Lecture 03-4: Physical Layer Fourier Series

CS 356R

Intro to Wireless Networks

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Please, interrupt and ask questions AT ANY TIME !

Adjusting schedue

- Prelab 1 due Monday before class
- Lab 1 due Friday (Feb 3) before class
- Project 1 assigned Friday (Feb 3)

Outline

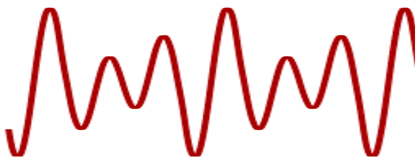
1. Fourier Series Definition

Fourier series of f(x)

- If f(x) is periodic, f(x) can be defined as
  - an infinite sum of cosines and sines of increasing frequency
- Both digital and analog works as long as they are periodic

"Digital" signal

Analog signal



- If f(x) is 2π periodic $[-\pi, \pi]$

Fourier series of f(x)

- If f(x) is 2π periodic [- π, π]
- $$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos(kx) + b_k \sin(kx))$$

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(kx) dx = \frac{1}{\| \cos(kx) \|^2} \langle f(x), \cos(kx) \rangle$$

Note $\| \cos(kx) \|^2 = \| \sin(kx) \|^2 = \pi$

$$b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(kx) dx = \frac{1}{\| \sin(kx) \|^2} \langle f(x), \sin(kx) \rangle$$

coefficient a_k b_k can be viewed as projecting f(x) on to orthogonal basis {cos(kx), sin(kx)}

Fourier series of f(x) using e^{ikx}

- If f(x) is 2π periodic $[-\pi, \pi]$
- Euler's formula says $e^{ikx} = \cos(kx) + i\sin(kx)$
- $$f(x) = \sum_{k=-\infty}^{\infty} c_k e^{ikx} = \sum_{k=-\infty}^{\infty} (\alpha_k + i\beta_k)(\cos(kx) + i\sin(kx))$$
  - After expanding all terms and dividing $k=[-\infty, -1]$, $k=0$, $k=[1, \infty]$
    we can show f(x) is a sum of cos/sin with complex coefficients (databook 57p)

Is e^{ikx} orthogonal with all other e^{ijx} when k != j?

What does orthogonal mean?

- Inner product is 0
  - Vector u, v
  - u = [0 1]
  - v = [1 0]
  - <<u, v>> = 0
- Function f(x) g(x) = $\int f(x) g(x) dx = 0$

Show $\int_{-\pi}^{\pi} e^{ikx} e^{-ijx} dx = 0$ (databook 58p)

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Show $\int_{-\pi}^{\pi} e^{ikx} e^{ijx} dx = 0$ (databook 58p)

What if f(x) is not 2 π periodic?

- We can generalize!
  - Just a matter of notation change

Fourier series f(x)

- If f(x) is $2L$ periodic $[-L, L]$
- $$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$$

$$a_m = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{m\pi x}{L}\,dx, \quad m = 0, 1, 2, 3, \ldots$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx, \quad n = 1, 2, 3, \ldots$$

What if just L periodic?

- If f(x) is L periodic [0, L) the equation becomes

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left(a_k \cos\left(\frac{2\pi k x}{L}\right) + b_k \sin\left(\frac{2\pi k x}{L}\right) \right)$$

$$a_k = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{2\pi k x}{L}\right) dx$$

$$b_k = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{2\pi k x}{L}\right) dx.$$

Outline

1. Fourier Series Definition
2. Matlab Example

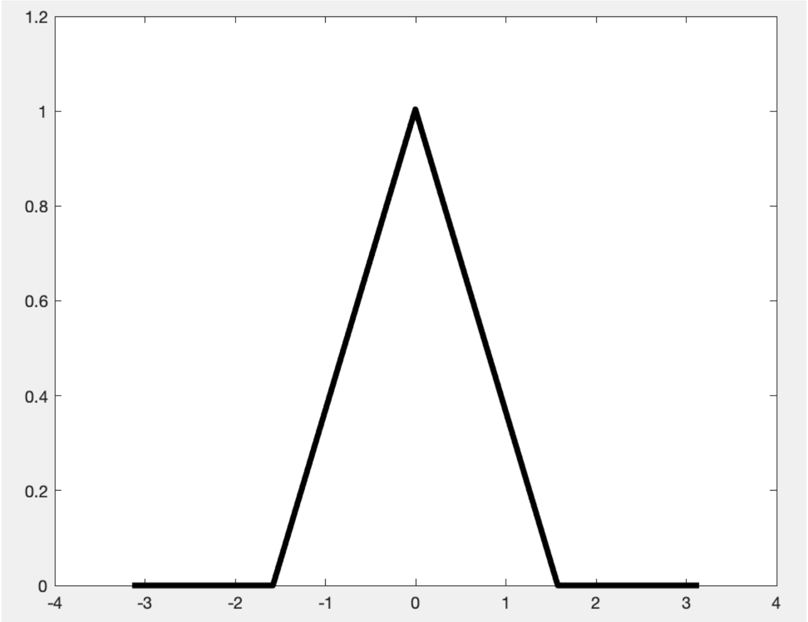
_/_ hat wave

- f(x) is 2 π periodic with range [- π, π]
  - Let L = π
- f(x) = 0 (- π<=x<=- 0.5π)

  2/pi x+1 (- 0.5π<=x<=0)

  -2/pi x+1 (0<=x<=0.5π)

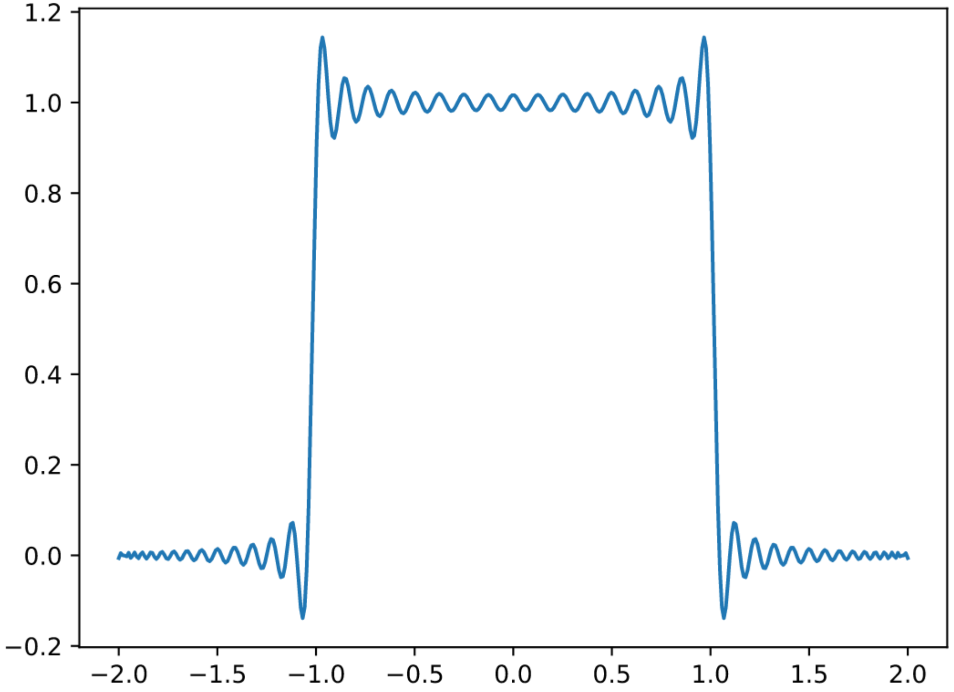
  0 (0.5π<=x<=0)



- N = 1024 : num samples
- Divide 2L so that we have exactly N samples
- Each scale dx = 2L/(N-1)

Square wave

- What is different that you see?



This is called Gibb’s phenomenon