Correctness

Theorem Snapshot I produces a consistent cut

Proof Need to prove $e_j \in C \wedge e_i \rightarrow e_j \Rightarrow e_i \in C$

< Definition >
$0.\ e_j \in C \equiv T(e_j) < t_{ss}$

< Assumption >
$1.\ e_j \in C$

< Assumption >
$2.\ e_i \rightarrow e_j$

< 0 and 1>
$3.\ T(e_j) < t_{ss}$

< Property of real time>
$4.\ e_i \rightarrow e_j \Rightarrow T(e_i) < T(e_j)$

< 2 and 4>
$5.\ T(e_i) < T(e_j)$

< 5 and 3>
$6.\ T(e_i) < t_{ss}$

< Definition >
$7.\ e_i \in C$

Clock Condition

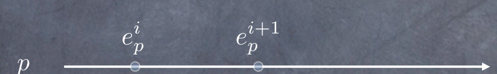
< Property of real time>
$4.\ e_i \rightarrow e_j \Rightarrow T(e_i) < T(e_j)$

Can the Clock Condition be implemented some other way?

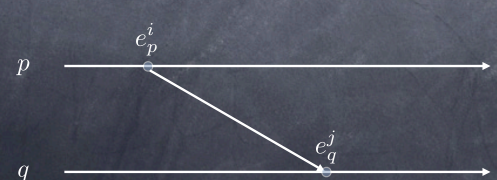
Lamport Clocks

Each process maintains a local variable LC

$LC(e) \equiv$ value of LC for event e

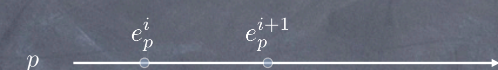


$LC(e_p^i) < LC(e_p^{i+1})$

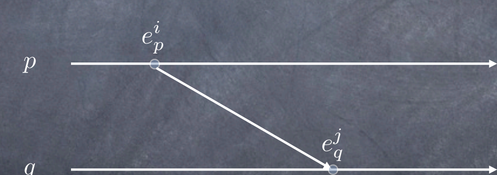


$LC(e_p^i) < LC(e_q^j)$

Increment Rules



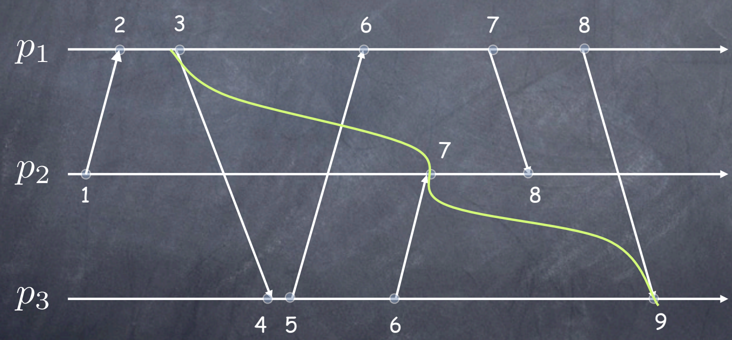
$LC(e_p^{i+1}) = LC(e_p^i) + 1$



$LC(e_q^j) = max(LC(e_q^{j-1}), LC(e_p^i)) + 1$

Timestamp m with $TS(m) = LC(send(m))$

Space-Time Diagrams and Logical Clocks



A subtle problem

when $LC = t$ do S

doesn't make sense for Lamport clocks!

- there is no guarantee that LC will ever be t
- S is anyway executed <u>after</u> $LC = t$

Fixes:

- if e is internal/send and $LC = t - 2$
  - execute e and then S
- if $e = receive(m) \wedge (TS(m) \geq t) \wedge (LC \leq t - 1)$
  - put message back in channel
  - re-enable e ; set $LC = t - 1$; execute S

An obvious problem

- No t_{ss}!
- Choose Ω large enough that it cannot be reached by applying the update rules of logical clocks

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mmmmhhhh...

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mmmmhhhh...

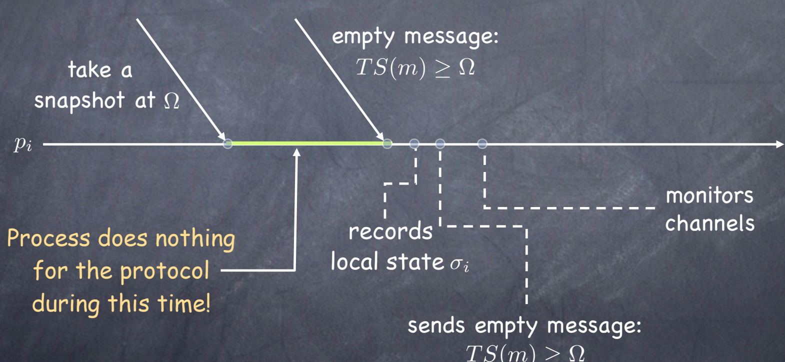
- Doing so assumes
  - upper bound on message delivery time
  - upper bound relative process speeds

We better relax it...

Snapshot II

- processor p_0 selects Ω
- p_0 sends "take a snapshot at Ω" to all processes; it waits for all of them to reply and then sets its logical clock to Ω
- when clock of p_i reads Ω then p_i
  - records its local state σ_i
  - sends an empty message along its outgoing channels
  - starts recording messages received on each incoming channel
  - stops recording a channel when receives first message with timestamp greater than or equal to Ω

Relaxing synchrony



Use empty message to announce snapshot!

Snapshot III

- processor p_0 sends itself "take a snapshot "
- when p_i receives "take a snapshot" <u>for the first time</u> from p_j:
  - records its local state σ_i
  - sends "take a snapshot" along its outgoing channels
  - sets channel from p_j to empty
  - starts recording messages received over each of its other incoming channels
- when p_i receives "take a snapshot" beyond the first time from p_k:
  - stops recording channel from p_k
- when p_i has received "take a snapshot" on all channels, it sends collected state to p_0 and stops.

Snapshots: a perspective

- The global state Σ^s saved by the snapshot protocol is a consistent global state

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  - many total orders (runs) are compatible with that partial order
  - all we know is that Σ^s could have occurred

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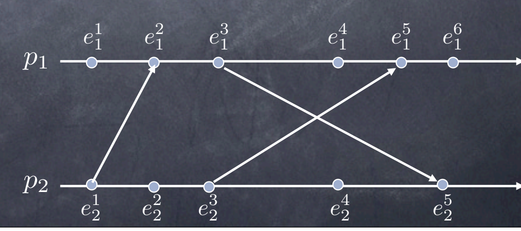
  - all we know is that Σ^s could have occurred
- We are evaluating predicates on states that may have never occurred!

An Execution and its Lattice



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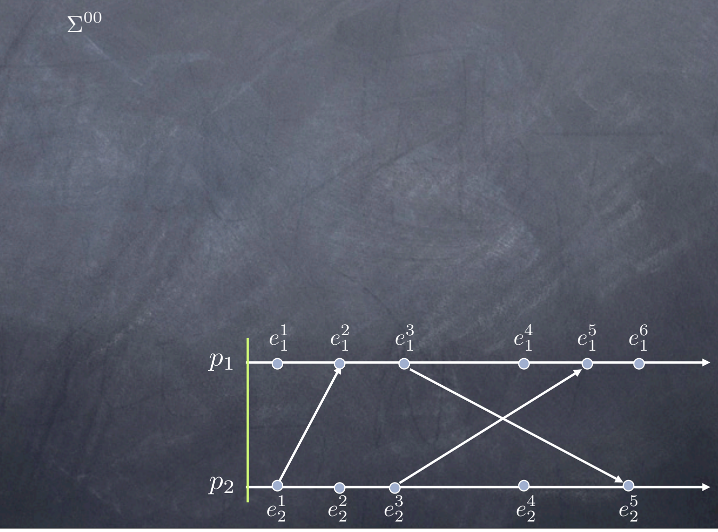

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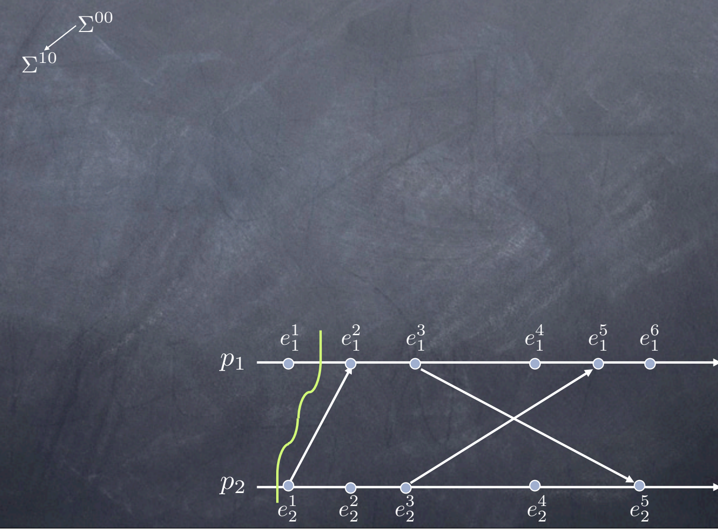

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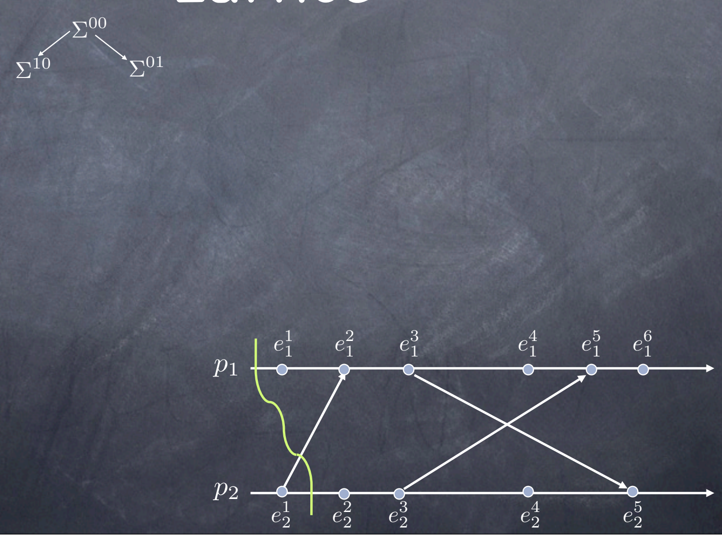

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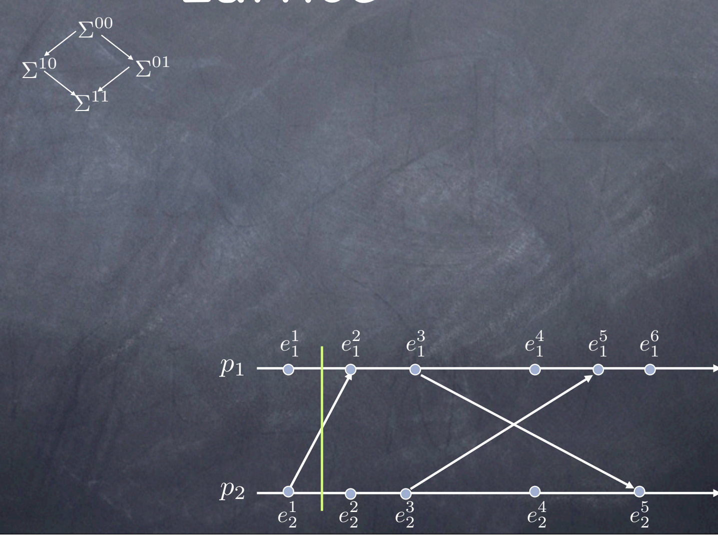
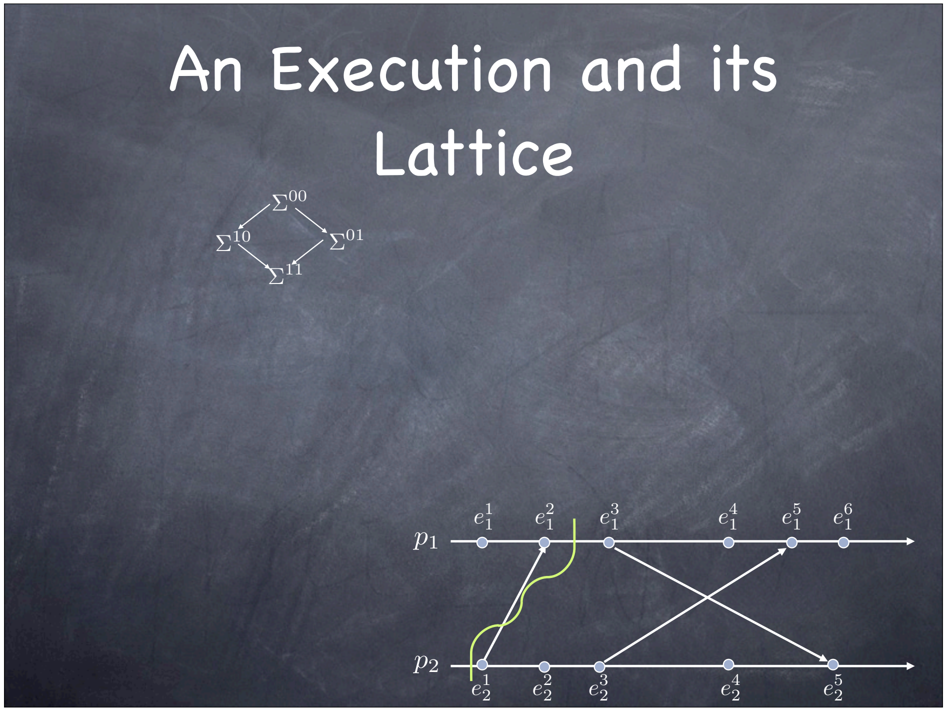
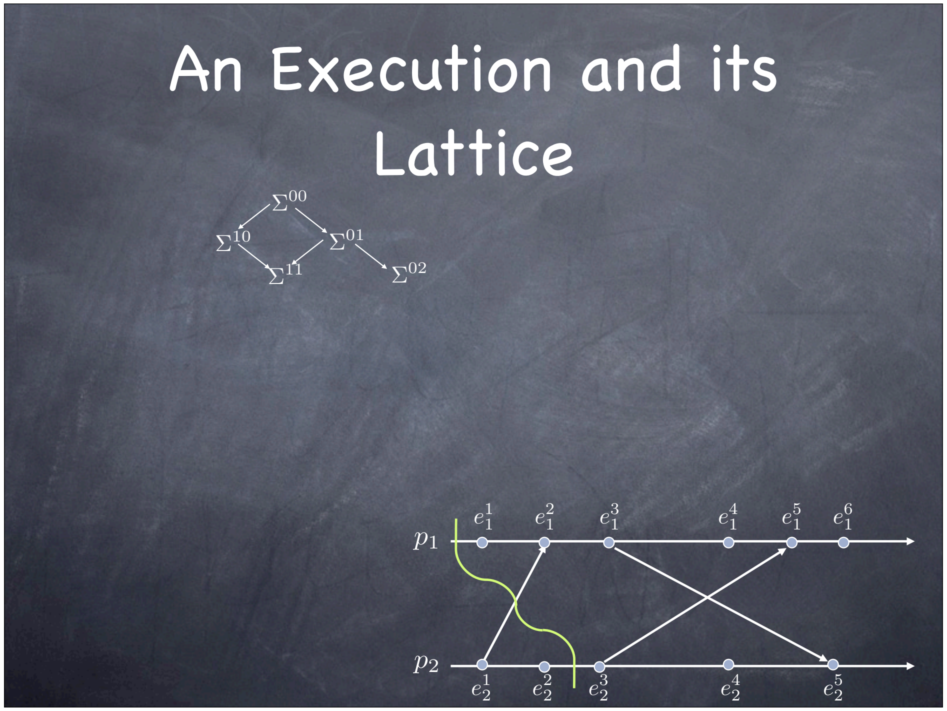



An Execution and its Lattice
Σ^{00}
Σ^{10}
Σ^{01}
Σ^{11}
p_1
p_2
e_1^1 e_1^2 e_1^3 e_1^4 e_1^5 e_1^6
e_2^1 e_2^2 e_2^3 e_2^4 e_2^5


An Execution and its Lattice
Σ^{00}
Σ^{10}
Σ^{01}
Σ^{11}
Σ^{02}
p_1
p_2
e_1^1 e_1^2 e_1^3 e_1^4 e_1^5 e_1^6
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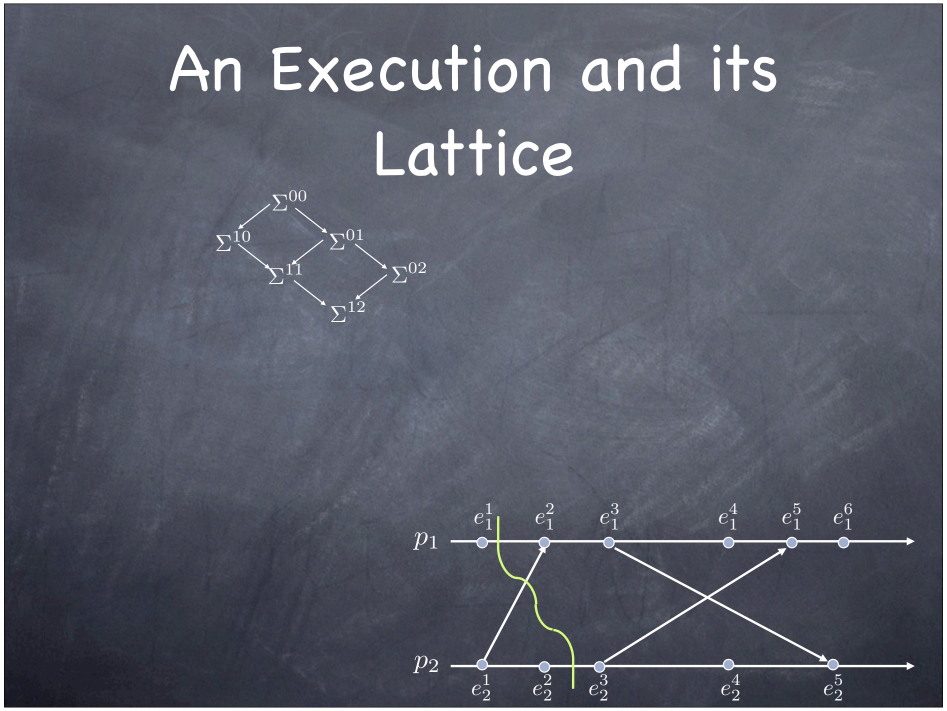
An Execution and its Lattice
Σ^{00}
Σ^{10}
Σ^{01}
Σ^{11}
Σ^{02}
Σ^{12}
p_1
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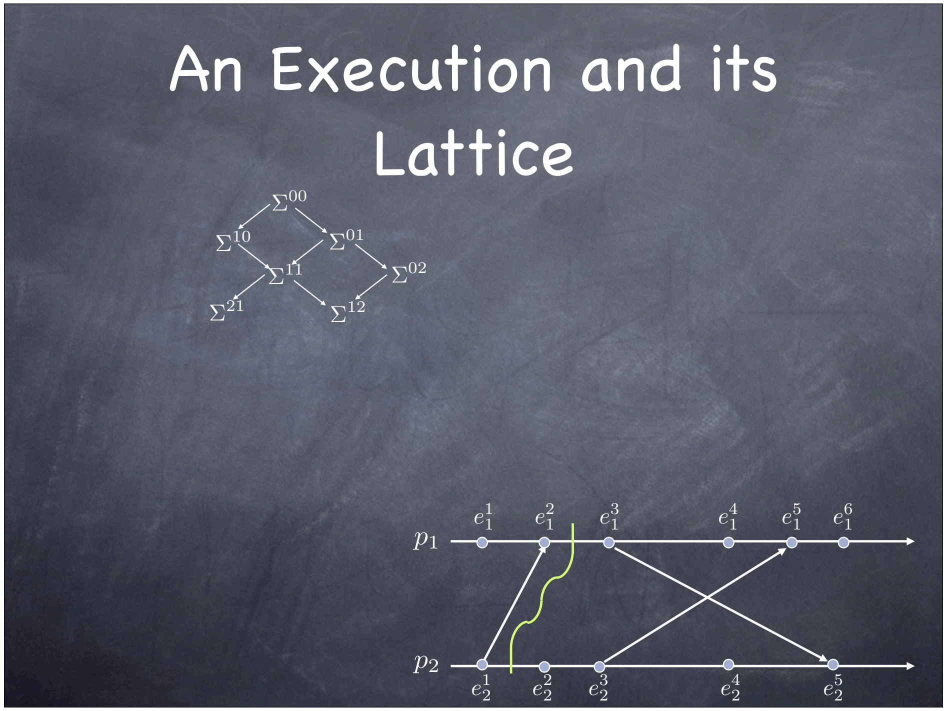
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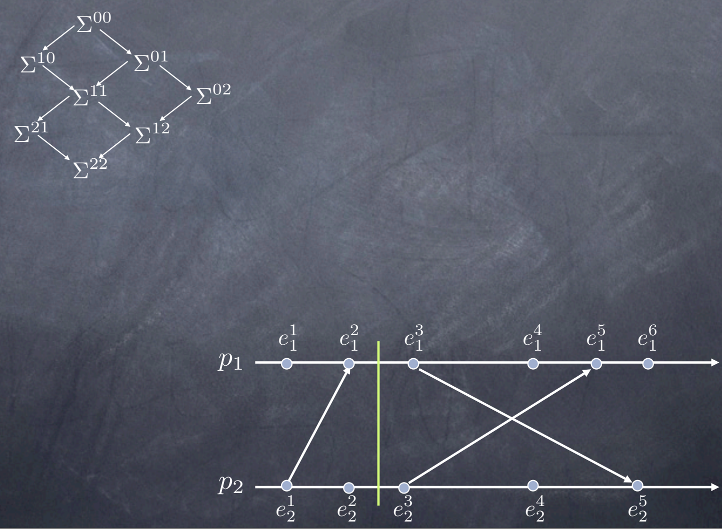

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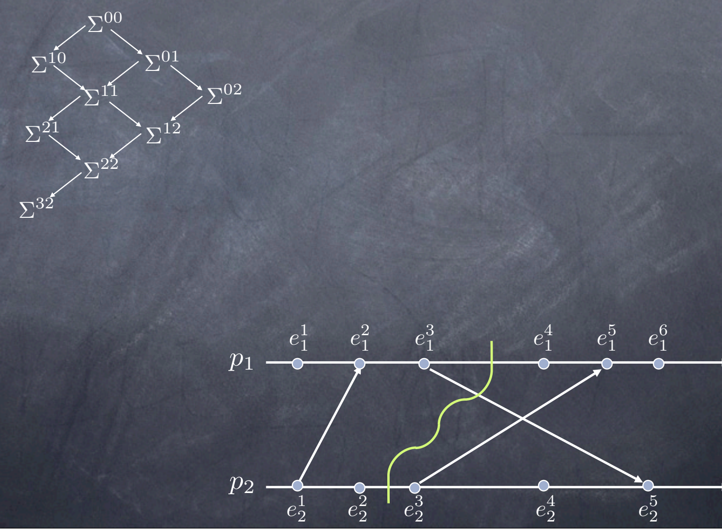

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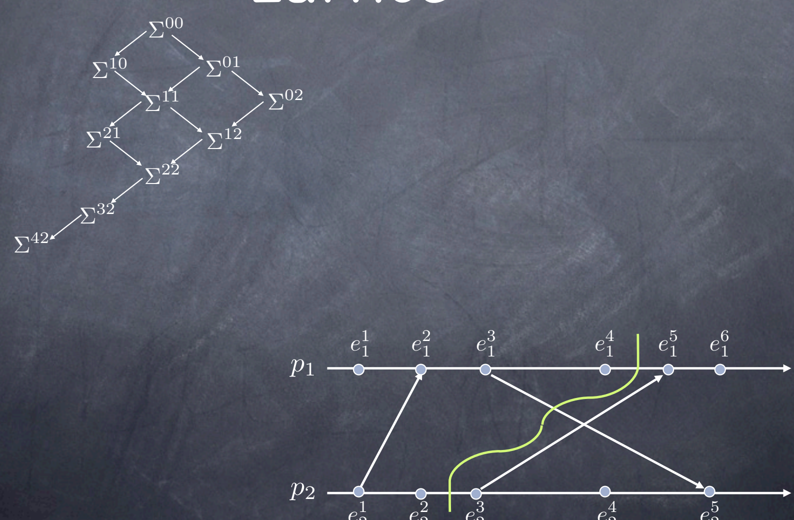

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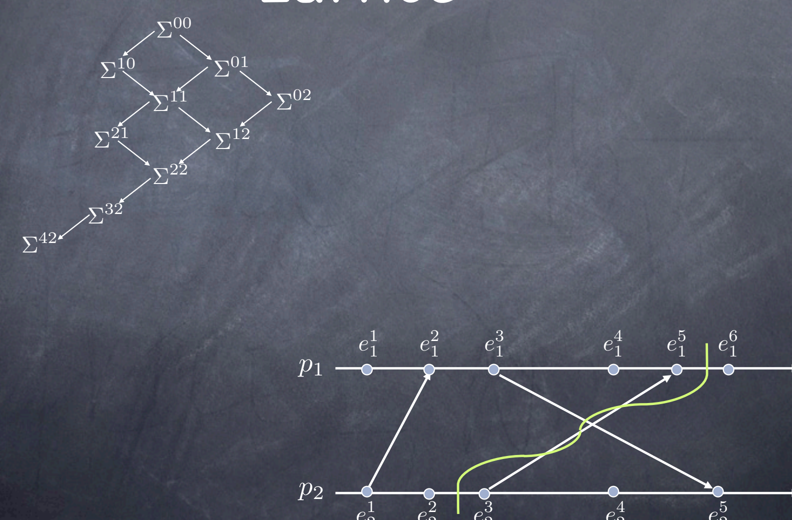

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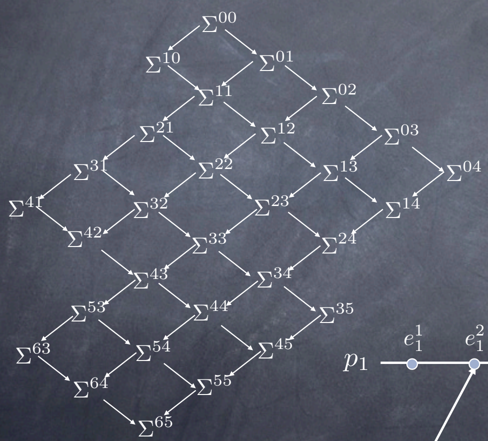
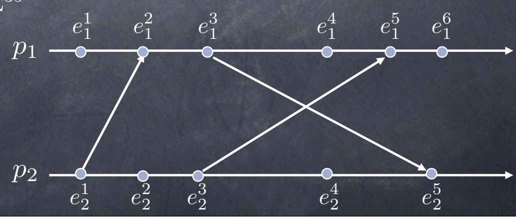
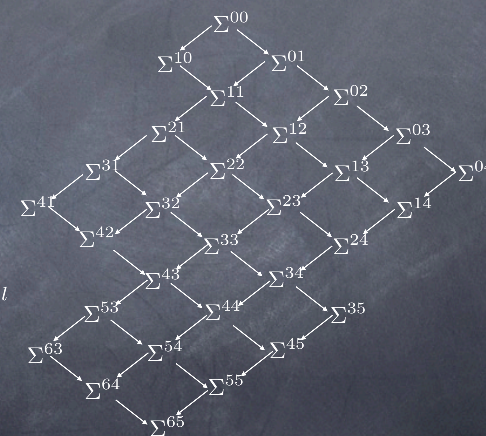




Reachability

Σ^{kl} is reachable from Σ^{ij} if there is a path from Σ^{ij} to Σ^{kl} in the lattice



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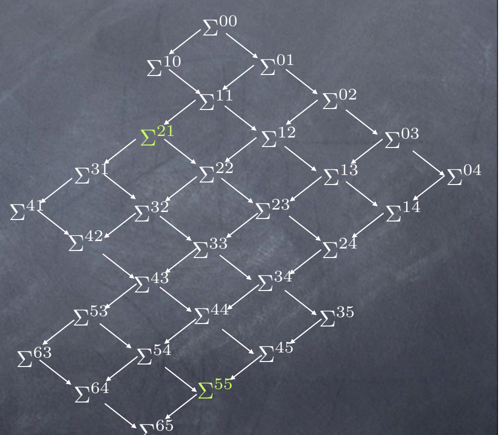

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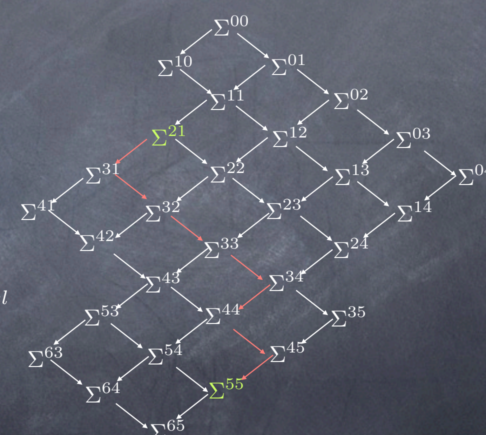

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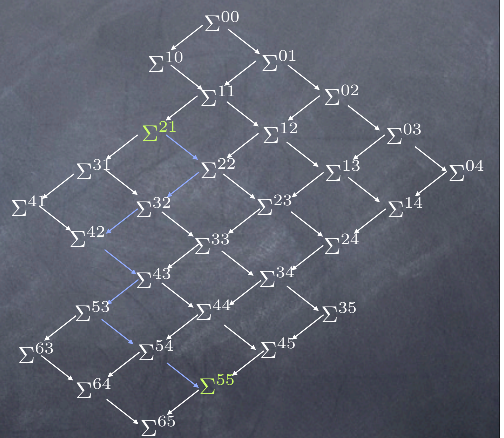

Σ^{kl} is reachable from Σ^{ij} if there is a path from Σ^{ij} to Σ^{kl} in the lattice

$$\Sigma^{ij} \rightsquigarrow \Sigma^{kl}$$

So, why do we care about Σ^s again?

- Deadlock is a stable property

  Deadlock $\Rightarrow \Box$ Deadlock

- If a run R of the snapshot protocol starts in Σ^i and terminates in Σ^f, then $\Sigma^i \rightsquigarrow_R \Sigma^f$

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- No deadlock in Σ^s implies no deadlock in Σ^i

Same problem, different approach

- Monitor process does not query explicitly
- Instead, it passively collects information and uses it to build an observation.
  (reactive architectures, Harel and Pnueli [1985])

An observation is an ordering of event of the distributed computation based on the order in which the receiver is notified of the events.

Observations: a few observations

- An observation puts no constraint on the order in which the monitor receives notifications

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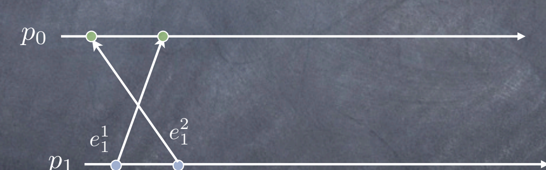

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To obtain a run, messages must be delivered to the monitor in FIFO order

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To obtain a run, messages must be delivered to the monitor in FIFO order
What about consistent runs?

Causal delivery

FIFO delivery guarantees:

$$send_i(m) \rightarrow send_i(m') \Rightarrow deliver_j(m) \rightarrow deliver_j(m')$$

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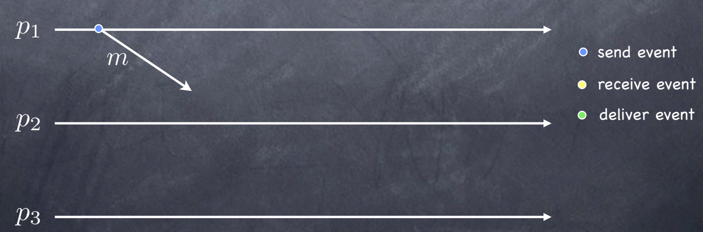

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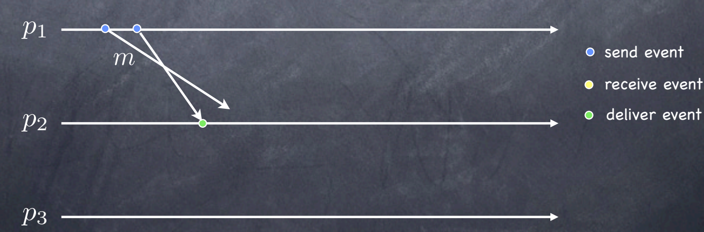

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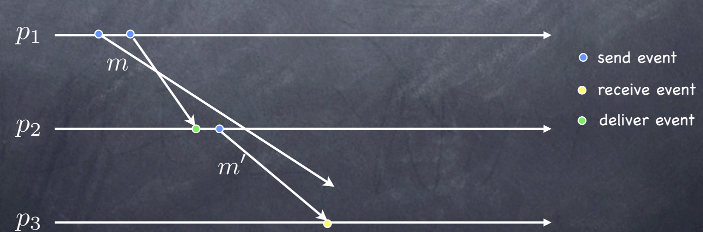

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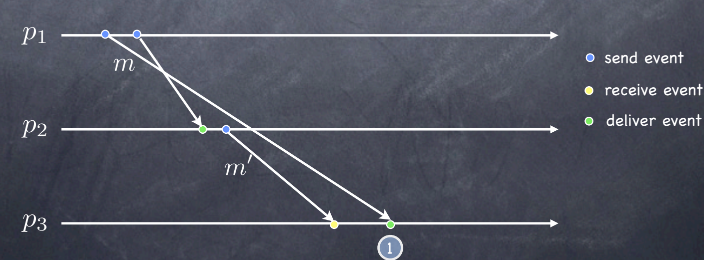

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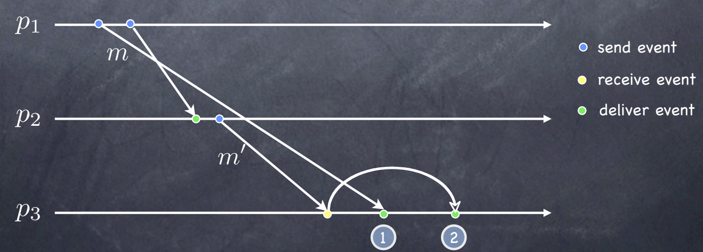

Causal Delivery in Synchronous Systems

We use the upper bound Δ on message delivery time

Causal Delivery in Synchronous Systems

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DR1: At time t, p_0 delivers all messages it received with timestamp up to $t - \Delta$ in increasing timestamp order

Causal Delivery with Lamport Clocks

DR1.1: Deliver all received messages in increasing (logical clock) timestamp order.

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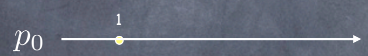

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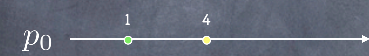

Should p_0 deliver?

Causal Delivery with Lamport Clocks

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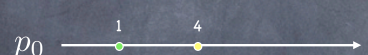

Should p_0 deliver?

Problem: Lamport Clocks don't provide gap detection

Given two events e and e' and their clock values $LC(e)$ and $LC(e')$—where $LC(e) < LC(e')$ determine whether some event e''exists s.t.

$$LC(e) < LC(e'') < LC(e')$$

Stability

DR2: Deliver all received stable messages in increasing (logical clock) timestamp order.

A message m received by p is stable at p if p will never receive a future message m's.t.

$$TS(m') < TS(m)$$

Implementing Stability

- Real-time clocks
  - wait for Δ time units

Implementing Stability

- Real-time clocks
  - wait for Δ time units
- Lamport clocks
  - wait on each channel for m s.t. $TS(m) > LC(e)$
- Design better clocks!

Clocks and STRONG Clocks

- Lamport clocks implement the clock condition:

$$e \rightarrow e' \Rightarrow LC(e) < LC(e')$$

- We want new clocks that implement the strong clock condition:

$$e \rightarrow e' \equiv SC(e) < SC(e')$$

Causal Histories

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  $\theta(e) = \{e' \in H \mid e' \rightarrow e\} \cup \{e\}$

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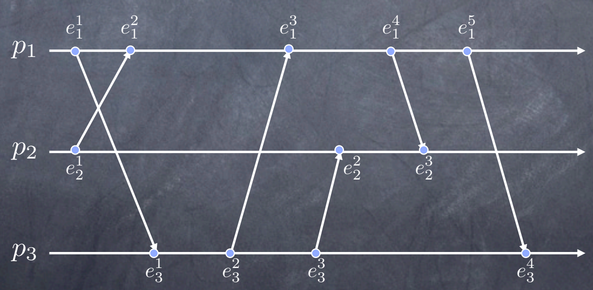

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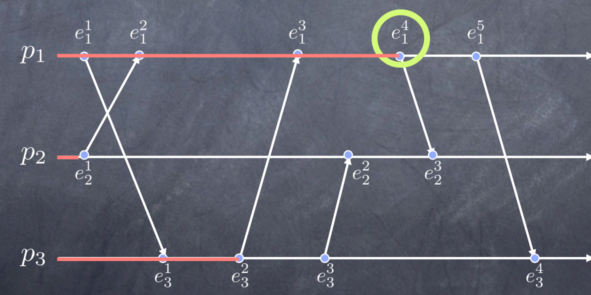


$$e \rightarrow e' \equiv \theta(e) \subset \theta(e')$$

How to build $\theta(e)$

Each process p_i:

- initializes θ : $\theta := \emptyset$
- if e_i^k is an internal or send event, then
  $$\theta(e_i^k) := \{e_i^k\} \cup \theta(e_i^{k-1})$$
- if e_i^k is a receive event for message m, then
  $$\theta(e_i^k) := \{e_i^k\} \cup \theta(e_i^{k-1}) \cup \theta(send(m))$$

Pruning causal histories

- Prune segments of history that are known to all processes (Peterson, Bucholz and Schlichting)
- Use a more clever way to encode $\theta(e)$

Vector Clocks

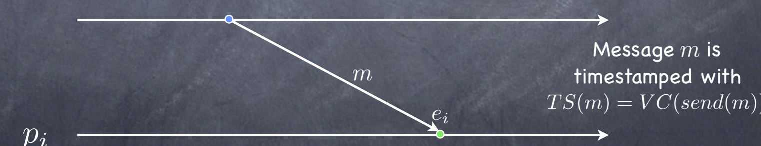
- Consider $\theta_i(e)$, the projection of $\theta(e)$ on p_i
- $\theta_i(e)$ is a prefix of h^i: $\theta_i(e) = h_i^{k_i}$ – it can be encoded using k_i
- $\theta(e) = \theta_1(e) \cup \theta_2(e) \cup \ldots \cup \theta_n(e)$ can be encoded using $k_1, k_2, \ldots, k_n$

Represent θ using an n-vector VC such that

$$VC(e)[i] = k \Leftrightarrow \theta_i(e) = h_i^{k_i}$$

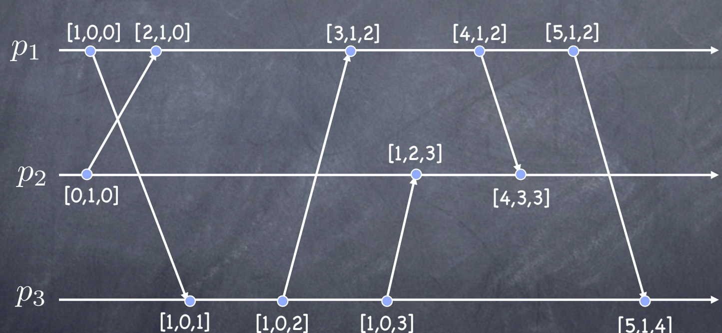
Update rules

$$VC(e_i)[i] := VC[i] + 1$$

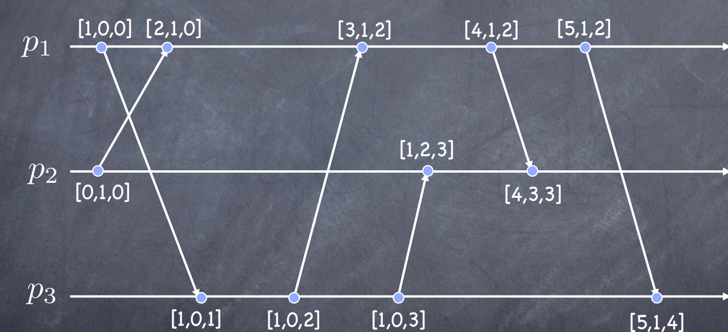


$$VC(e_i) := max(VC, TS(m))$$
$$VC(e_i)[i] := VC[i] + 1$$

Example



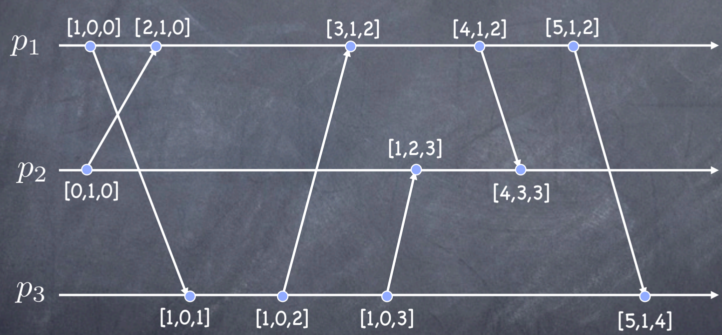
Operational interpretation



$VC(e_i)[i]$ =

$VC(e_i)[j]$ =

Operational interpretation



$VC(e_i)[i]$ = no. of events executed by p_i up to and including e_i

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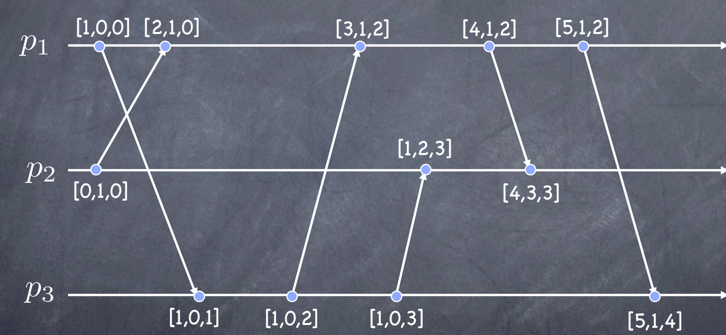
Operational interpretation



$VC(e_i)[i]$ = no. of events executed by p_i up to and including e_i

$VC(e_i)[j]$ = no. of events executed by p_j that happen before e_i of p_i

VC properties: event ordering

Given two vectors V and V', less than is defined as:

$$V < V' \equiv (V \neq V') \wedge (\forall k : 1 \leq k \leq n : V[k] \leq V'[k])$$

- Strong Clock Condition: $e \rightarrow e' \equiv VC(e) < VC(e')$
- Simple Strong Clock Condition:
  Given e_i of p_i and e_j of p_j, where $i \neq j$
  $$e_i \rightarrow e_j \equiv VC(e_i)[i] \leq VC(e_j)[i]$$
- Concurrency
  Given e_i of p_i and e_j of p_j, where $i \neq j$
  $$e_i \parallel e_j \equiv (VC(e_i)[i] > VC(e_j)[i]) \wedge (VC(e_j)[j] > VC(e_i)[j])$$

VC properties: consistency

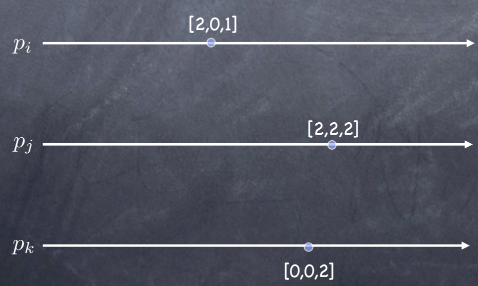
- Pairwise inconsistency
  Events e_i of p_i and e_j of p_j $(i \neq j)$ are pairwise inconsistent (i.e. can't be on the frontier of the same consistent cut) if and only if
  $$(VC(e_i)[i] < VC(e_j)[i]) \vee (VC(e_j)[j] < VC(e_i)[j])$$
- Consistent Cut
  A cut defined by $(c_1, \ldots, c_n)$ is consistent if and only if
  $$\forall i, j : 1 \leq i \leq n, 1 \leq j \leq n : (VC(e_i^{c_i})[i] \geq VC(e_j^{c_j})[i])$$

VC properties: weak gap detection

- Weak gap detection

  Given e_i of p_i and e_j of p_j, if $VC(e_i)[k] < VC(e_j)[k]$ for some $k \neq j$, then there exists e_k s.t

  $$\neg(e_k \to e_i) \wedge (e_k \to e_j)$$



VC properties: weak gap detection

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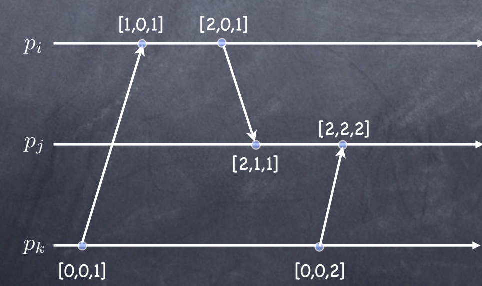

VC properties: strong gap detection

- Weak gap detection

  Given e_i of p_i and e_j of p_j, if $VC(e_i)[k] < VC(e_j)[k]$ for some $k \neq j$, then there exists e_k s.t

  $$\neg(e_k \to e_i) \wedge (e_k \to e_j)$$

- Strong gap detection

  Given e_i of p_i and e_j of p_j, if $VC(e_i)[i] < VC(e_j)[i]$ then there exists e_i' s.t.

  $$(e_i \to e_i') \wedge (e_i' \to e_j)$$

VCs for Causal Delivery

- Each process increments the local component of its VC only for events that are notified to the monitor
- Each message notifying event e is timestamped with $VC(e)$
- The monitor keeps all notification messages in a set M

Stability

Suppose p_0 has received m_j from p_j.
When is it safe for p_0 to deliver m_j?

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- There is no earlier message from p_j

$TS(m_j)[j] = 1+$ no. of p_j messages delivered by p_0

- There is no earlier message m_k'' from p_k, $k \neq j$

  see next slide...

Checking for m''_k

- Let m'_k be the last message p_0 delivered from p_k
- By strong gap detection, m''_k exists only if

$$TS(m'_k)[k] < TS(m_j)[k]$$

- Hence, deliver m_j as soon as

$$\forall k : TS(m'_k)[k] \geq TS(m_j)[k]$$

The protocol

- p_0 maintains an array $D[1, \ldots, n]$ of counters
- $D[i] = TS(m_i)[i]$ where m_i is the last message delivered from p_i

DR3: Deliver m from p_j as soon as both of the following conditions are satisfied:

$D[j] = TS(m)[j] - 1$

2. $D[k] \geq TS(m)[k], \forall k \neq j$

Properties

Property: a predicate that is evaluated over a run of the program

"every message that is received was previously sent"

Not everything you may want to say about a program is a property:

"the program sends an average of 50 messages in a run"

Safety properties

- "nothing bad happens"
  - no more than k processes are simultaneously in the critical section
  - messages that are delivered are delivered in causal order
  - Windows never crashes
- A safety property is "prefix closed":
  - if it holds in a run, it holds in every prefix

Liveness properties

- "something good eventually happens"
  - a process that wishes to enter the critical section eventually does so
  - some message is eventually delivered
  - Windows eventually boots
- Every run can be extended to satisfy a liveness property
  - if it does not hold in a prefix of a run, it does not mean it may not hold eventually

A really cool theorem

Every property is a combination of a safety property and a liveness property

(Alpern & Schneider)