Lecture 03-11: Physical Layer OFDM

CS 356R

Intro to Wireless Networks

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Outline

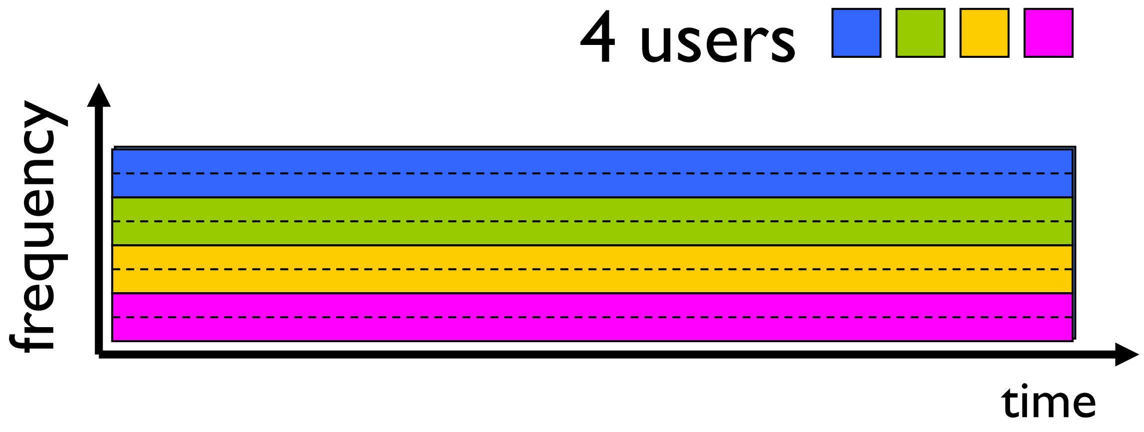
1. Multiplexing

Multiplexing

- Capacity of the transmission medium usually exceeds the capacity required for a single signal
  - Single signal can never occupy the whole medium
- Thus, sharing is necessary
- Since the spectrum is huge multiplexing is a must for wireless

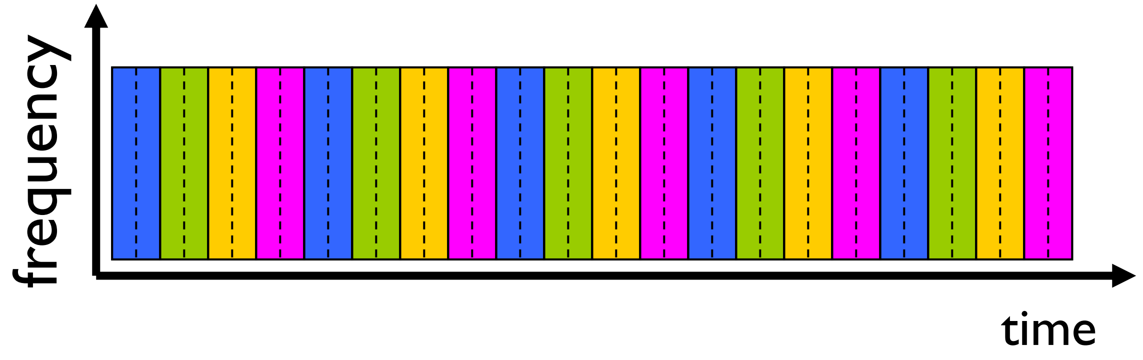
Multiplexing enables multiple signals to share the same transmission medium

Multiplexing Techniques: FDM and TDM



Frequency Division Multiplexing (FDM)

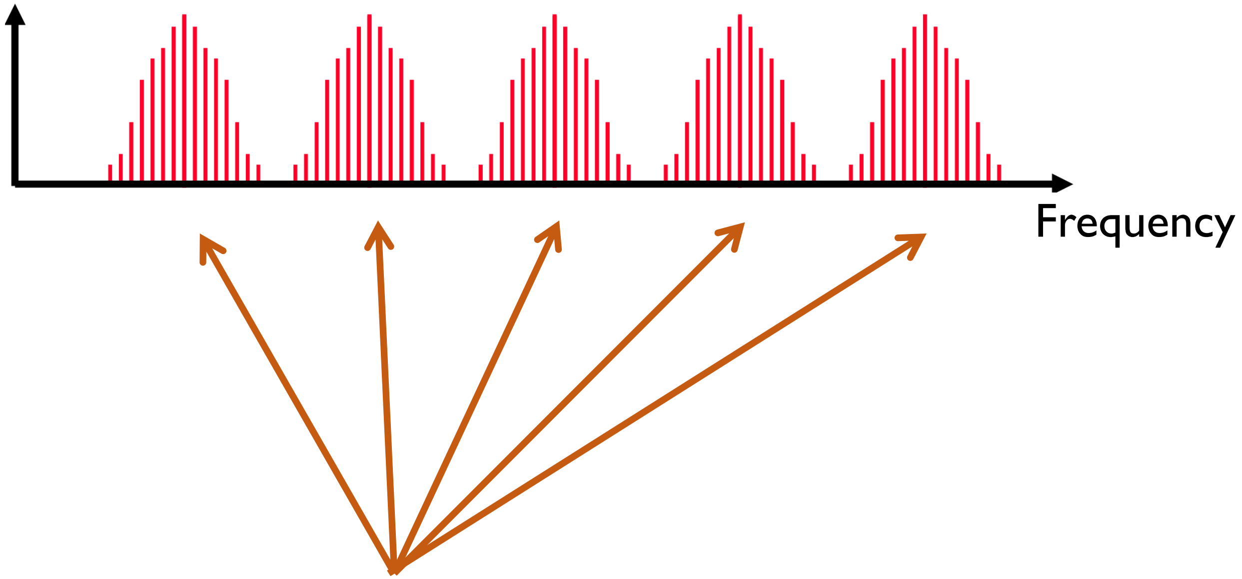
- optical, electromagnetic frequencies divided into (narrow) frequency bands
- each call allocated its own band, can transmit at max rate of that narrow band



Time Division Multiplexing (TDM)

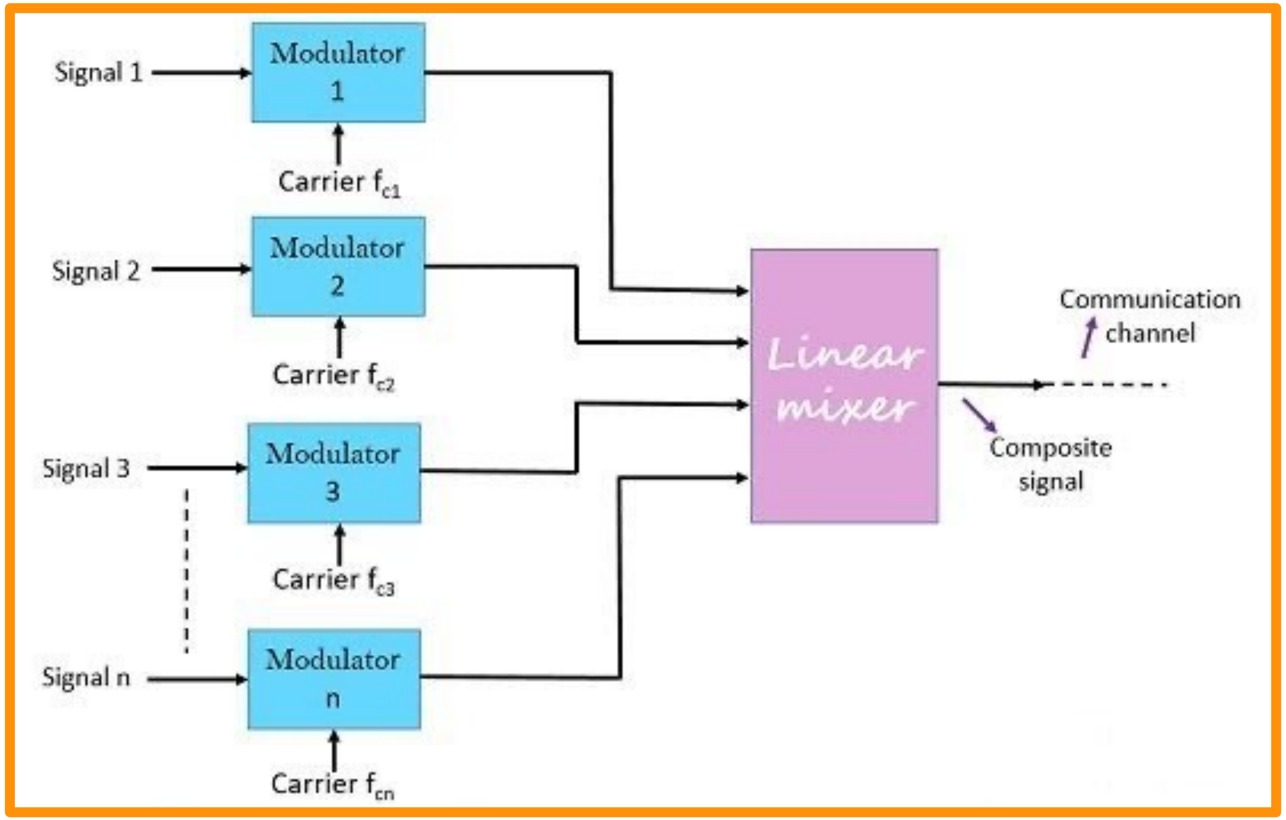
- time divided into slots
- each call allocated periodic slot(s), can transmit at maximum rate of (wider) frequency band (only) during its time slot(s)

Multiple users can share the wireless spectrum using FDM



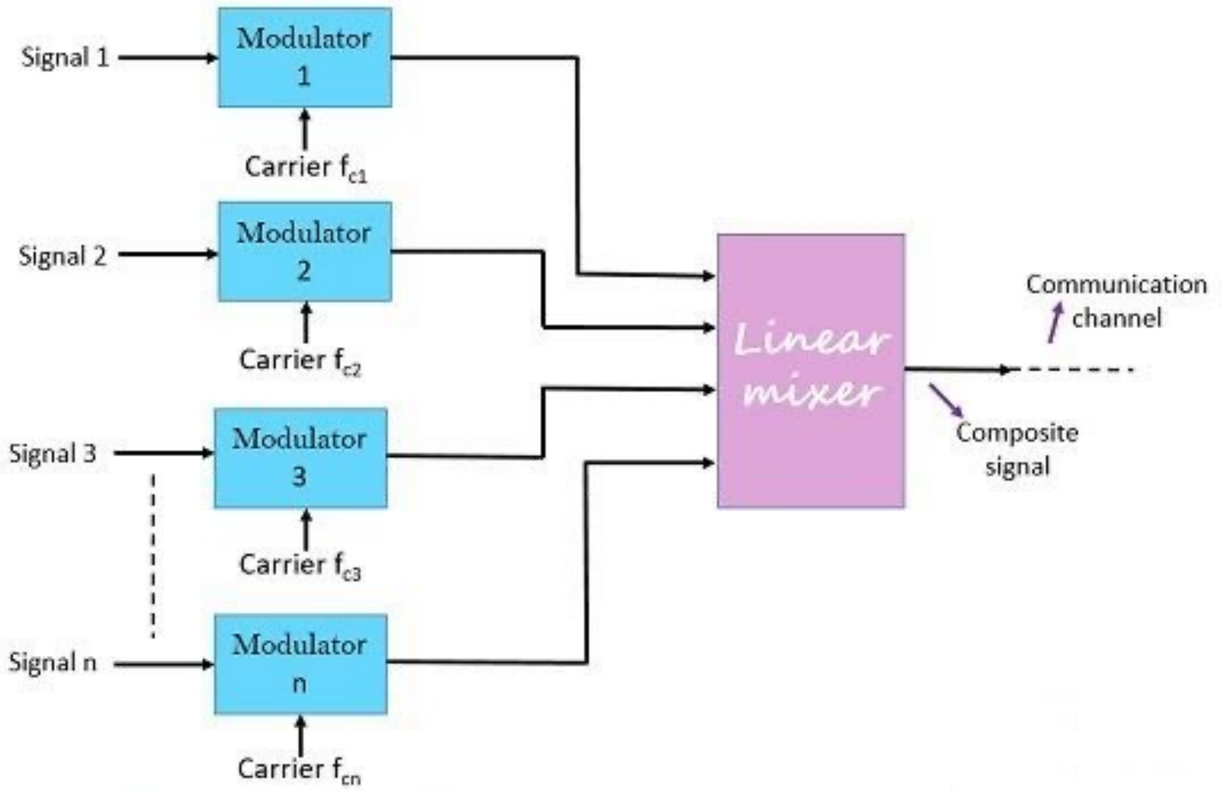
Different users use different carrier frequency

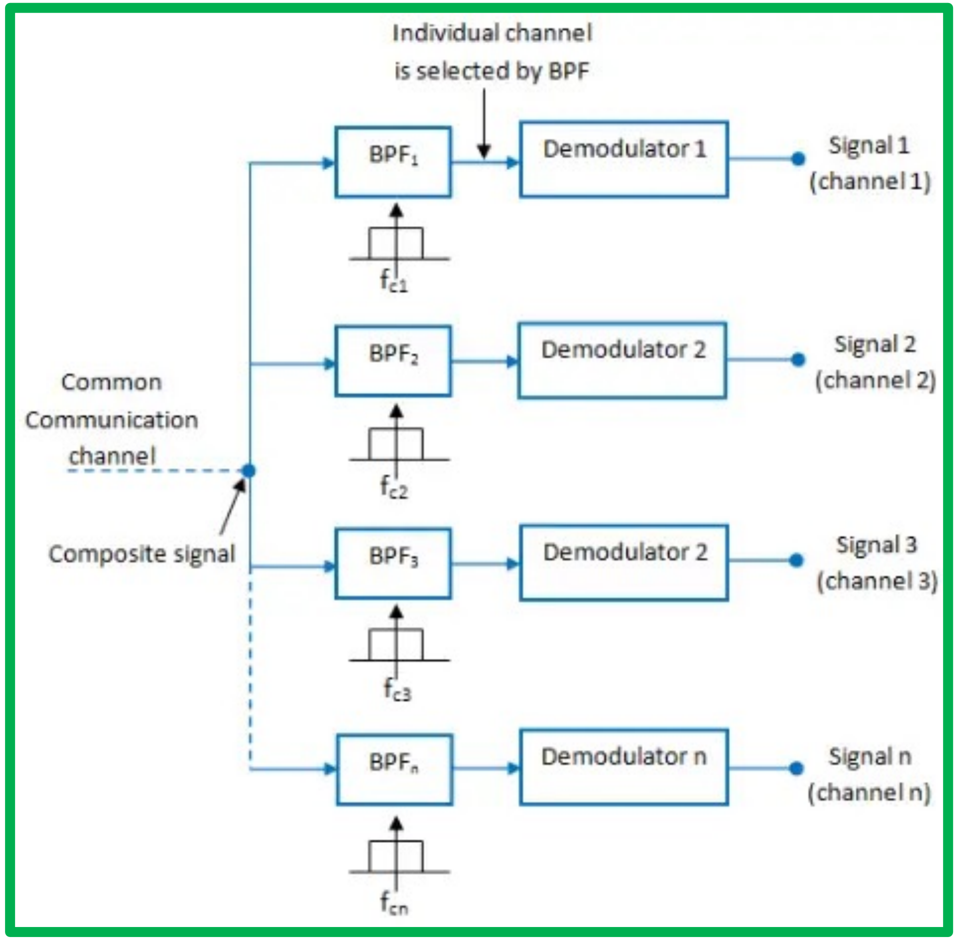
Sender-side: how general FDM multiplexing works



Sender gathers each signal from a different user, modulates each with a different frequency, add them all up, and transmit

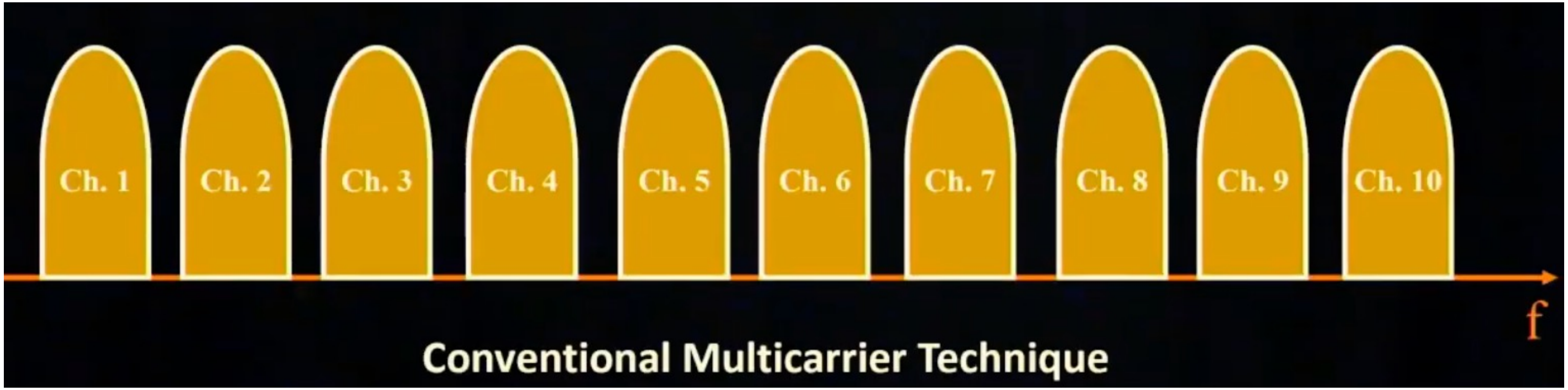
Receiver-side: how general FDM demultiplexing works





Receiver uses band pass filters (BFP) to filter out non relevant frequency band and modulate each to separate out each signal

In FDM it is important to have guard band in between so that it's easier to separate each channel



- Guard band: an unused part of the radio spectrum between radio bands (aka channel) to prevent interference.
- Guard band is essential for filtering and detection

Spatial frequency reuse

Reuse the same frequency band in a geographically separated area

Outline

1. Multiplexing
2. OFDM basics

Orthogonal Frequency Division Multiplexing (OFDM) employs both modulation and multiplexing

- Modulation: mapping information (bits) to changes in the carrier phase, frequency, and/or amplitude
- Frequency Multiplexing: method of sharing a frequency bandwidth among independent data channels

OFDM is a special case of FDM

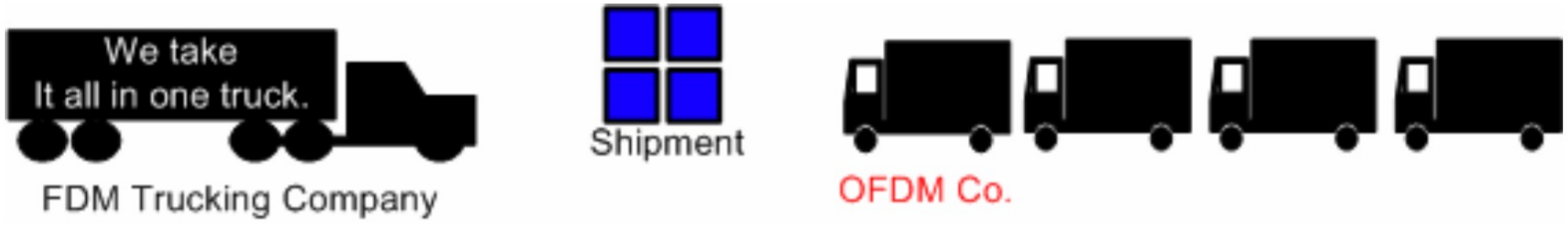
- OFDM serves single user not multiple users sharing
- OFDM "splits" one main signal into independent sub-signals to do multiplexing
- Sub-signals are orthogonal to each other

OFDM is popular and powerful technology

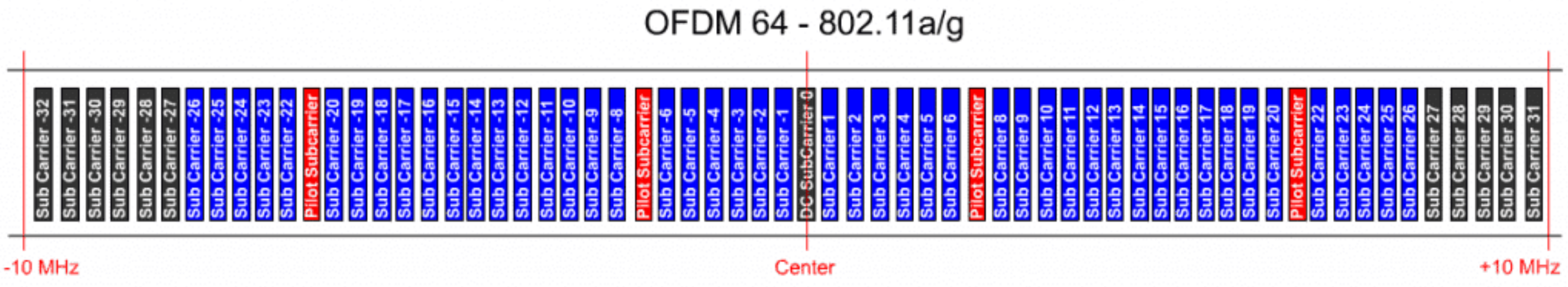
- **Used by a wide variety of systems**
  - Cellular systems (3GLTE, WiMAX),
  - Wireless local area networks (LANs)
  - digital audio radio
  - underwater communications
  - optical light modulation
- **Why so great?**
  - Greater spectral efficiency
  - Fights against inter-symbol interference
  - Resilient to multi-path distortion

Which option do you prefer?

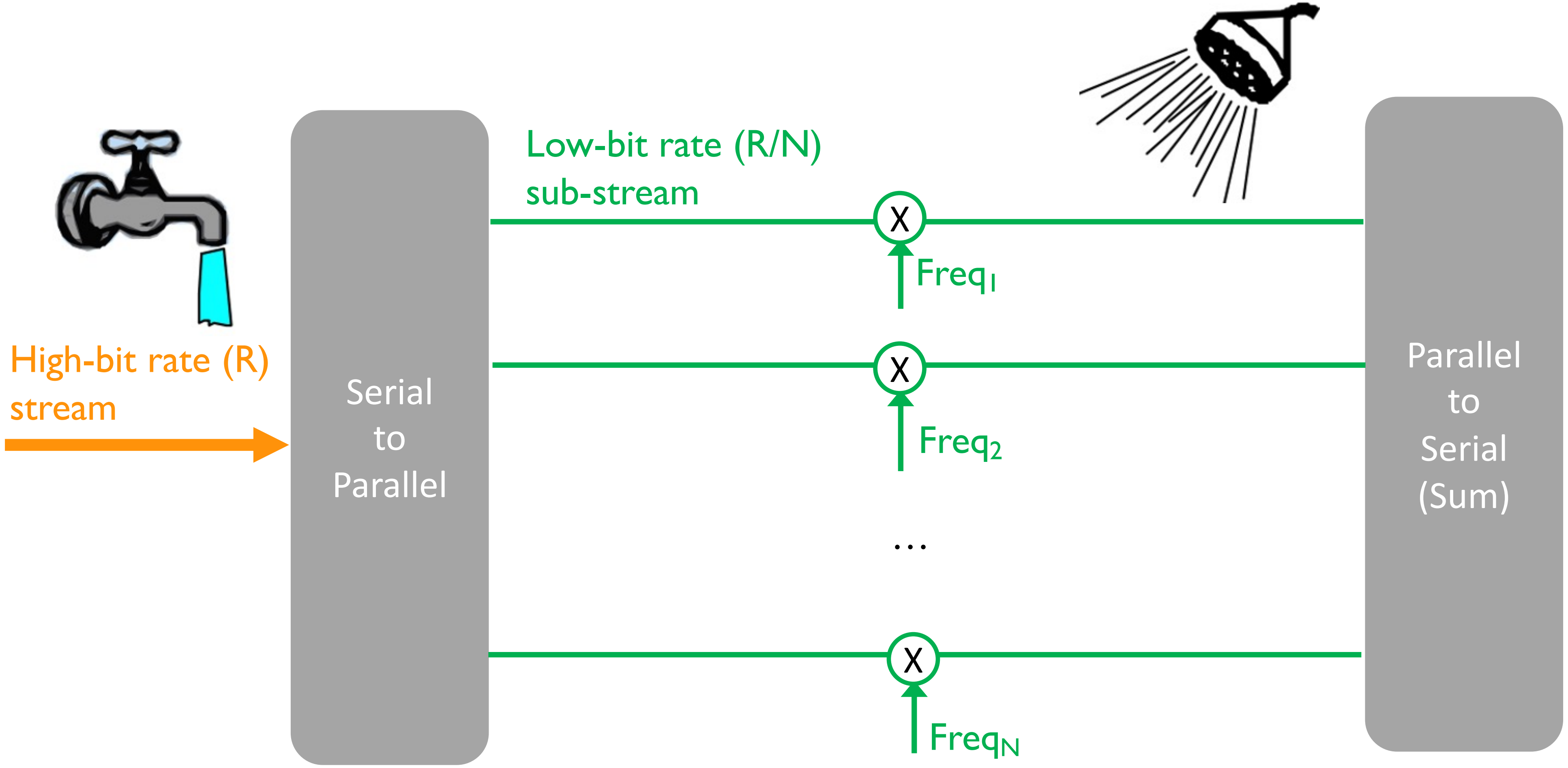
(a) (b)



OFDM divides a frequency band into sub-band called subcarriers

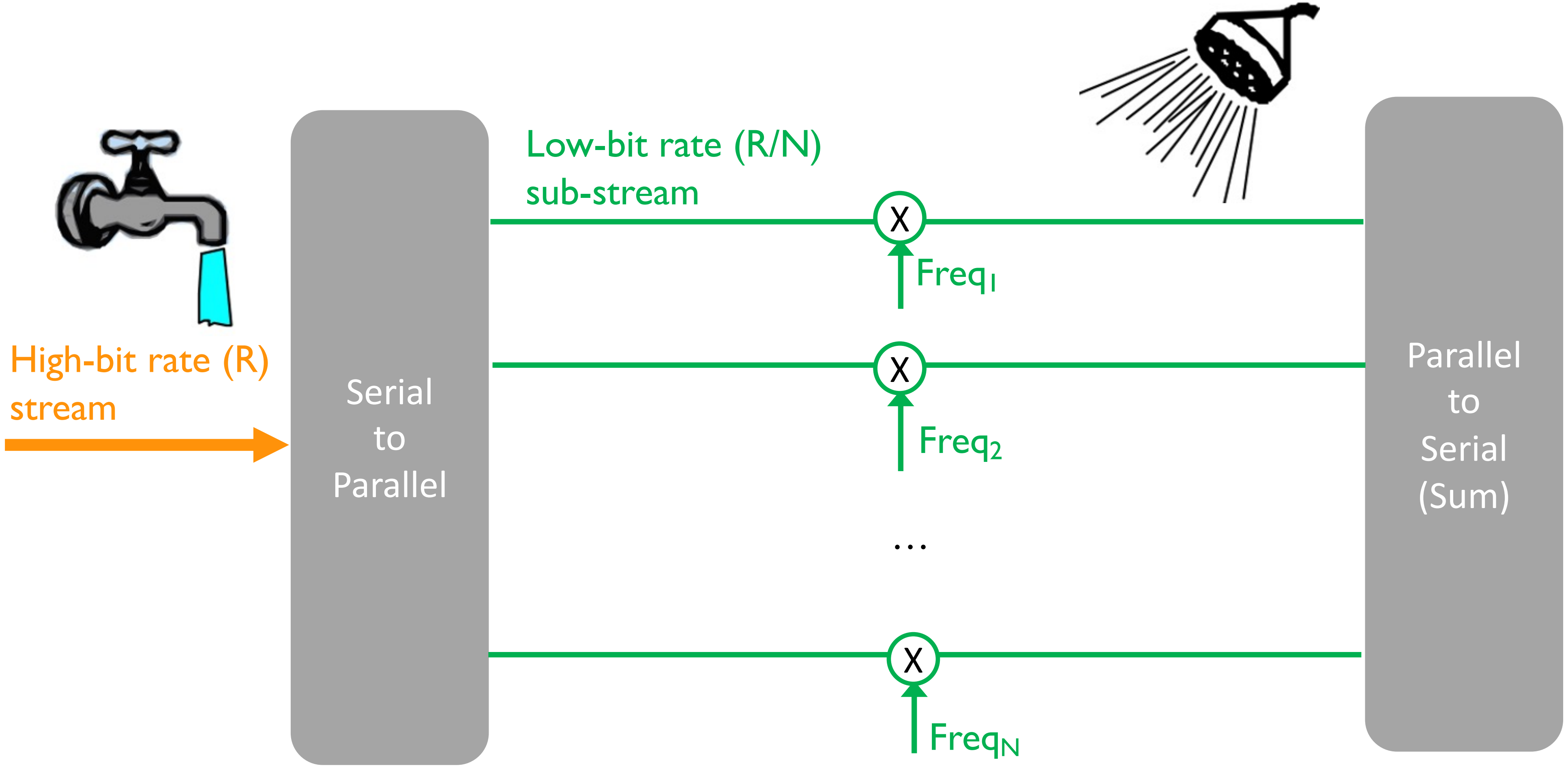


OFDM also splits user data stream into several sub-streams where each sub-streams sent in parallel on each subcarrier



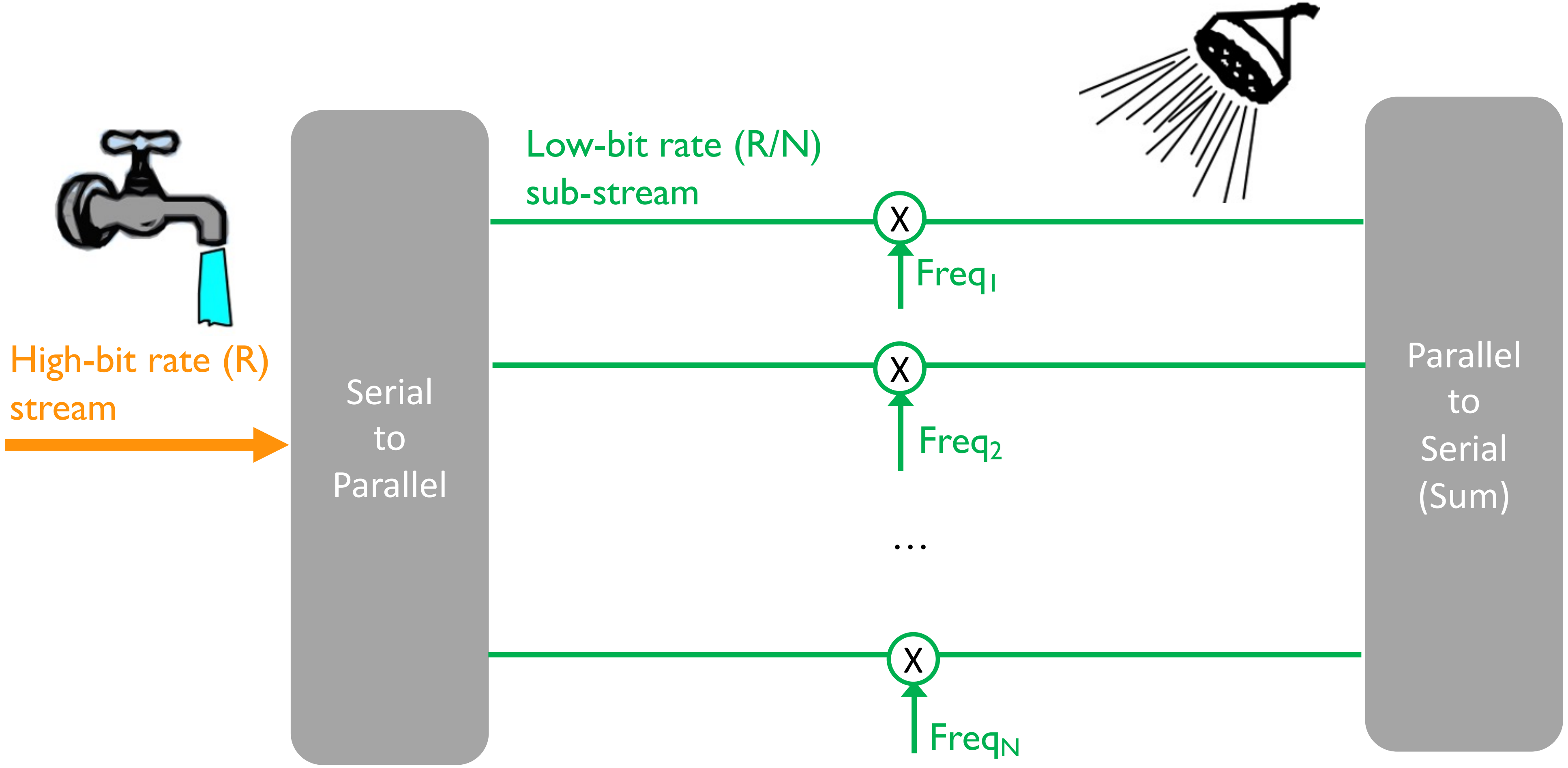
Each sub-stream of data is independent

OFDM also splits user data stream into several sub-streams where each sub-streams sent in parallel on each subcarrier



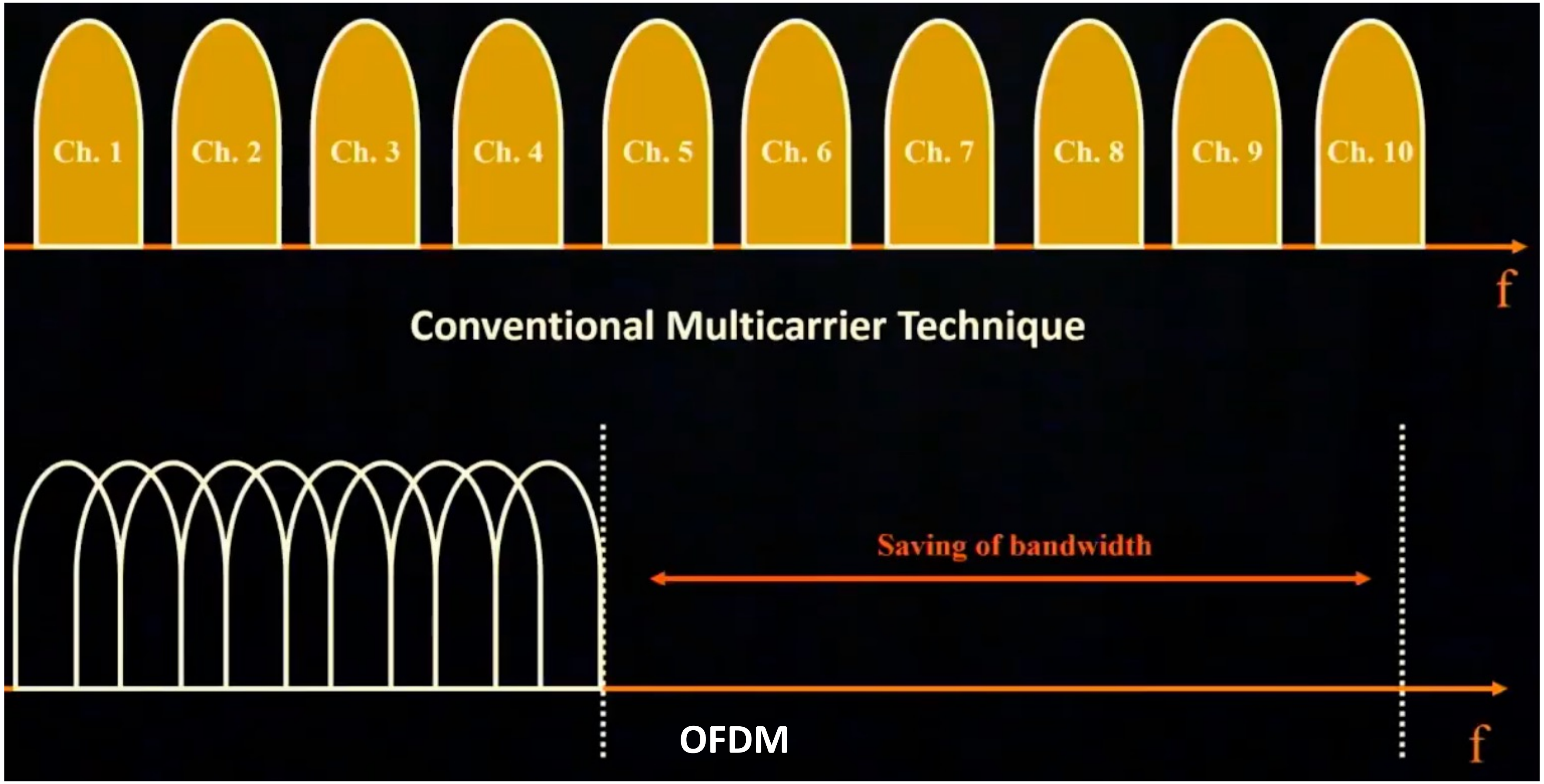
Having lower bit rate in each carrier makes it less susceptible to frequency selective fading

OFDM also splits user data stream into several sub-streams where each sub-streams sent in parallel on each subcarrier



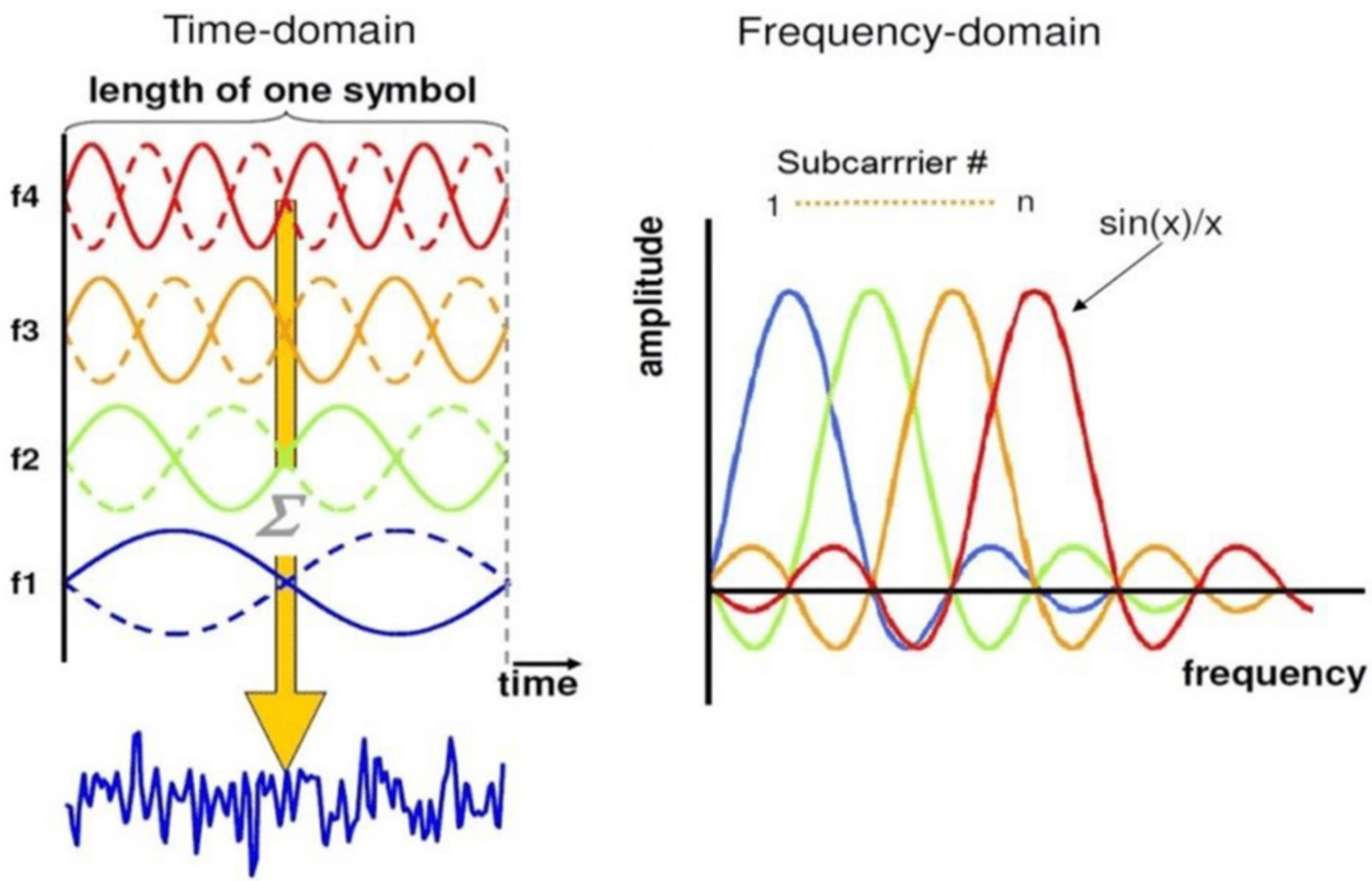
For modulating each sub-stream
BPSK, QPSK, 16QAM, 256 QAM can be used

FDM vs OFDM frequency band usage



In OFDM we place each subcarrier as tightly spaced as possible resulting in much higher spectrum efficiency

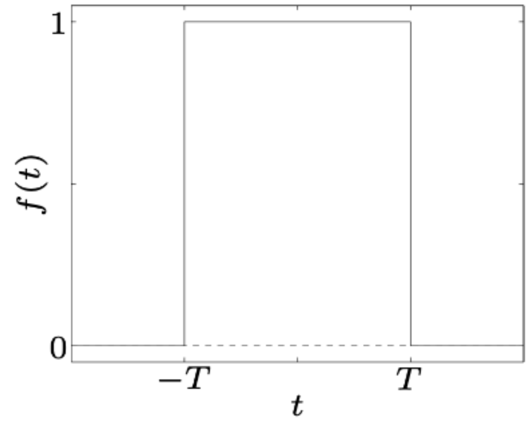
OFDM spectrum



Why each carrier is shaped sinc function?

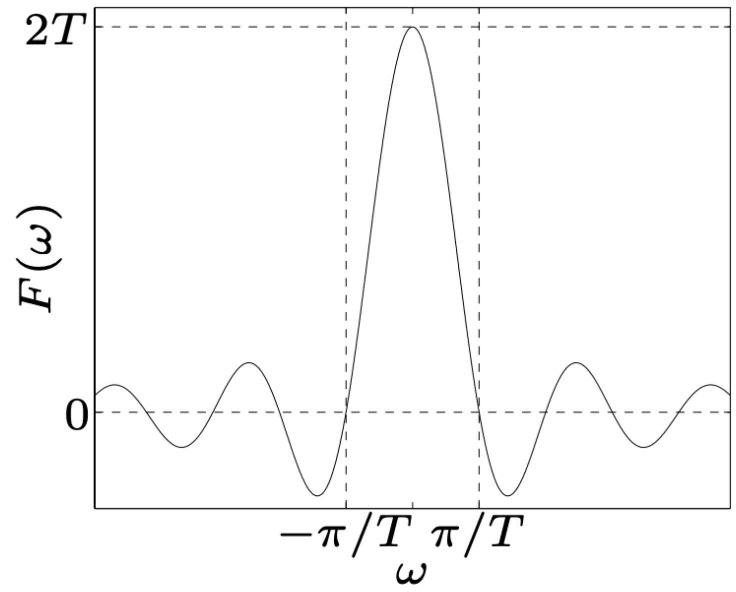
Fourier transform of rectangular pulse is a sinc function

rectangular pulse: $f(t) = \begin{cases} 1 & -T \le t \le T \\ 0 & |t| > T \end{cases}$

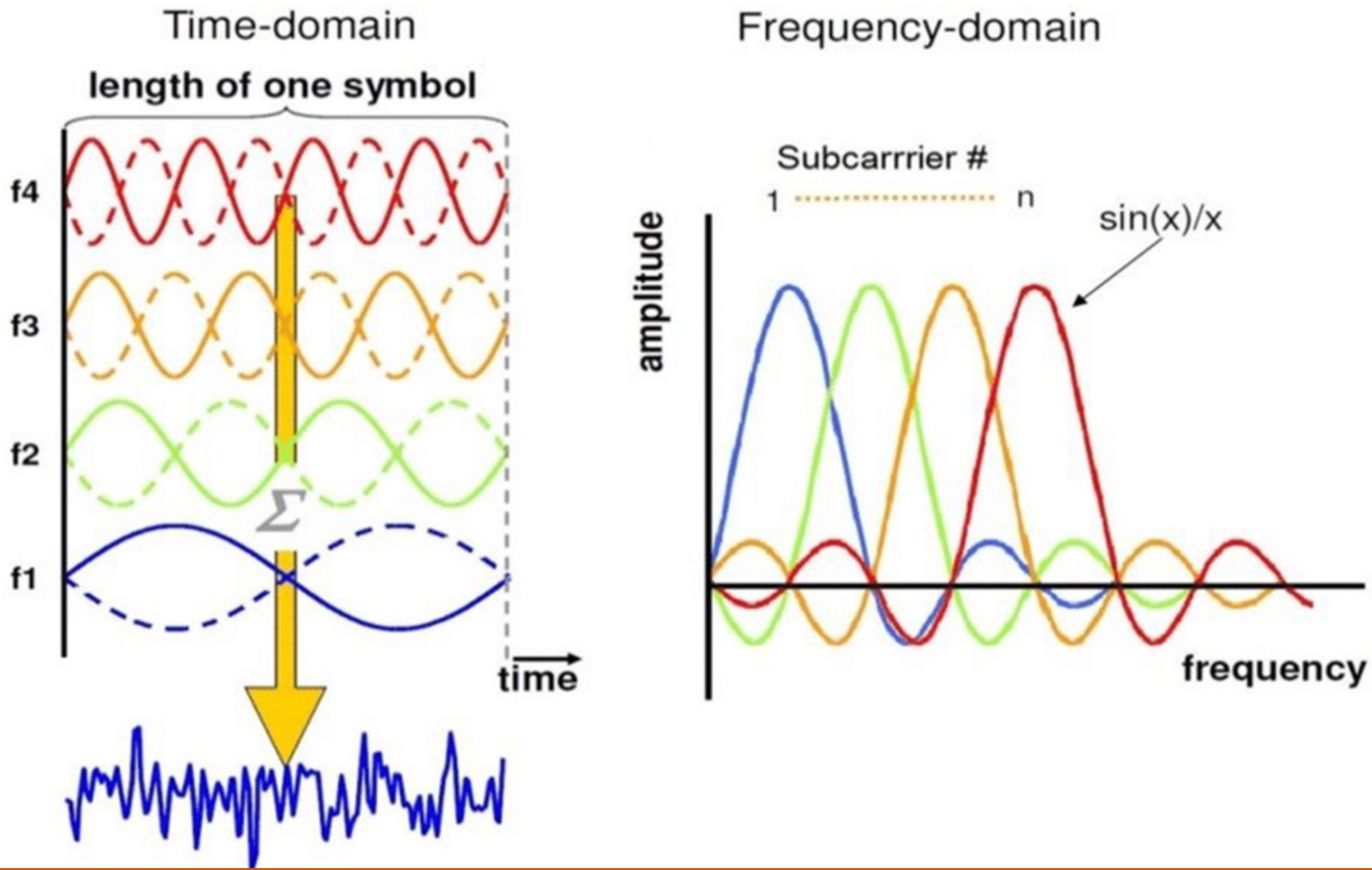


$$F(\omega) = \int_{-T}^{T} e^{-j\omega t}\, dt = \frac{-1}{j\omega}\left(e^{-j\omega T} - e^{j\omega T}\right) = \frac{2\sin\omega T}{\omega}$$

where $\omega = k\,\pi/T$

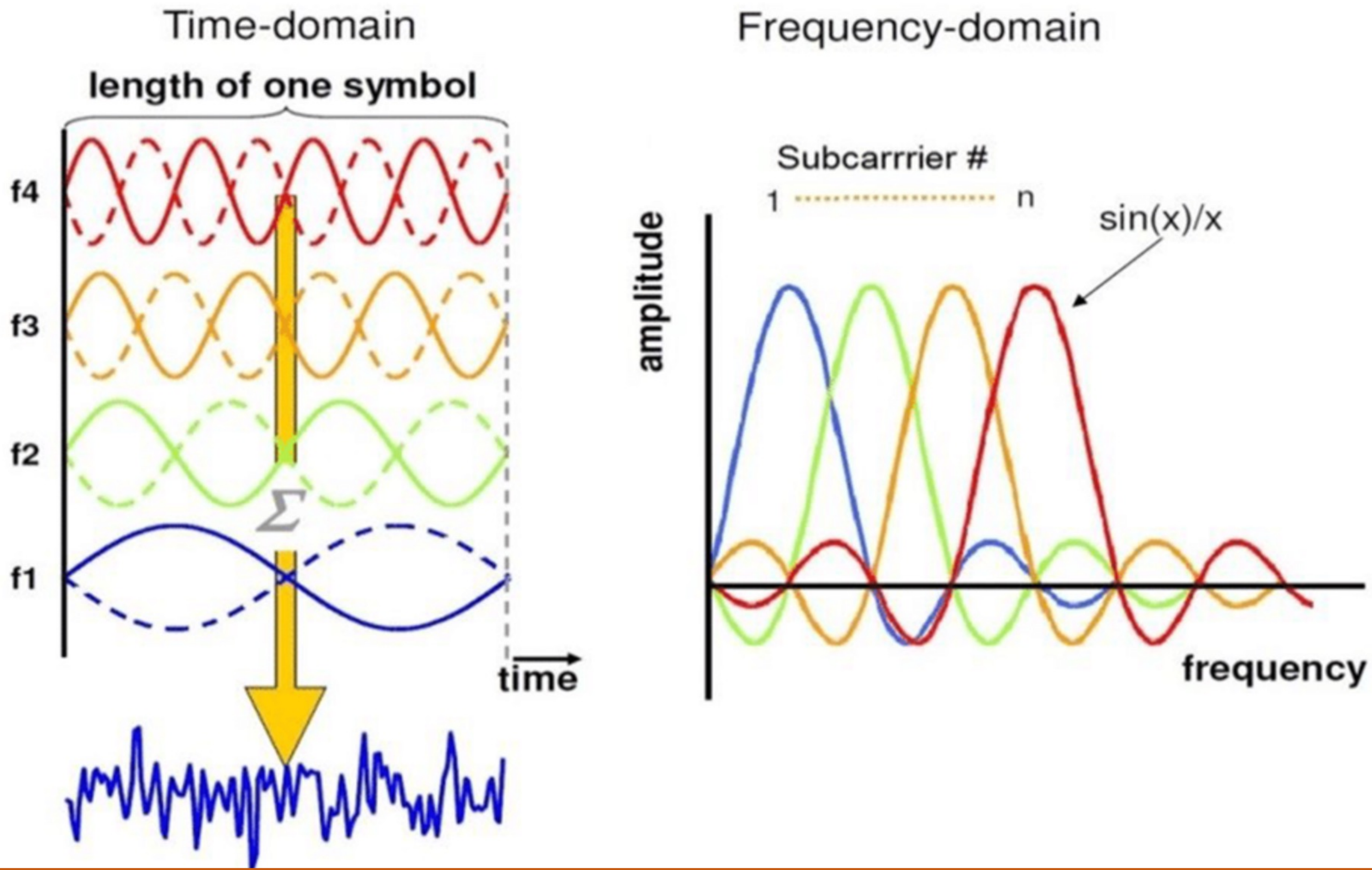


OFDM spectrum



Each subcarriers must be orthogonal to one another

OFDM spectrum



Where/when have we seen bunch of orthogonal signals?

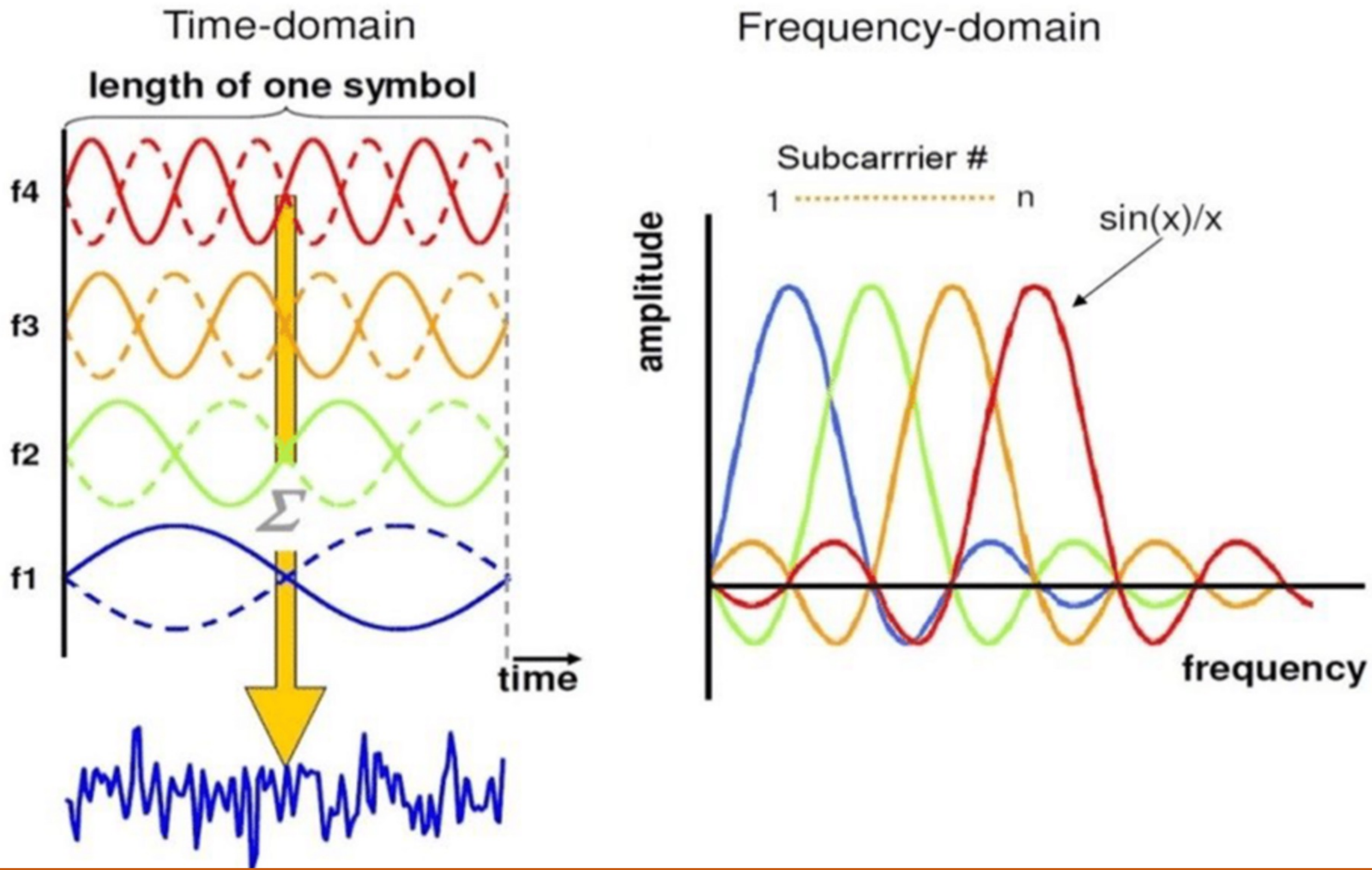
Fourier series!

- When f(t) is 2 π periodic, $f(t) = \sum_{k=-\infty}^{\infty} c_k \; e^{ikt}$
  - Where where $c_k = \langle\langle f(t), e^{ikt} \rangle\rangle$
- We have proved each e^{ikt} is orthogonal to one another
  - How? by showing their inner product is 0

$$\int_{-\pi}^{\pi} e^{ijx} e^{-ikx} dx = \int_{-\pi}^{\pi} e^{i(j-k)x} dx = \left[\frac{e^{i(j-k)x}}{i(j-k)} \right]_{-\pi}^{\pi} = 0 \quad \text{(when } j \neq k\text{)}$$

e^{iwt} are orthogonal to one another where $w = k\,\pi/L$

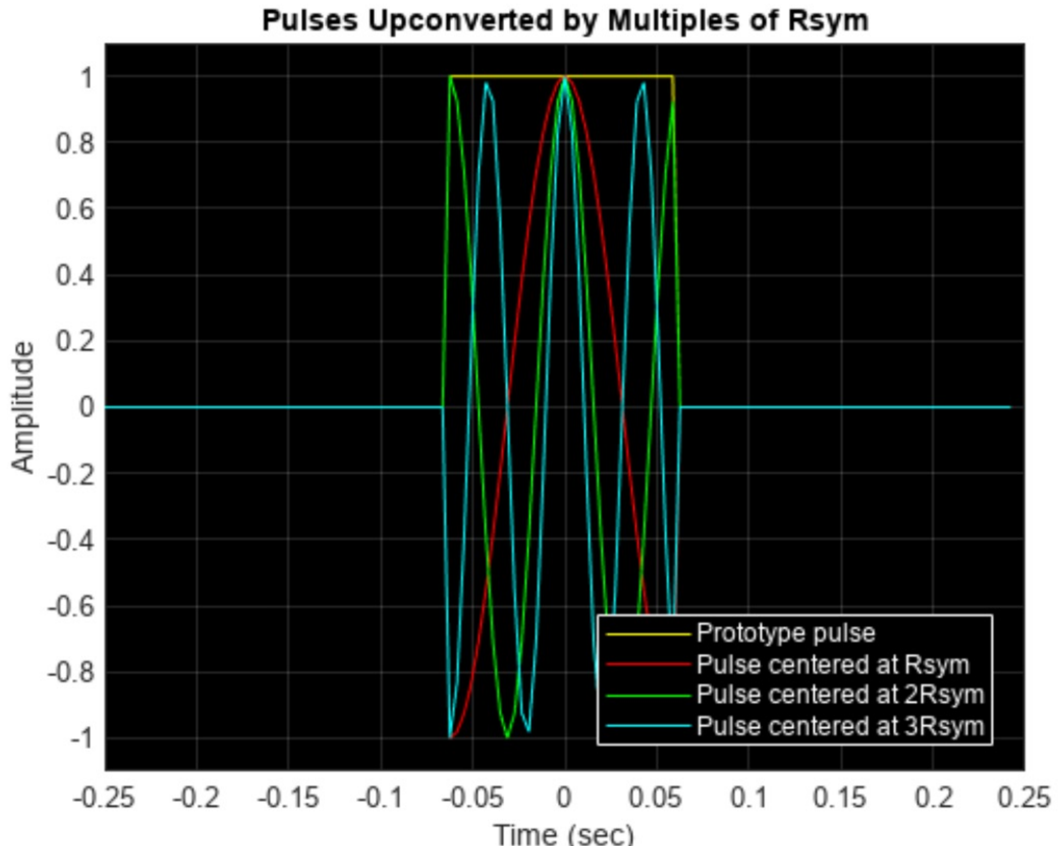
OFDM spectrum

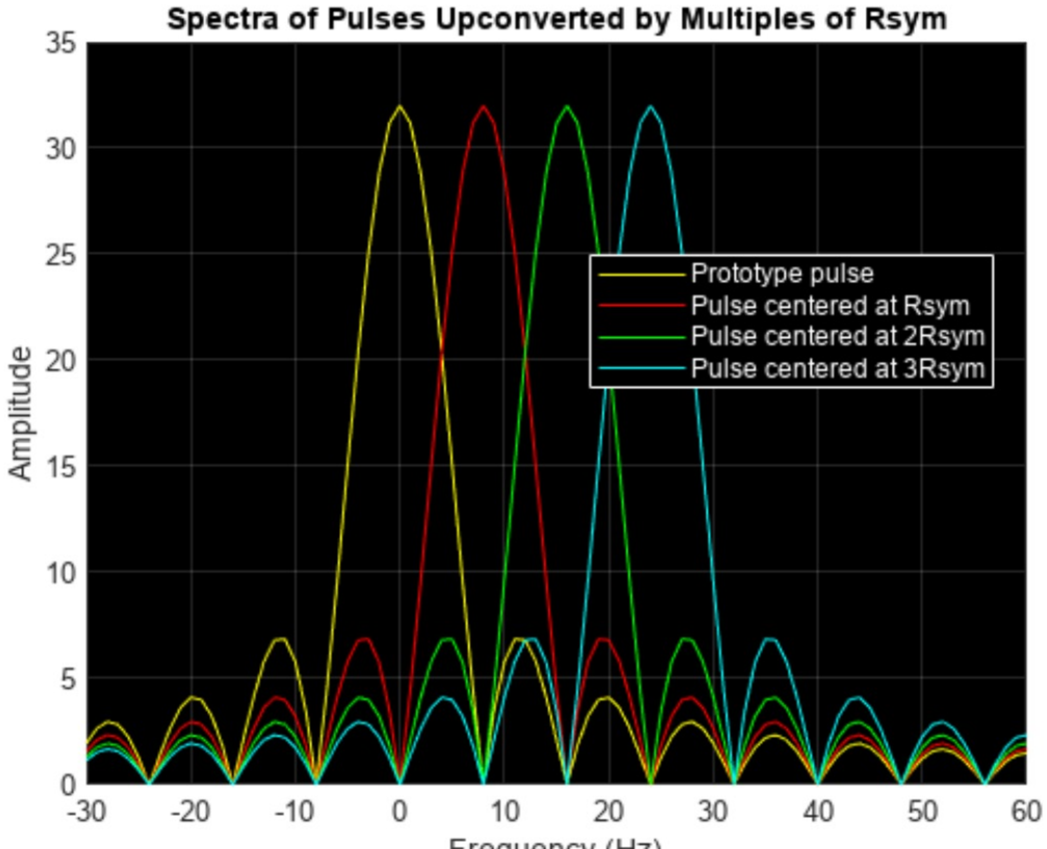


f1 is the fundamental frequency and f2, f3, f4 are its harmonics

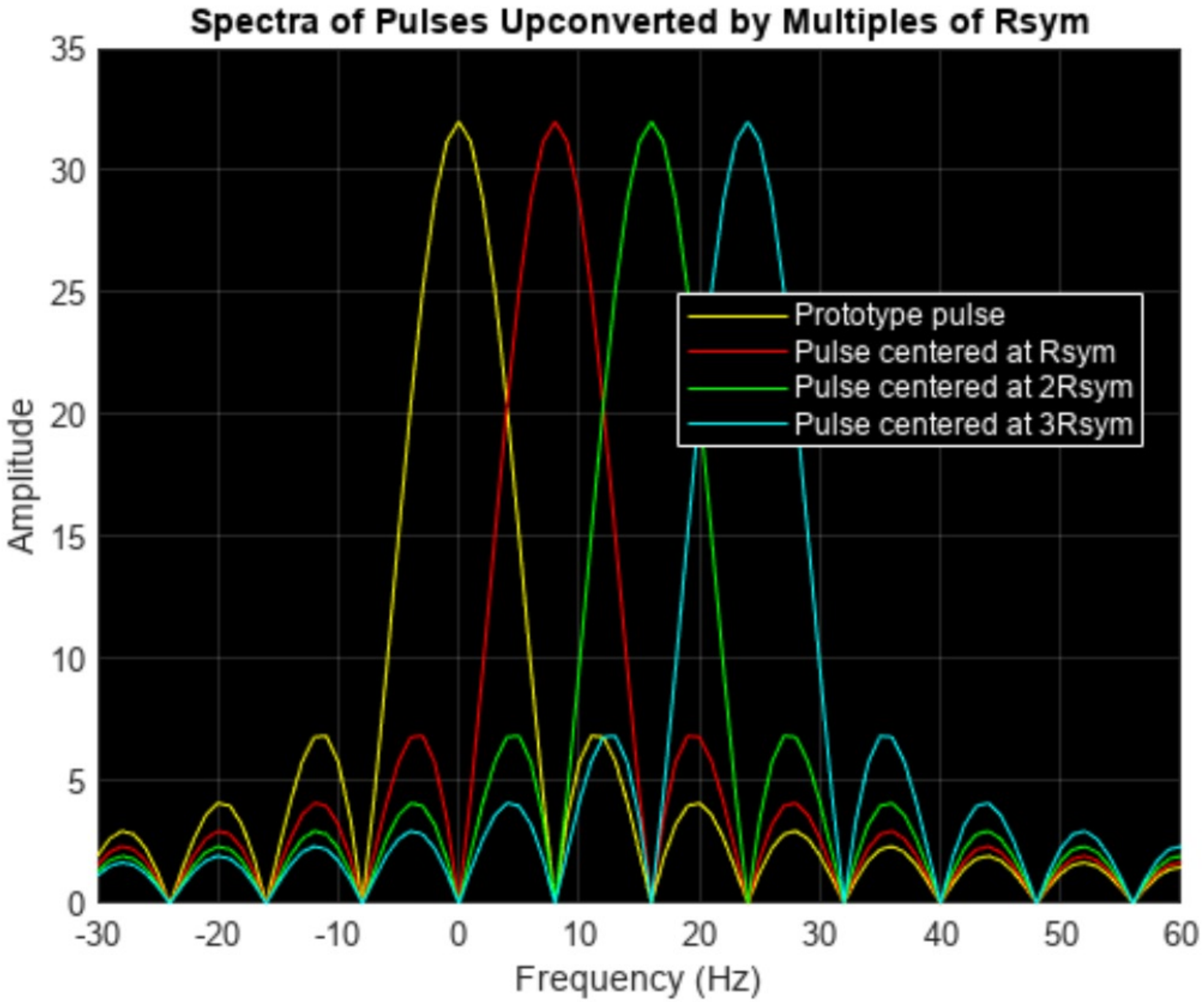
Matlab example

- Symbol duration T_{sym} = 0.125 sec
  - From -0.0625 till +0.0625
- Symbol rate $R_{sym} = 1/T_{sym}$
  - 1 R_{sym} = 8 Hz (fundamental frequency)
  - 2 R_{sym} = 16 Hz
  - 3 R_{sym} = 24 Hz
  - 4 R_{sym} = 32 Hz





Orthogonality of OFDM

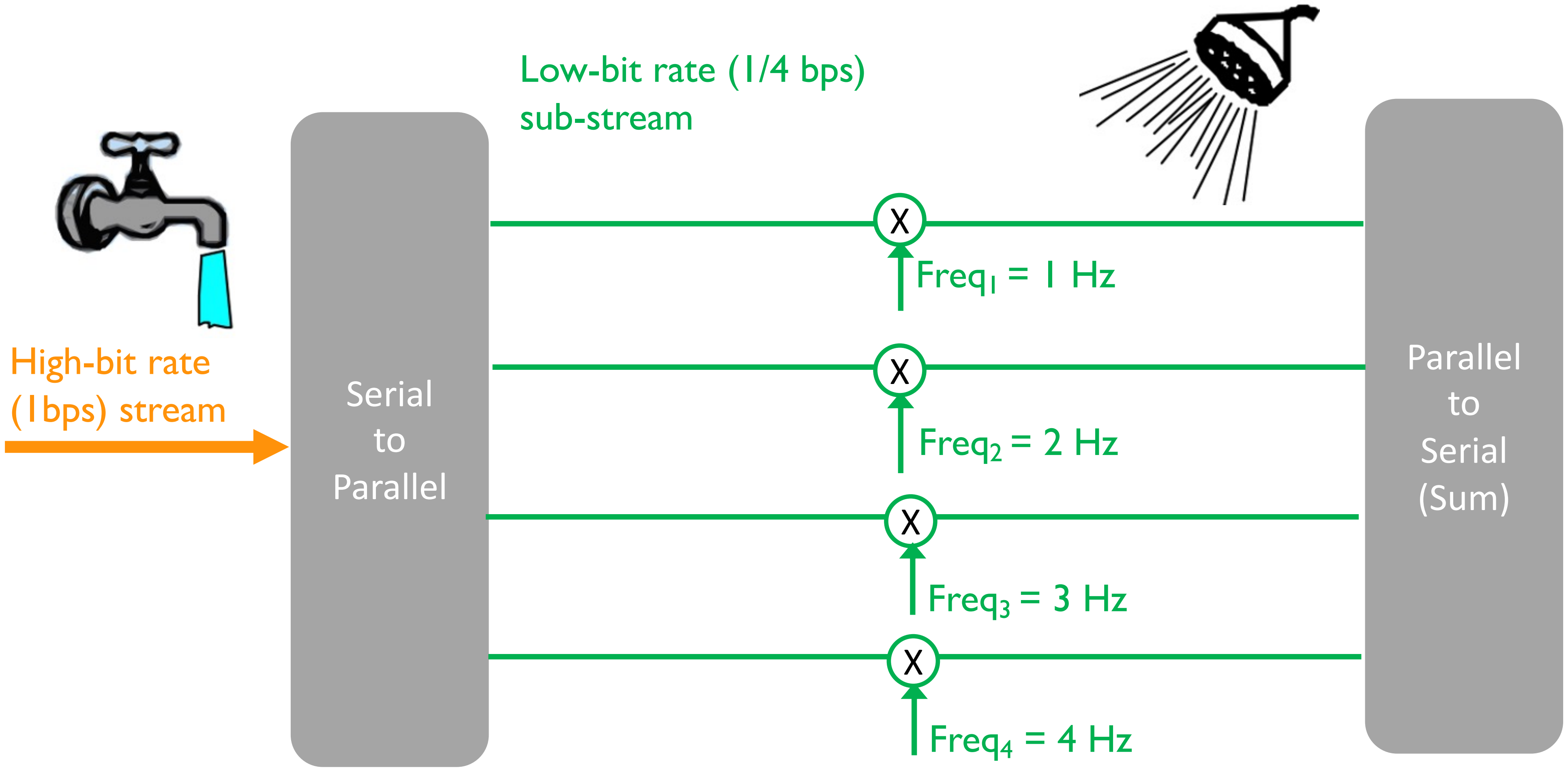


spectral peaks of each subcarrier occurs
at the zero crossings of all the other pulses!

Outline

1. Multiplexing
2. OFDM basics
3. OFDM sender step-by-step

OFDM sender



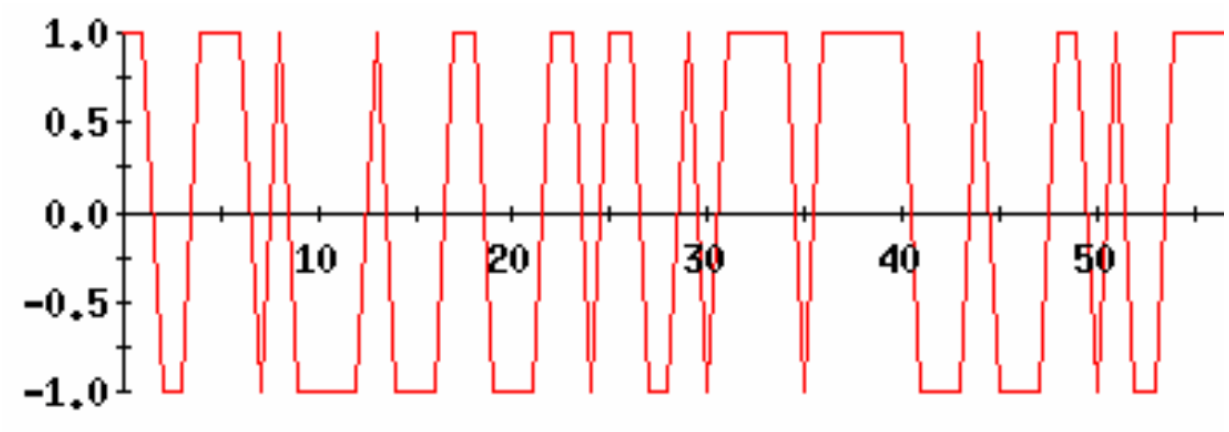
Let’s keep this picture in mind

Simple OFDM scenario

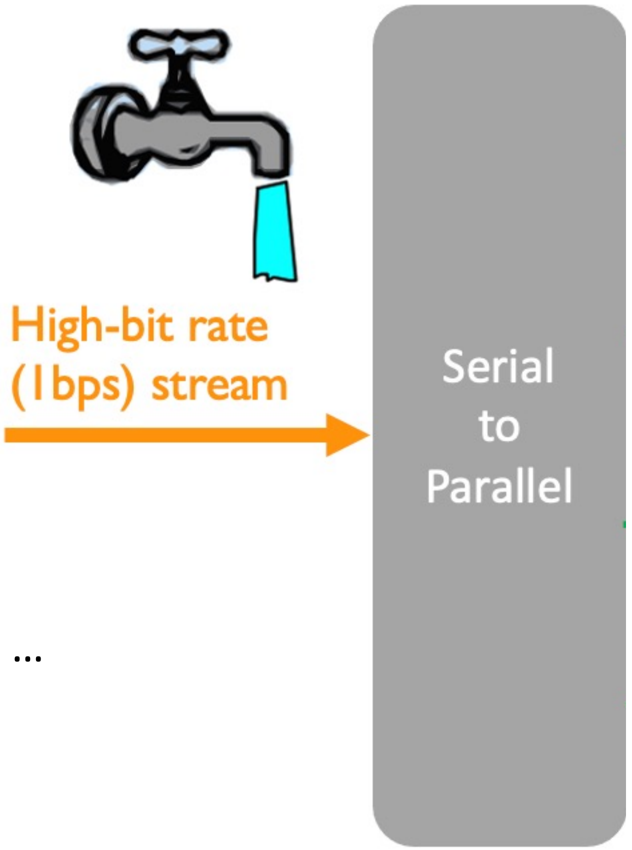
- 4 carriers
- Use BPSK for each subcarrier
- Original signal has a symbol rate of 1 symbol/sec
- Sampling rate 1 Hz (1 sample/sec)
- 1 symbol/sec for each subcarrier

Step 1: Let's have original bit stream

- Original bit stream



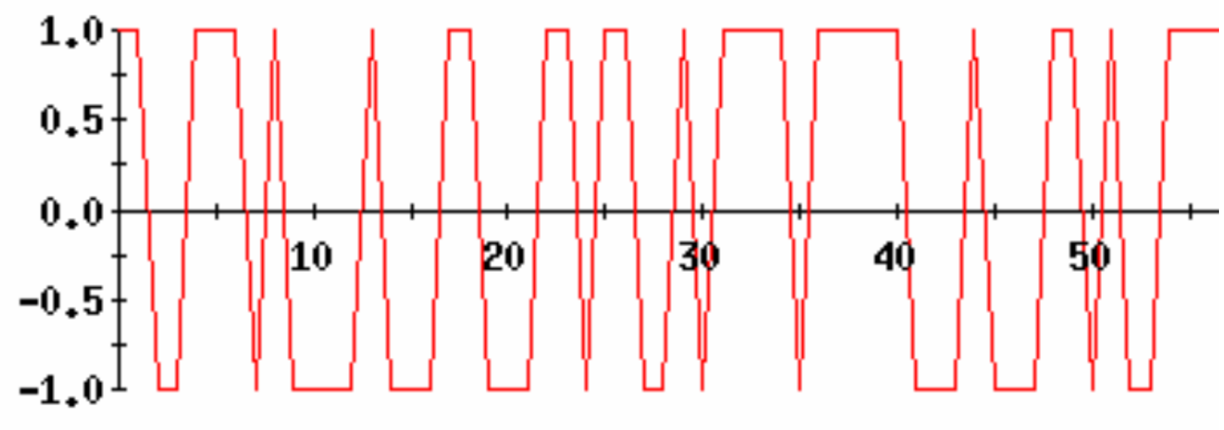
1 1 -1 -1 1 1 1 -1 1 -1 -1 -1 -1 1 -1 -1 -1 1 …



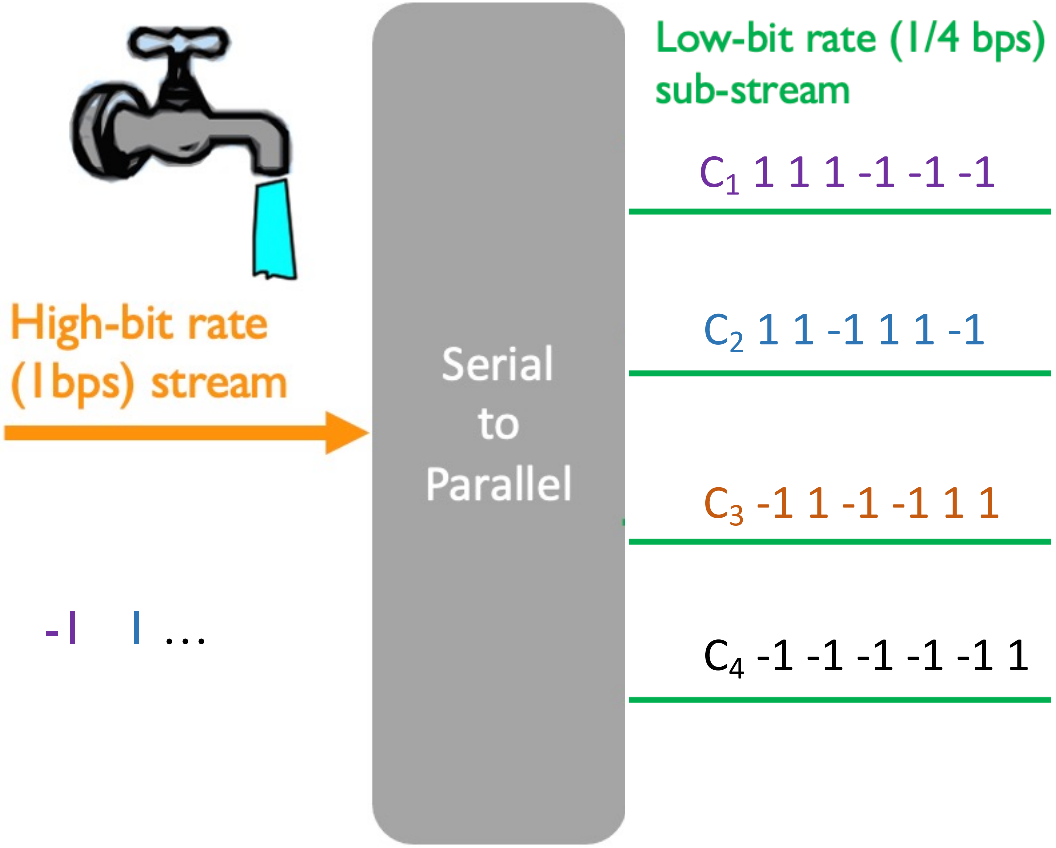
Each transition is a bit

Step 2: Let’s divide it up into 4 sub-stream

- Original bit stream



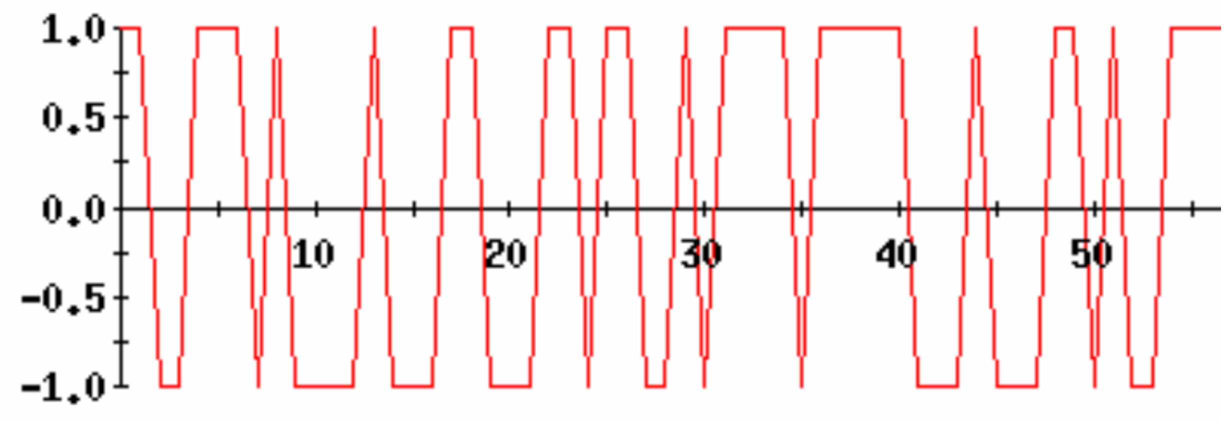
1 1 -1 -1 1 1 1 -1 1 -1 -1 -1 -1 1 -1 -1 -1 1 …



Note symbol rate is 1 symbol per sec over all 4 carriers thus, each carrier has symbol rate of ¼ symbol per sec

Step 2: Let's divide it up into 4 sub-stream

- Original bit stream



1 1 -1 -1 1 1 1 -1 1 -1 -1 -1 -1 1 -1 -1 -1 1 …

High-bit rate (1bps) stream

Serial to Parallel

Low-bit rate (1/4 bps) sub-stream

C_1 1 1 1 -1 -1 -1

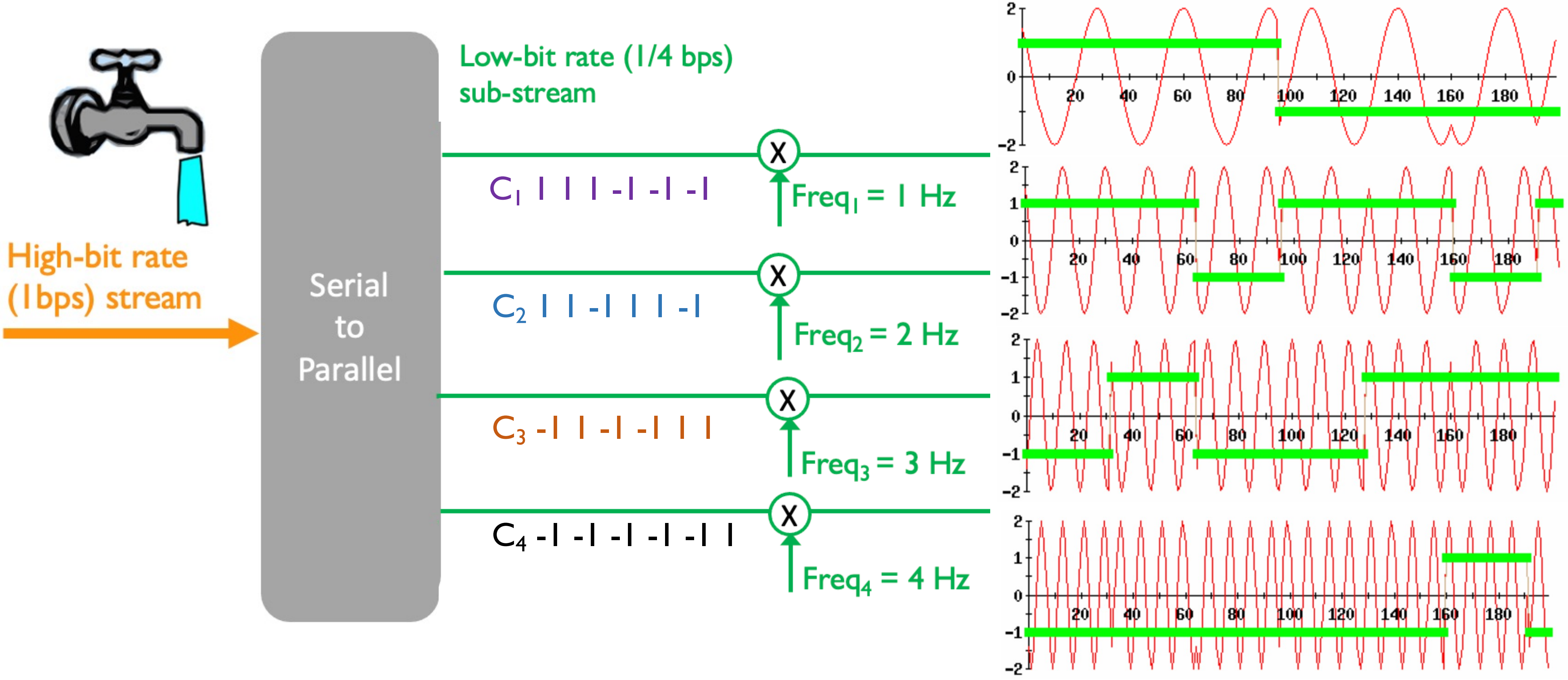
C_2 1 1 -1 1 1 -1

C_3 -1 1 -1 -1 1 1

C_4 -1 -1 -1 -1 -1 1

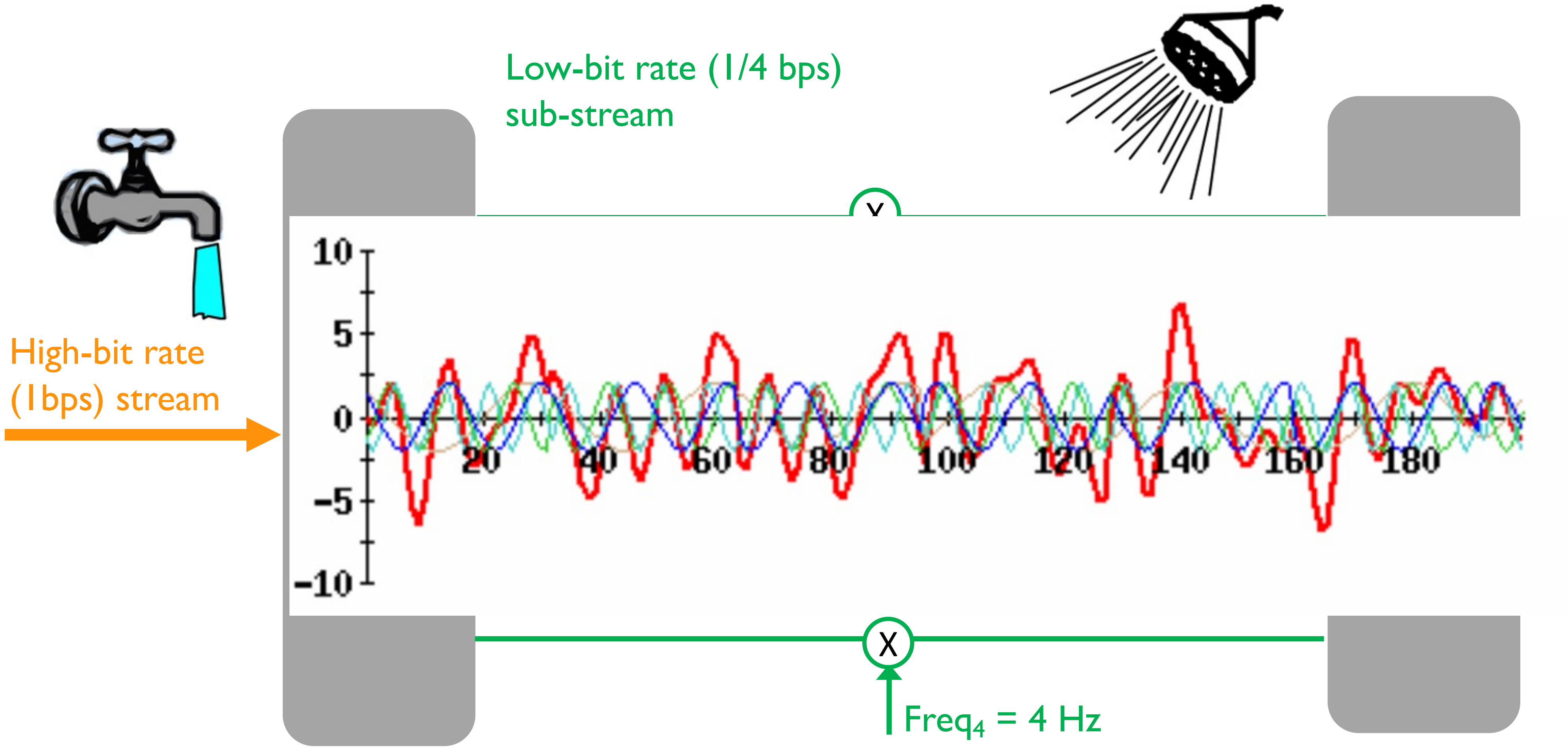
Given 1 bits/symbol for BPSK,
1 bps data rate is divided into ¼ bps to each sub-stream

Step 3: BPSK modulation happens at each subcarrier



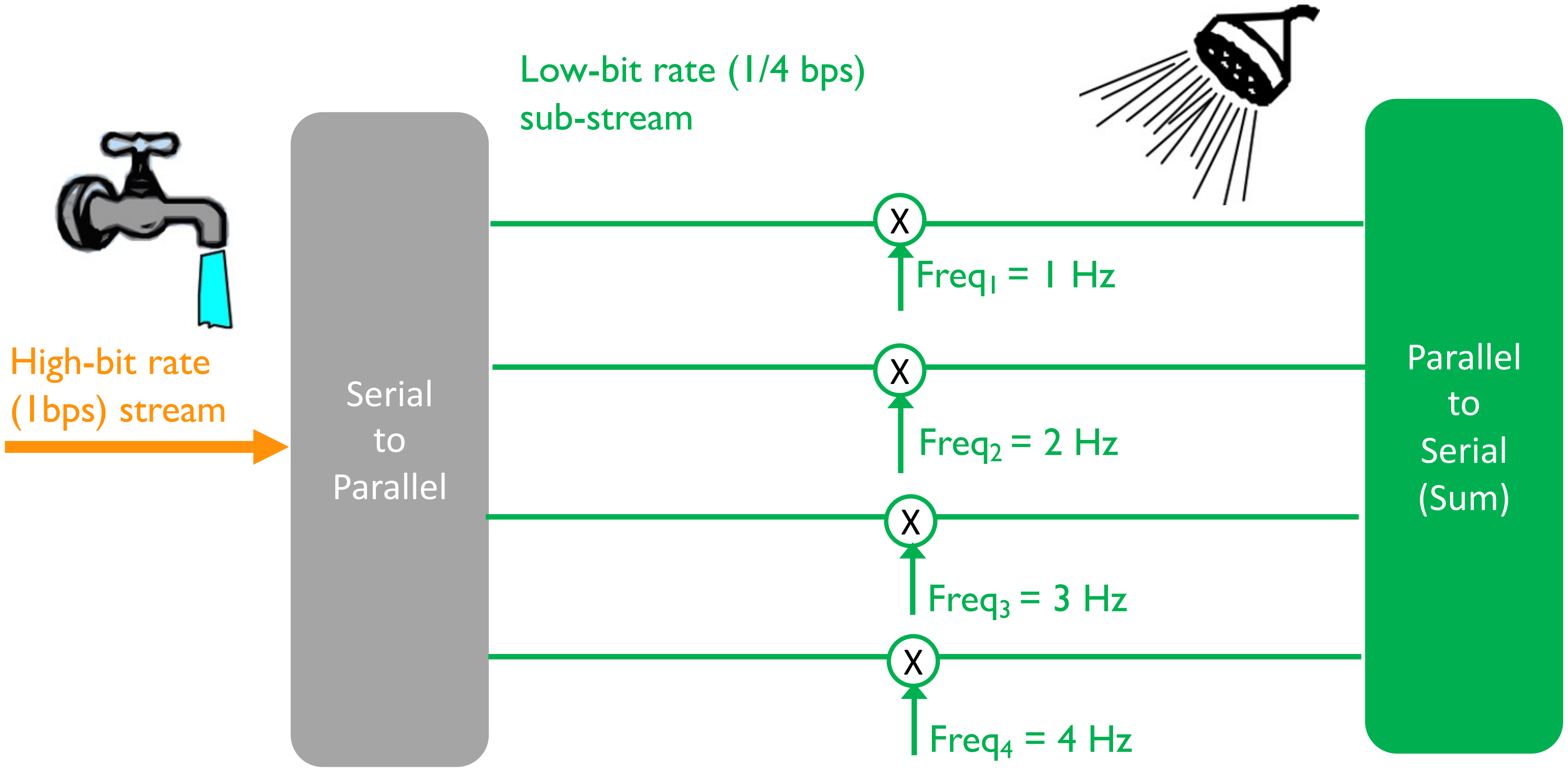
Each sub carrier frequencies are picked
(1, 2, 3, 4 Hz picked for simplicity)

Step 4: Add up the symbols across all subcarriers



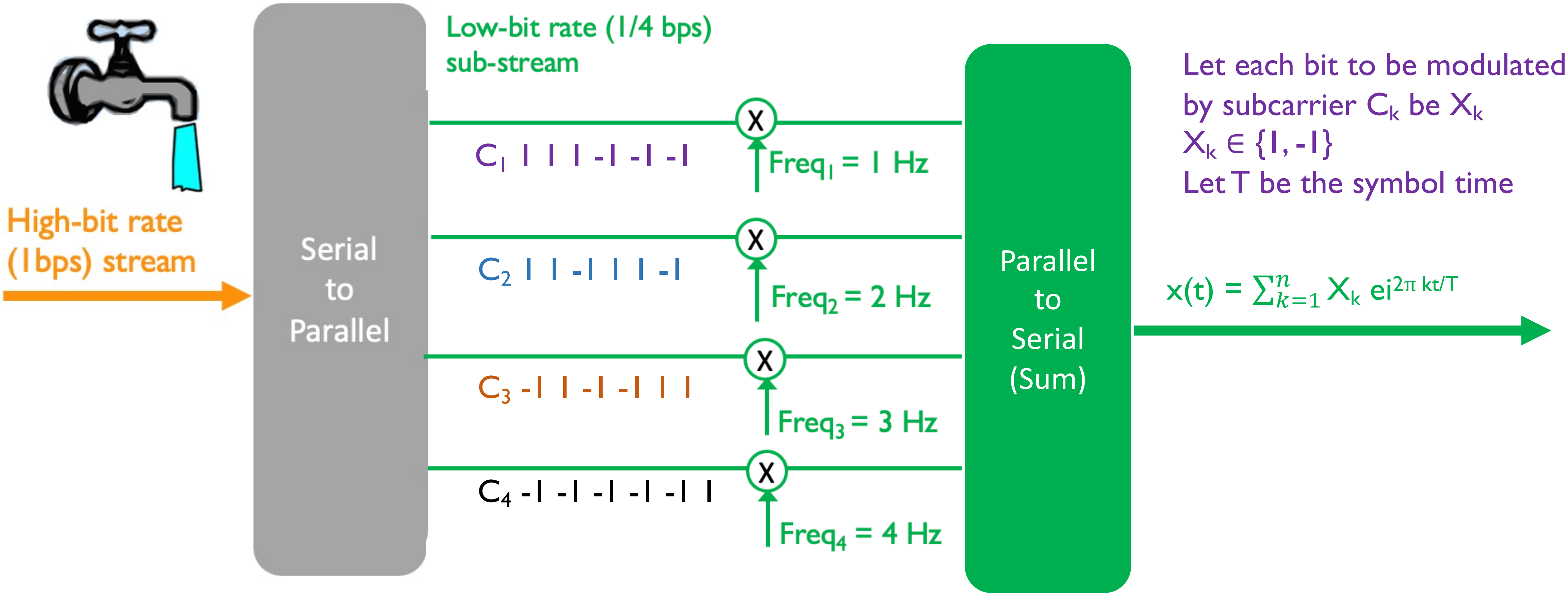
What would the equation look like?

Step 4: Add up the symbols across all subcarriers



What would the equation look like?

Step 4: What is this summation process?



Let each bit to be modulated by subcarrier C_k be X_k

$X_k \in \{1, -1\}$

Let T be the symbol time

Does $\sum_{k=1}^{n} X_k e^{i2\pi kt/T}$ look familiar?

Recap: DFT

$$F[n] = \sum_{k=0}^{N-1} f[k] e^{-j\frac{2\pi}{N}nk} \qquad (n = 0 : N-1)$$

- N is the number of discrete samples taken over one period
- k is the index for time domain samples
- n is the index of frequency bins (aka freq bin number)

freq-domain N of DTFT   NxN DFT Matrix   time-domain N samples of f(t)

$$\begin{pmatrix} F[0] \\ F[1] \\ F[2] \\ \vdots \\ F[N-1] \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & \dots & 1 \\ 1 & W & W^2 & W^3 & \dots & W^{N-1} \\ 1 & W^2 & W^4 & W^6 & \dots & W^{N-2} \\ 1 & W^3 & W^6 & W^9 & \dots & W^{N-3} \\ \vdots & & & & & \\ 1 & W^{N-1} & W^{N-2} & W^{N-3} & \dots & W \end{pmatrix} \begin{pmatrix} f[0] \\ f[1] \\ f[2] \\ \vdots \\ f[N-1] \end{pmatrix}$$

where $W = \exp(-j2\pi/N)$

Recap: IDFT

- The inverse of $F[n] = \sum_{k=0}^{N-1} f[k] e^{-j\frac{2\pi}{N}nk}$ is $f[k] = \frac{1}{N} \sum_{n=0}^{N-1} F[n] e^{+j\frac{2\pi}{N}nk}$

time-domain
N samples of f(t)

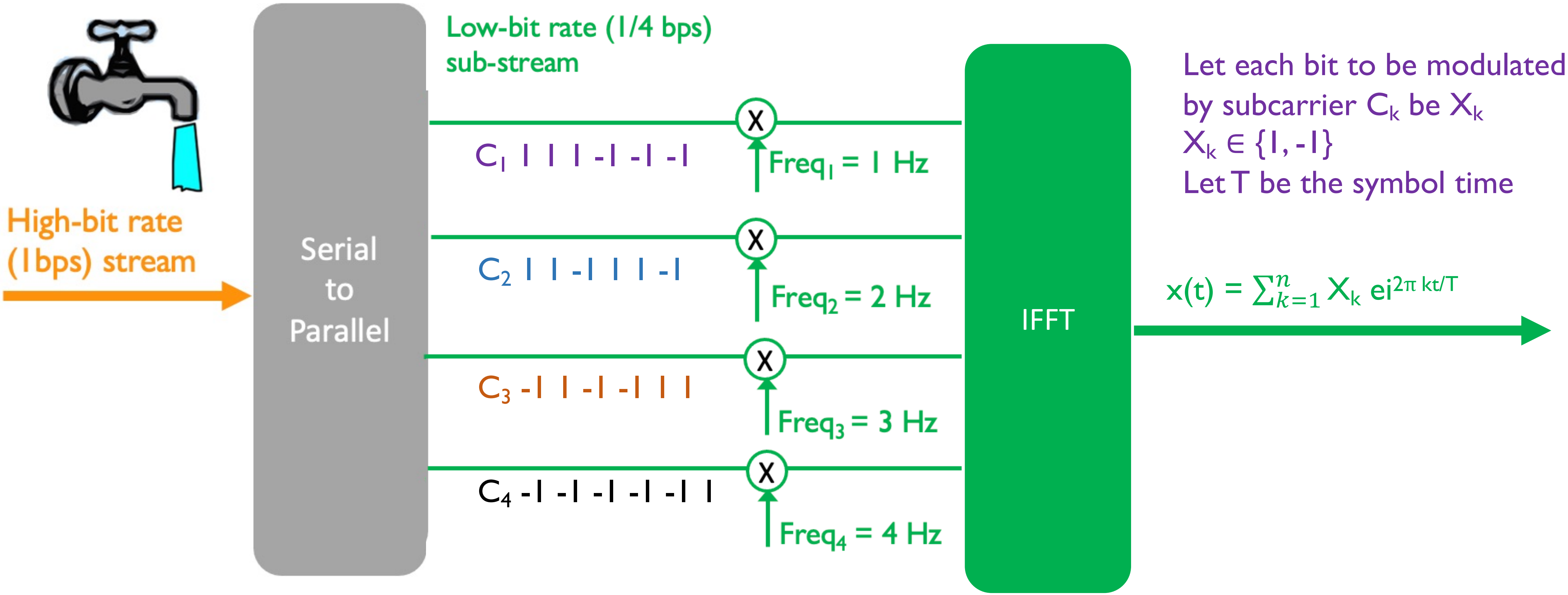
IDFT Matrix

freq-domain N of DTFT

$$\begin{pmatrix} f[0] \\ f[1] \\ f[2] \\ \vdots \\ f[N-1] \end{pmatrix} = \frac{1}{N} \begin{bmatrix} \omega^{-0\cdot 0} & \omega^{-0\cdot 1} & \omega^{-0\cdot 2} & \cdots & \omega^{-0\cdot(N-1)} \\ \omega^{-1\cdot 0} & \omega^{-1\cdot 1} & \omega^{-1\cdot 2} & \cdots & \omega^{-1\cdot(N-1)} \\ \omega^{-2\cdot 0} & \omega^{-2\cdot 1} & \omega^{-2\cdot 2} & \cdots & \omega^{-2\cdot(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \omega^{-(N-1)\cdot 0} & \omega^{-(N-1)\cdot 1} & \omega^{-(N-1)\cdot 2} & \cdots & \omega^{-(N-1)\cdot(N-1)} \end{bmatrix} \begin{pmatrix} F[0] \\ F[1] \\ F[2] \\ \vdots \\ F[N-1] \end{pmatrix}$$

where $W = \exp(-j2\pi/N)$

We can implement OFDM sender with IFFT fast



Let each bit to be modulated by subcarrier C_k be X_k

$X_k \in \{1, -1\}$

Let T be the symbol time

Since OFDM modulation is fast,
it was able to widely adapted and implemented in wireless chipset

Compare with BPSK demodulation

- Matlab bpsk_manual_example.m

Backup Slides