break;  }
  return k;  }
```

- key_new_private MAY successfully return a valid key no matter what the type of the requested cryptographic key is.

SNC Example 2

- Consider the constraint from the `queryUser` function:

$$\Pi_{\alpha,\mathsf{true}} \;=\; ((\alpha = \mathsf{true}) \;\wedge\; (\beta = \text{'y'} \;\vee\; (\beta \neq \text{'n'} \wedge \Pi_{\alpha,\mathsf{true}}[\mathsf{true}/\alpha][\beta'/\beta])))$$

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- The strongest necessary condition for $\Pi_{\alpha,\mathrm{true}}$ is $\alpha = \mathrm{true}$.

SNC Example 2

```
bool queryUser(bool featureEnabled) {
   if(!featureEnabled) return false;
   char userInput = getUserInput();
   if(userInput == 'y') return true;
   if(userInput=='n') return false;
   printf("Input must be y or n!  Please try again");
   return queryUser(featureEnabled);
}
```

- If `feature_enabled` is true in the calling context, queryUser MAY return true
- If `feature_enabled` is false, queryUser will not return true.

Weakest Sufficient Conditions

- Assuming we have a way of computing the strongest necessary condition in a given theory, are we done?

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Weakest Sufficient Conditions

- Assuming we have a way of computing the strongest necessary condition in a given theory, are we done?
- Unfortunately, if we only compute strongest necessary conditions, we can no longer safely negate our constraints...

$$\lceil \neg \phi \rceil \not\Leftrightarrow \neg \lceil \phi \rceil$$

- Therefore, we need a dual notion of strongest necessary conditions, i.e. weakest sufficient conditions.

Weakest Sufficient Conditions

- The weakest sufficient condition $\lfloor \phi \rfloor$ of formula ϕ not containing any choice variables satisfies:

$$\begin{array}{ll} (1) & \lfloor \phi \rfloor \Rightarrow \phi \\ (2) & \forall \phi'.((\phi' \Rightarrow \phi) \Rightarrow (\phi' \Rightarrow \lfloor \phi \rfloor)) \end{array}$$

Weakest Sufficient Conditions

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- Just as strongest necessary conditions preserve satisfiability, weakest sufficient conditions preserve validity:

$$\mathrm{VALID}(\phi) \Leftrightarrow \mathrm{VALID}(\lfloor \phi \rfloor)$$

WSC Example 1

- Consider the constraint from key_new_private:

$$\begin{array}{c}(1 \leq \alpha \leq 2 \wedge (\beta_1 \neq 0 \wedge \beta_2 \neq 0 \wedge \beta_3 \neq 0 \wedge \beta_4 \neq 0 \wedge \beta_5 \neq 0) \\ \vee(\alpha = 3 \wedge \beta_6 \neq 0) \vee \alpha \leq 0 \vee \alpha \geq 4)\end{array}$$

WSC Example 1

- Consider the constraint from `key_new_private`:

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- The weakest sufficient condition for this formula is $\alpha \leq 0 \vee \alpha \geq 4$.

WSC Example 1

```
Key * key_new_private(int type) {
  Key *k = key_new(type);
  switch (type) {
    case KEY_RSA1:
    case KEY_RSA:
      if ((k->rsa->d = BN_new()) == NULL) fatal("BN_new failed");
      if ((k->rsa->iqmp = BN_new()) == NULL) fatal("BN_new failed");
      if ((k->rsa->q = BN_new()) == NULL) fatal("BN_new failed");
      if ((k->rsa->p = BN_new()) == NULL) fatal("BN_new failed");
      if ((k->rsa->dmq1 = BN_new()) == NULL) fatal("BN_new failed");
      if ((k->rsa->dmp1 = BN_new()) == NULL) fatal("BN_new failed");
      break;
    case KEY_DSA:
      if ((k->dsa->priv_key = BN_new()) == NULL) fatal("BN_new failed");
    default:
      break;  }
  return k;  }
```

- key_new_private MUST successfully return a valid key if the type of the requested cryptographic key is neither KEY_RSA1, nor KEY_RSA, nor KEY_DSA

WSC Example 2

- Consider the constraint from the `queryUser` function:

$$\Pi_{\alpha,\mathsf{true}} = ((\alpha = \mathsf{true}) \land (\beta = \text{'y'} \lor (\beta \neq \text{'n'} \land \Pi_{\alpha,\mathsf{true}}[\mathsf{true}/\alpha][\beta'/\beta])))$$

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- Consider the constraint from the `queryUser` function:

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- The weakest sufficient condition for this formula is false.

WSC Example 2

```
bool queryUser(bool featureEnabled) {
   if(!featureEnabled) return false;
   char userInput = getUserInput();
   if(userInput == 'y') return true;
   if(userInput=='n') return false;
   printf("Input must be y or n!  Please try again");
   return queryUser(featureEnabled);
}
```

- No condition on `feature_enabled` is sufficient to guarantee `queryUser` will return true.
- Hence, the weakest sufficient condition is false.

Negation Revisited

- By having pairs of necessary and sufficient conditions, $(\lceil \phi \rceil, \lfloor \phi \rfloor)$, we can now make negation work:

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- By having pairs of necessary and sufficient conditions, $(\lceil \phi \rceil, \lfloor \phi \rfloor)$, we can now make negation work:

$$\neg(\lceil \phi \rceil, \lfloor \phi \rfloor) = (\neg \lfloor \phi \rfloor, \neg \lceil \phi \rceil)$$

- The strongest necessary condition for $\neg\phi$ is given by the negation of its weakest sufficient condition, $\neg\lfloor \phi \rfloor$.

- Similarly, the weakest sufficient condition for $\neg\phi$ is given by the negation of ϕ's strongest necessary condition, $\neg\lceil \phi \rceil$.

Negation Example

- Consider once more the constraint:

$$\boxed{\begin{array}{c}(1 \leq \alpha \leq 2 \wedge (\beta_1 \neq 0 \wedge \beta_2 \neq 0 \wedge \beta_3 \neq 0 \wedge \beta_4 \neq 0 \wedge \beta_5 \neq 0) \\ \vee(\alpha = 3 \wedge \beta_6 \neq 0) \vee \alpha \leq 0 \vee \alpha \geq 4)\end{array}}$$

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- The strongest necessary and weakest sufficient conditions for success:

$$(\mathrm{true}, \alpha \leq 0 \vee \alpha \geq 4)$$

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$$(?, ?)$$

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- The strongest necessary and weakest sufficient conditions for success:

$$(\text{true}, \alpha \leq 0 \vee \alpha \geq 4)$$

- Strongest necessary and weakest sufficient conditions for failure:

$$(\neg(\alpha \leq 0 \vee \alpha \geq 4), ?)$$

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- Nothing guarantees `key_new_private` will fail; i.e. weakest sufficient condition is false.

Negation Example

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- The strongest necessary and weakest sufficient conditions for success:

$$(\text{true}, \alpha \leq 0 \vee \alpha \geq 4)$$

- Strongest necessary and weakest sufficient conditions for failure:

$$(1 \leq \alpha \leq 3, \text{false})$$

- Requested key must have type `KEY_RSA1`, `KEY_RSA`, or `KEY_DSA` for function to fail.

What Have We Done So Far?

- We identified a special class of variables, called choice variables that model uncertainty and imprecision in constraint-based analysis.

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- We argued that computing pairs of strongest necessary and weakest sufficient conditions not containing choice variables allows us:

  - ✓ to overcome termination problems
  - ✓ to mitigate scalability problems
  - ✓ to negate constraints in a sound way
  - ✓ and preserve satisfiability and validity

What Have We *Not* Done So Far?

We have not shown how to compute strongest necessary and weakest sufficient conditions in any specific theory

Rest of This Talk

- We show how to compute strongest necessary and weakest sufficient conditions for a system of recursive constraints representing the exact path- and context-sensitive conditions under which a property holds

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  - No choice variables

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- We show how to compute strongest necessary and weakest sufficient conditions for a system of recursive constraints representing the exact path- and context-sensitive conditions under which a property holds
- We use these strongest necessary and weakest sufficient conditions to perform sound and complete path- and context-sensitive program analysis for answering may and must queries
  - Completeness assumes a user-provided finite abstraction
  - No choice variables $\Rightarrow$ Small formulas $\Rightarrow$ Good scalability

Where Does This Approach Fit In?

There are many proposed techniques for path- and context-sensitive program analysis.

- Model checking tools: Bebop, BLAST, SLAM, ...

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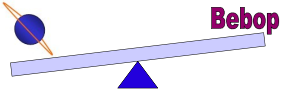
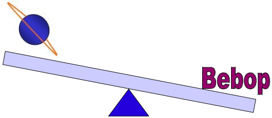


The Scalability Scale



Sound & Complete Scale

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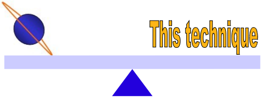

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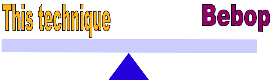

Sound & Complete Scale

Contributions

- A sound and complete algorithm for path- and context-sensitive program analysis that scales to multi-million line programs

Contributions

- A sound and complete algorithm for path- and context-sensitive program analysis that scales to multi-million line programs
- Key Insight: 
  - While choice variables are useful within their scoping boundary, they can be eliminated without losing completeness for answering may and must queries about program properties outside of this scoping boundary.

Choice Variables and Scope

```
void process_file(File* f) {
  printf(``Open new file?\n'');
  char user_input = getUserInput();
  if(user_input == 'y')
     f = fopen(NEW_FILE_NAME);
  process_file_internal(f);
  if(user_input == 'y')
     fclose(f);
}
```

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User input is represented by a choice variable

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}
```

Branch correlation arises from test on choice variable

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```

Correct matching of `fopen()/fclose()`
depends on this branch correlation

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     fclose(f);
}
```

Since this user input is not visible in calling contexts of `process_file`, the choice variable is only useful within this scope

Choice Variables and Scope

```
void process_file(File* f) {
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  char user_input = getUserInput();
  if(user_input == 'y')
     f = fopen(NEW_FILE_NAME);
  process_file_internal(f);
  if(user_input == 'y')
     fclose(f);
}
```

If we are interested in answering may and must queries, we can safely eliminate choice variables at their scoping boundaries

Choice Variables and Scope

```
void process_file(File* f) {
  printf(``Open new file?\n'');
  char user_input = getUserInput();
  if(user_input == 'y')
     f = fopen(NEW_FILE_NAME);
  process_file_internal(f);  /* dereference f */
  if(user_input == 'y')
     fclose(f);
}
```

May the original input file `f` be dereferenced by `process_file`?

Choice Variables and Scope

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void process_file(File* f) {
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  if(user_input == 'y')
     f = fopen(NEW_FILE_NAME);
  process_file_internal(f);  /* dereference f */
  if(user_input == 'y')
     fclose(f);
}
```

Must the original input file `f` be dereferenced by `process_file`?
NO!

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5. Solve using standard fixed-point computation

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- Constraints ϕ_{ij} are boolean combinations of $\alpha = C_i$, $\beta = C_i$, Π_{f_i,α,C_j} and $C_i = C_j$.

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- α's represent function inputs, provided by the calling context.

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- β's represent choice variables. The scope of each β is the function body in which it is introduced.

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- Π's on the right hand side result from function calls.

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int f(int x) {
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- Since each variable v_i must have exactly one abstract value C_j, the boolean constraints must satisfy the following additional existence and uniqueness constraints:

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$$\mathrm{SAT}^*(\phi) \equiv \mathrm{SAT}(\phi \wedge \psi_{\text{exist}} \wedge \psi_{\text{unique}})$$
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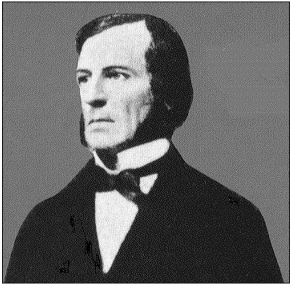
- For instance, using the variables in the previous example,

$$\begin{array}{l} \text{UNSAT}^*(\alpha_1 \wedge \alpha_2) \\ \text{VALID}^*(\beta_1 \vee \beta_2 \vee \beta_3) \end{array}$$

Step 3: Eliminate Choice Variables

$$\mathsf{SNC}(\phi, \beta) = \phi[\mathsf{true}/\beta] \vee \phi[\mathsf{false}/\beta]$$

$$\mathsf{WSC}(\phi, \beta) = \phi[\mathsf{true}/\beta] \wedge \phi[\mathsf{false}/\beta]$$



Resulting Constraints

$$E = \begin{bmatrix} [\vec{\Pi}_{f_1,\alpha,C_i}] & = & [\vec{\phi}_{1i}(\vec{\alpha}_1, \vec{\beta}_1, \vec{\Pi}[\vec{b}_1/\vec{\alpha}][\vec{\beta}'/\vec{\beta}])] \\ \vdots & & \vdots \\ [\vec{\Pi}_{f_k,\alpha,C_i}] & = & [\vec{\phi}_{ki}(\vec{\alpha}_k, \vec{\beta}_k, \vec{\Pi}[\vec{b}_k/\vec{\alpha}][\vec{\beta}'/\vec{\beta}])] \end{bmatrix}$$

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No more choice variables

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But still recursive

Example

- If we eliminate the choice variables from the constraint

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  - Recall from earlier: $\neg\lceil\phi\rceil \not\Leftrightarrow \lceil\neg\phi\rceil$ and $\neg\lfloor\phi\rfloor \not\Leftrightarrow \lfloor\neg\phi\rfloor$
  - Contradictions and tautologies must be explicitly enforced when applying substitution
    - Consider $\Pi_{f,\alpha,C_1} \wedge \Pi_{f,\alpha,C_2}$ where $\lceil\Pi_{f,\alpha,C_1}\rceil$ and $\lceil\Pi_{f,\alpha,C_2}\rceil$ are both true

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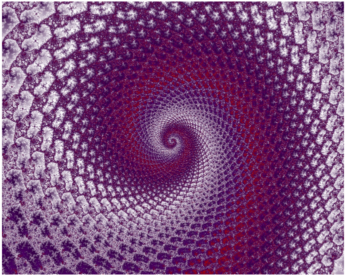
The resulting constraints preserve strongest necessary and weakest sufficient conditions under syntactic substitution.

Step 5: Compute fixed point



Since the modified system of constraints preserve strongest necessary and weakest sufficient conditions under syntactic substitution, compute a fixed-point solution by repeated substitution

Example

```
int f(int x) {
   int y = getUserInput();
   if(x == 1 || y == 2) return 1;
   return f(1);
}
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Original constraint:

$$\begin{array}{rcl} \Pi_{f,\alpha,C_1} & = & (\alpha = 1 \vee \beta = 2 \vee \\ & & ((\neg\alpha = 1 \wedge \neg\beta = 2 \wedge \Pi_{f,\alpha,C_1}[1/\alpha][\beta'/\beta])) \end{array}$$

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In the previous step, we computed:

$$\begin{aligned}\lceil \Pi_{f,\alpha,C_1} \rceil &= \text{true} \\ \lfloor \Pi_{f,\alpha,C_1} \rfloor &= \alpha_1 \vee \lfloor \Pi_{f,\alpha,C_1} \rfloor [\text{true}/\alpha_1]\end{aligned}$$

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Compute greatest fixed-point:

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- The sufficient condition expresses that the function MUST return 1 because $\mathrm{VALID}(\lfloor \Pi_{\mathrm{f},\alpha,\mathrm{C}_1} \rfloor)$ holds.

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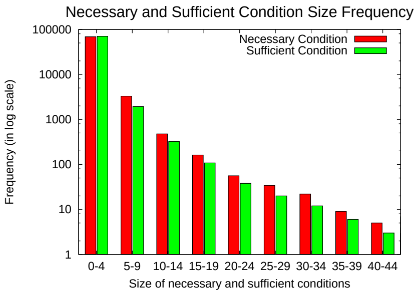

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Bugs	1	17	134	1	17	134
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- Observed close to an order of magnitude reduction of false positives without resorting to (potentially unsound) ad-hoc heuristics.

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- Analysis of contents of position dependent data structures, such as arrays, linked lists etc., is one of our current projects.

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  - Closely related to *cover algorithms* for existential quantifier elimination ("Cover Algorithms and Their Combination" by Gulwani and Musuvathi)

Related Work

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Thank You For Listening!