Logical Foundations for the ACL2 Theorem Prover

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Joint work with Bob Boyer, J Moore, and the ACL2 community

Presented at JAF 2019

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MY ANSWERS:

1. Introduce ACL2 as a **practical application** of logic.
2. Discuss **foundational issues** for ACL2.

OUTLINE

Outline

Overview and Context

The ACL2 home page begins with the following summary.

> *ACL2 is a logic and programming language in which you can model computer systems, together with a tool to help you prove properties of those models. "ACL2" denotes "**A** **C**omputational **L**ogic for **A**pplicative **C**ommon **L**isp".*

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But before we talk about ACL2, let's put it in context.

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FV tools include *equivalence checkers*, *model checkers*, various *static checkers*, and (occasionally) *interactive theorem provers* (ITPs) such as Coq, Isabelle, HOL4, PVS, Agda — and **ACL2**.

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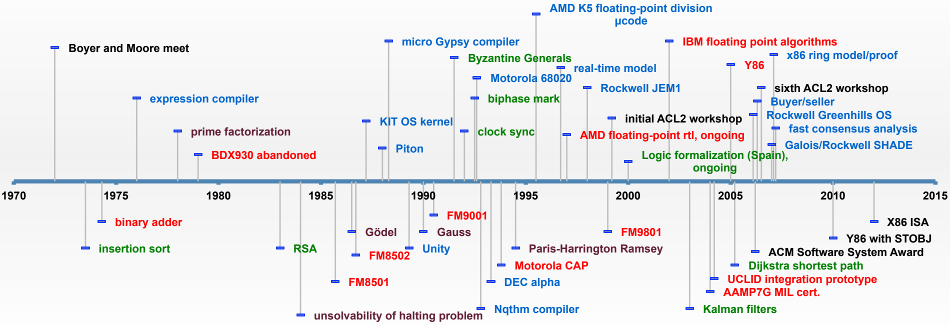
- Used in recent international SAT competitions
- Has checked 2-petabyte SAT proof of longstanding open problem (Schur number 5) [3]; $\sim$16 CPU years

Partial timeline



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Overview and Context

Introduction to the ACL2 System

Logical Foundations for ACL2

Conclusion

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  - *Boyer-Moore Theorem Provers* go back to their collaboration starting in 1971. [10]
  - The ACL2 community contributes with feature requests and (on occasion) prototype implementations.

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- ACL2 as an automated theorem prover
  [DEMO]: file `demo-2.lsp`
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  - ACL2 provides **automation** for induction, linear arithmetic, Boolean reasoning, rule application, ...
  - During a proof, each goal is replaced by a list of subgoals (possible empty) such that if they are all theorems, then that goal is a theorem.

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(Anyhow, it's nice to have Ken Kunen's Nqthm proof of the Paris-Harrington theorem. [9])

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- characters;
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- closure under an ordered pair operation, `cons`.

`Cons` provides lists, with the symbol `nil` for the empty list.

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ACL2 !>(cons 3 nil)
(3)
ACL2 !>(cons 2 (cons 3 nil))
(2 3)
ACL2 !>(cons 1 (cons 2 (cons 3 nil)))
(1 2 3)
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- We will see the importance of introducing new concepts **locally**: justified by conservativity.
- Theories *evolve* by introducing new function symbols using the *extension principles*. [6]

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A definition may be recursive if some *measure* into ε_0 is proved to decrease on each recursive call.

Extension principle: Choice (and $\exists$)

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$P(\vec{x}) = \exists \vec{y} A(\vec{x}, \vec{y})$

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Conservatively introduce a Skolem (witness) function $w(\vec{x})$ and a predicate $P(\vec{x})$:

$w(\vec{x}) = \varepsilon \vec{y} A(\vec{x}, \vec{y})$ [If any $\vec{y}$ satisfies $A(\vec{x}, \vec{y})$, then $w(\vec{x})$ does.]

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```
(defun-sk fermat-counterex (n)
  (exists (i j k)
    (and (posp i) (posp j) (posp k)
         (equal (+ (expt i n) (expt j n))
                (expt k n)))))
(defthm fermat
  (implies (and (integerp n) (< 2 n))
           (not (fermat-counterex n))))
```

EXTENSION PRINCIPLE: CHOICE (AND ∃) (2)

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Conservativity *with* induction follows from a model-theoretic forcing argument.

Extension principle: Constraints

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A derived inference rule, *functional instantiation* [2], is often useful with constrained functions.

Example:

```
(defun map2-fn (lst1 lst2)
  (if (consp lst1)
      (cons (fn (first lst1) (first lst2))
            (map2-fn (rest lst1) (rest lst2)))
    nil))
(defthm map2-fn-commutative
  (implies (equal (len lst1) (len lst2)) ; same length
           (equal (map2-fn lst2 lst1)
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(defun map2-* (lst1 lst2)
  (if (consp lst1)
      (cons (* (first lst1) (first lst2))
            (map2-* (rest lst1) (rest lst2)))
    nil))
(defthm map2-*-commutative
  (implies (equal (len lst1) (len lst2))
           (equal (map2-* lst2 lst1)
                  (map2-* lst1 lst2)))
  :hints (("Goal" :by (:functional-instance
                       map2-fn-commutative
                       (fn *) (map2-fn map2-*)))))
```

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Using `LOCAL` can dramatically speed up book inclusion!

```
(local ; Hence skipped when including this top-level book!
  (include-book "overspill-proof"))

(defstub overspill-p (n x) t)

(defun overspill-p* (n x)
  (if (zp n)
      (overspill-p 0 x)
    (and (overspill-p n x)
         (overspill-p* (1- n) x))))

(defchoose overspill-p-witness (n) (x)
  (or (and (natp n) (standardp n)
           (not (overspill-p n x)))
      (and (natp n) (i-large n)
           (overspill-p* n x))))

(defthm overspill-p-overspill
  (let ((n (overspill-p-witness x)))
    (or (and (natp n) (standardp n)
             (not (overspill-p n x)))
        (and (natp n) (i-large n)
             (implies (and (natp m)
                           (<= m n))
                      (overspill-p m x)))))
  :rule-classes nil)
```

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A comment in the ACL2 sources, the "Essay on Correctness of Meta Reasoning", works out the correctness argument.

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```

where `sum$` is defined essentially as follows.

```
(defun sum$ (fn lst)
  (if (endp lst) ; lst is empty
      0
    (+ (apply$ fn (list (first lst)))
       (sum$ fn (rest lst)))))
```

"HIGHER-ORDER" `Apply$` (1)

We cannot employ the usual two-sorted, weak second-order approach. **Example**: Not a theorem without the `defun`!

```
(local (defun f (x) x))
(thm (equal (apply$ 'f (list x)) x))
```

"HIGHER-ORDER" Apply$ (1)

We cannot employ the usual two-sorted, weak second-order approach. **Example**: Not a theorem without the `defun`!

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(local (defun f (x) x))
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Example successful use of `apply$`:

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(defun$ norm^2 (x y) (+ (* x x) (* y y)))
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But the following fails, as it should:

`apply$` is a constrained function with trivial constraints.

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(thm (equal (apply$ 'norm^2 (list 3 4))
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"HIGHER-ORDER" Apply$ (2)

However, the proof succeeds for the `thm` below, where the *warrant hypothesis*, `(warrant norm^2)`, asserts:

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(∀ x y) (equal (apply$ 'norm^2 (list x y))
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Warrant hypotheses are not vacuous!
There is a natural *evaluation theory* where every warrant is *attached* to the constant "true" function. [8]

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Details: see *Essay on Defattach* comment in the ACL2 sources.

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  `symbol-package-name('PKG1::F) = "PKG1"`
  Hence packages must be recorded.
- One can specify a *measure* in order to admit a recursive definition. But what if the measure is defined in terms of a function whose definition is `LOCAL`?
- *Congruence-based reasoning* allows replacing one subterm by another that is equivalent but not necessarily equal. [7]

Outline

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Overview and Context

Introduction to the ACL2 System

Logical Foundations for ACL2

Conclusion

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- For more information, see the ACL2 home page, in particular links to The Tours and Publications, which links to introductory material.

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THANK YOU!

EXTRA SLIDES

We can go on, time permitting....

Some ACL2 features *not* discussed further today:

- Prover algorithms
  - Waterfall, linear arithmetic, Boolean reasoning, ...
  - Rewriting: Conditional, congruence-based, rewrite cache, syntaxp, bind-free, ...
- Using the prover effectively
  - The-method and introduction-to-the-theorem-prover
  - Theories, hints, rule-classes, ...
  - Accumulated-persistence, brr, proof-checker, dmr, ...
- Programming support, including (just a few):
  - Guards
  - Hash-cons and function memoization
  - Packages
  - Mutable State, stobjs, arrays, applicative hash tables, ...
- System-level: Emacs support, books and certification, abbreviated printing, parallelism (ACL2(p)), ...

Meta-theoretic Reasoning (2)

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ON EFFICIENT EXECUTION

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- ACL2 definitions are actually programs in the Common Lisp programming language.
- *Guards* specify intended domains of functions and support sound, efficient Common Lisp evaluation.
- Several features support efficient computation by reusing storage, yet with a first-order logic foundation.
  - *Single-threaded objects* including *state*
  - *Arrays*
  - *Function memoization* (reuse of saved results)
  - *Fast alists* (applicative hash tables)