ACL2: Implementation of a Computational Logic

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June 10, 2015

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Today I'll discuss a logic and software tool, ACL2, which has been my focus off and on since the early 1990s.

(But my intention in Gothenburg is to return to my roots in model theory, especially models of set theory and arithmetic.)

OUTLINE

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Logical Foundations

Conclusion

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Let's start with some context.

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- UT Austin: x86 interpreter defined in ACL2, validation by co-simulation, proofs about x86 machine code

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- Fast execution
- Documentation (about 100,000 lines for just the system)

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That talk mentions this link to several demos and their logs:

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  - *Boyer-Moore Theorem Provers* go back to the start of their collaboration in 1971.

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- Interfaces include Emacs, ACL2 Sedan (Eclipse-based), none.

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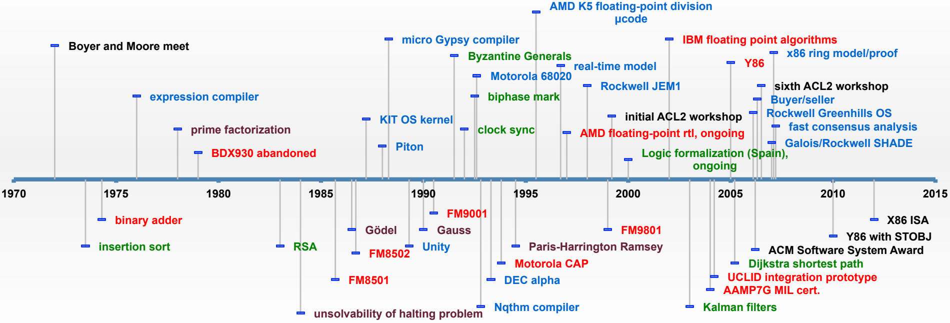
(We can expand on these topics if there is time and interest.)

Partial timeline



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But all ACL2 theories extend a given *ground-zero* theory, which is essentially Peano Arithmetic with ε_0-induction, extended with data types for:

- characters,
- strings,
- symbols,
- complex numbers with rational coefficients, and
- closure under a pairing operation (`cons`).

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  - M. Kaufmann and J Moore, "Structured Theory Development for a Mechanized Logic." *Journal of Automated Reasoning* 26, no. 2 (2001) 161-203.
- Importance: One may want to introduce new concepts to carry out some proofs, but this must be done conservatively in order to believe the results.

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But for now, let's just show how `LOCAL` and conservativity apply: 25 lines in `overspill-proof.lisp` correspond to 256 lines in `overspill-proof.lisp`.

Key parts of the book `overspill.lisp`:

```
(local (include-book "overspill-proof"))
(set-enforce-redundancy t)
(defstub overspill-p (n x) t)

(defun overspill-p* (n x)
  (if (zp n)
      (overspill-p 0 x)
    (and (overspill-p n x)
         (overspill-p* (1- n) x))))

(defchoose overspill-p-witness (n) (x)
  (or (and (natp n) (standardp n)
           (not (overspill-p n x)))
      (and (natp n) (i-large n)
           (overspill-p* n x))))

(defthm overspill-p-overspill
  (let ((n (overspill-p-witness x)))
    (or (and (natp n) (standardp n)
             (not (overspill-p n x)))
        (and (natp n) (i-large n)
             (implies (and (natp m)
                           (<= m n))
                      (overspill-p m x)))))
  :rule-classes nil)
```

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We'll look at just a few on the next slides.

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- **Define** "implementation function" `impl`: $\texttt{impl}(x) = 10 * x$
- Attach `impl` to `spec`:
  `(defattach spec impl)`

Result not provable from axioms for `f` and `spec`:

```
ACL2 !>(f 3 4)
```

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- **Define** "implementation function" `impl`: $\texttt{impl}(x) = 10 * x$
- Attach `impl` to `spec`:
  `(defattach spec impl)`

Result not provable from axioms for `f` and `spec`:

```
ACL2 !>(f 3 4)
70
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```

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For details, including issues pertaining to evaluation, see the *Essay on Defattach* comment in the ACL2 sources.

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Conservatively introduce $w(y,z)$ and $r(y,z)$ *using local witness*
$w(y,z) = (\varepsilon\, x)(p(x,y,z) \wedge q(x,y,z))$
to prove these axioms:

- $r(y,z) = (p(w(y,z),y,z) \wedge q(w(y,z),y,z))$
- $(p(x,y,z) \wedge q(x,y,z)) \implies r(y,z)$

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Conservativity *with* induction follows from a model-theoretic forcing argument.

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We can return to this on an extra slide, if there is time and interest.

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- *Packages* provide namespaces — *e.g.*, `PKG1::F` and `PKG2::F` are distinct. But packages introduce axioms such as `symbol-package-name(PKG1::F) = "PKG1"`. So package introduction is *not conservative* and hence must be recorded.

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- One can specify a *measure* in order to admit a recursive definition. But what if the measure is defined in terms of a function whose definition is `LOCAL`?

OUTLINE

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THANK YOU!

EXTRA SLIDES

We can go on, time permitting....

Some ACL2 features *not* discussed further today:

- Prover algorithms
  - Waterfall, linear arithmetic, Boolean reasoning, ...
  - Rewriting: Conditional, congruence-based, rewrite cache, syntaxp, bind-free, ...
- Using the prover effectively
  - The-method and introduction-to-the-theorem-prover
  - Theories, hints, rule-classes, ...
  - Accumulated-persistence, brr, proof-checker, dmr, ...
- Programming support, including (just a few):
  - Guards
  - Hash-cons and function memoization
  - Packages
  - Mutable State, stobjs, arrays, applicative hash tables, ...
- System-level: Emacs support, books and certification, abbreviated printing, parallelism (ACL2(p)), ...

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