Lecture 03-12: Physical Layer OFDM 2

CS 356R

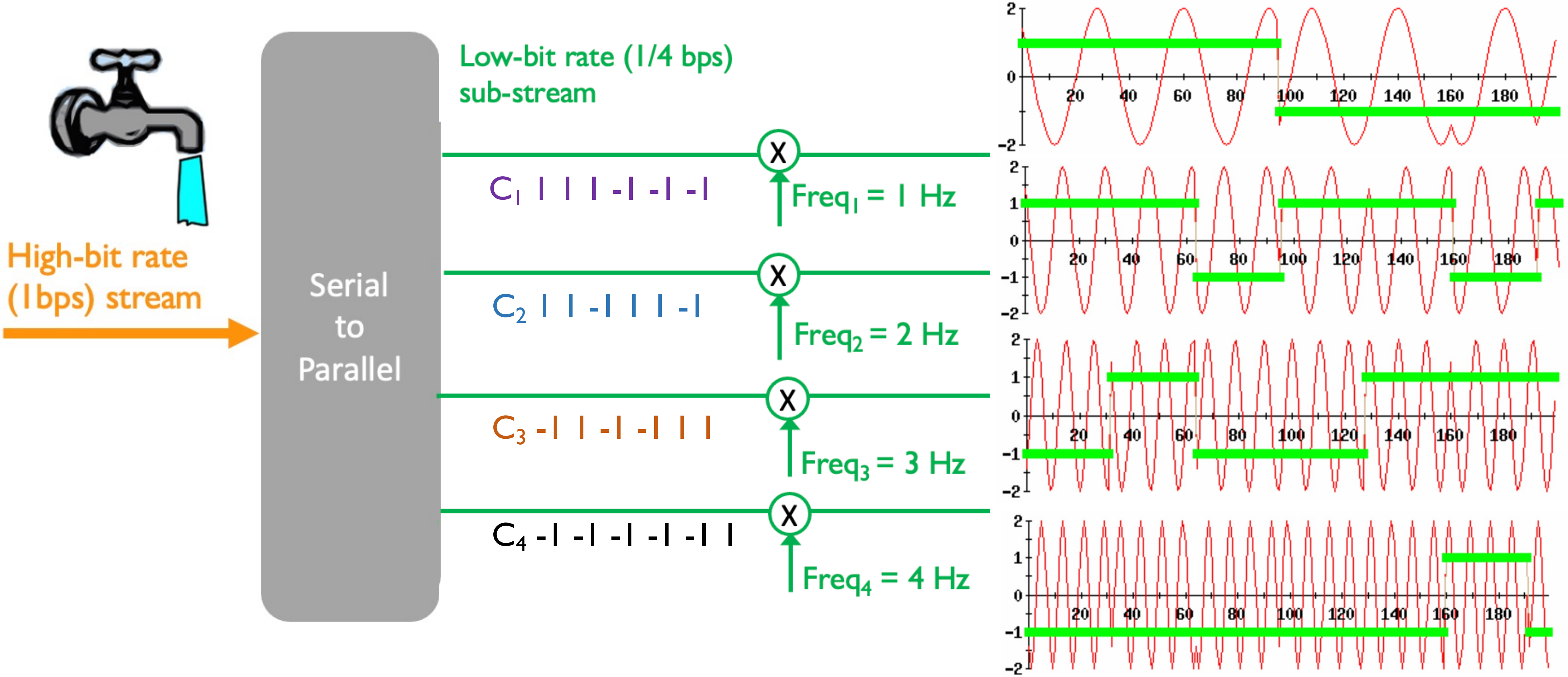
Intro to Wireless Networks

Mikyung Han

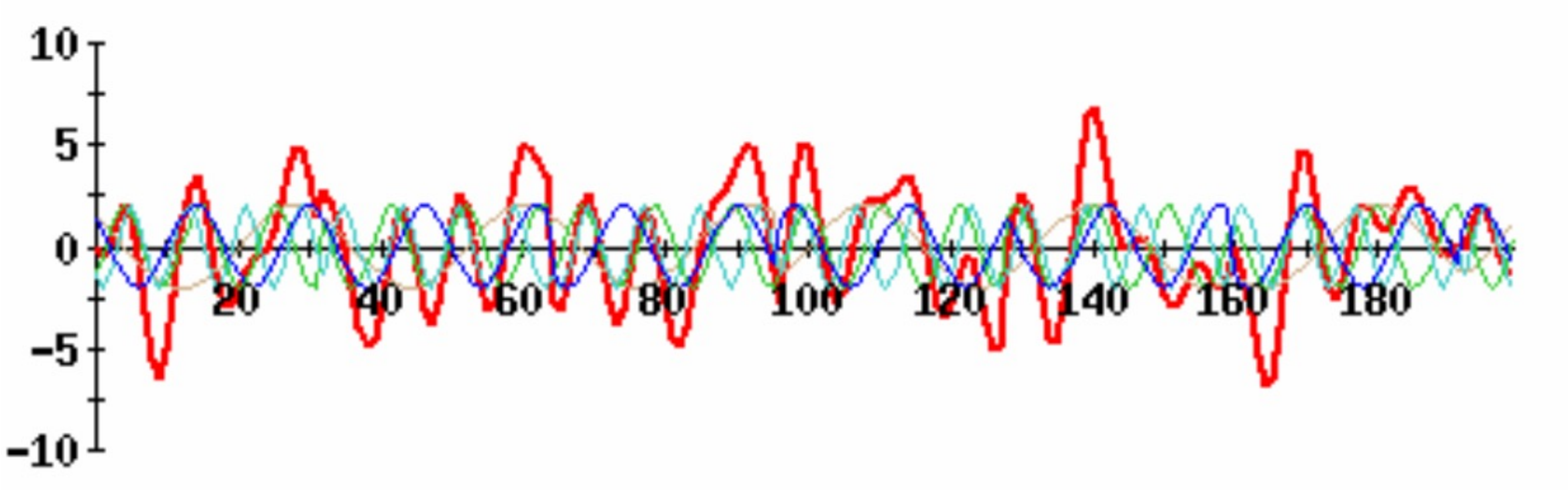
Outline

1. Recap OFDM basics

Simple OFDM example

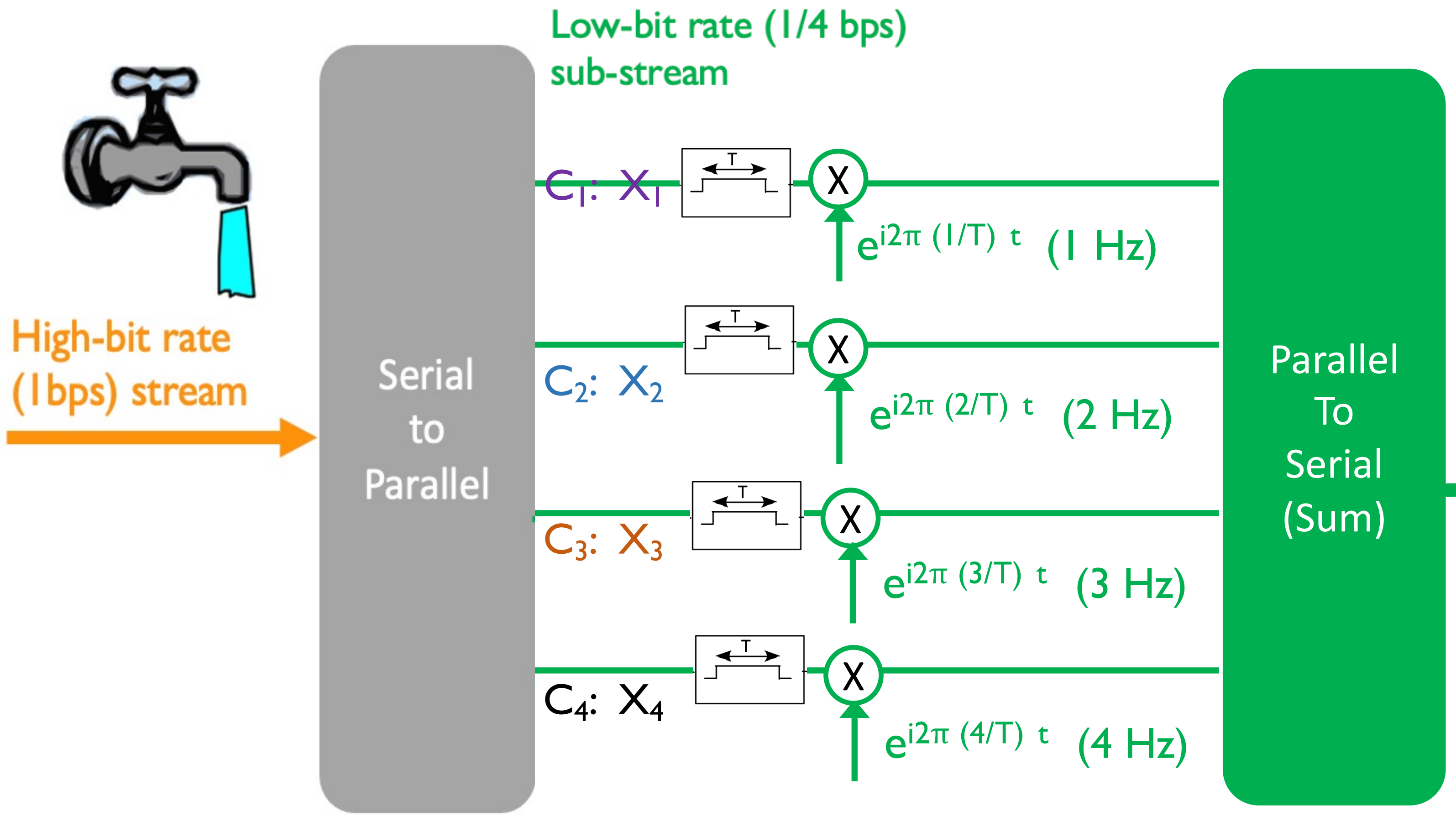


Final step is to add everything up in time domain and send!



What would OFDM modulation equation look like?

OFDM sender equation



- Let each bit in subcarrier C_k be X_k
- In BPSK, $X_k \in \{1, -1\}$
- Let T be the symbol time
- Modulate them with orthogonal frequencies

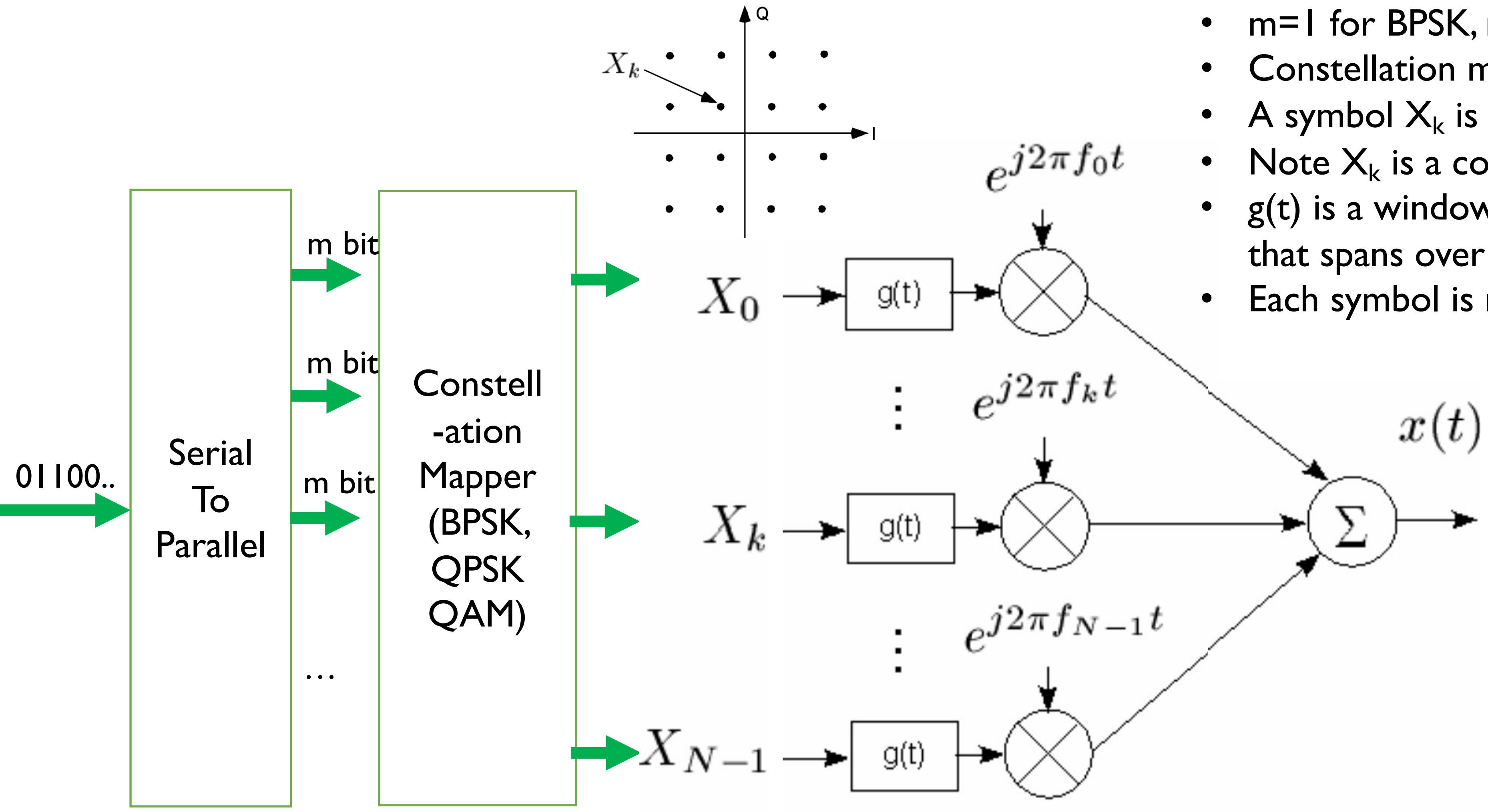
$$x(t) = \sum_{k=1}^{n} X_k\, e^{i2\pi\,(k/T)\,t}$$

Equation for the final signal x(t) should look familiar ☺

Outline

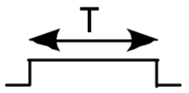
1. Recap OFDM basics
2. Implementing OFDM

Generalized Equation for OFDM



$$= \sum_{k=0}^{n-1} X_k \, e^{i2\pi \, (k/T) \, t}$$

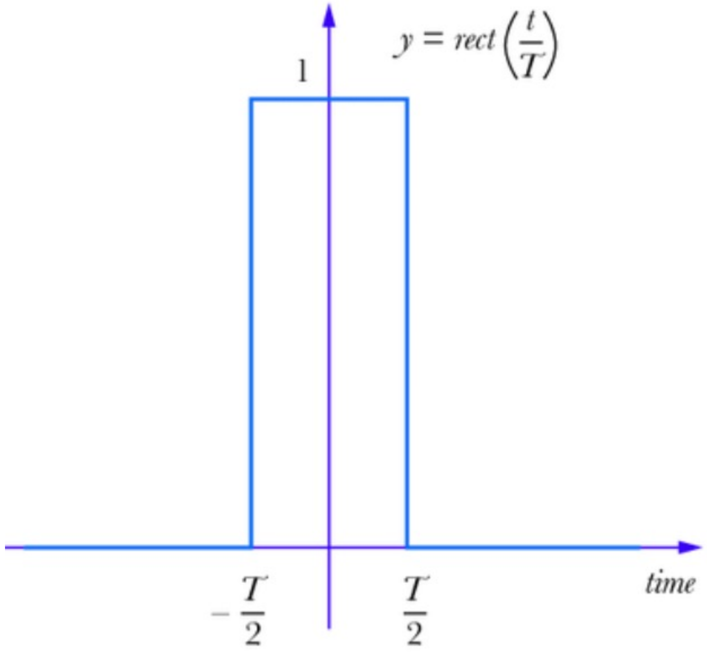
- m is modulation order
- m=1 for BPSK, m=2 for QPSK, m=4 for 16 QAM
- Constellation mapper maps m bits into a symbol
- A symbol X_k is a point in constellation diagram
- Note X_k is a complex value
- g(t) is a window (pulse function) that spans over symbol time T

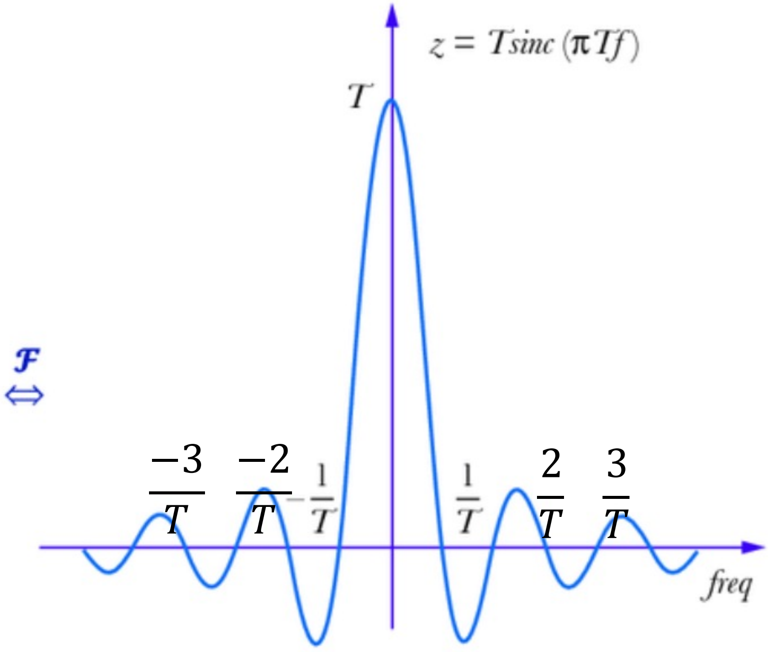


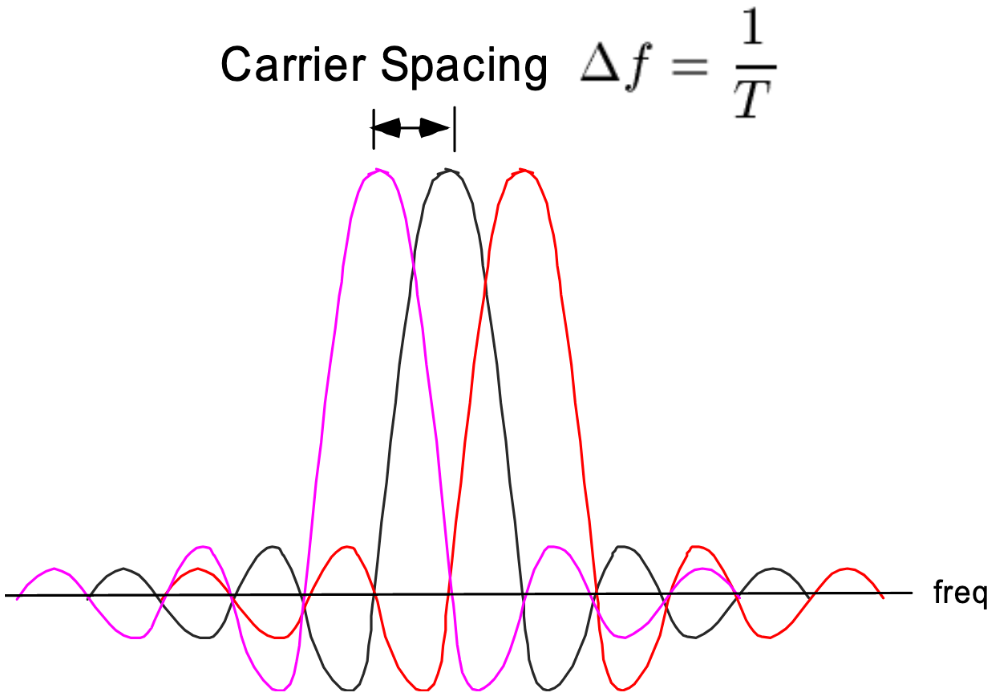
- Each symbol is modulated with orthogonal freq

Note FFT of rectangular pulse is a sinc function

- Rectangular pulse function with period period T from [-T/2, T/2]
- FFT of this pulse is sinc(πTf) where sinc(x) is defined as sin(x)/x
    - 0 crossings whenever f = k/T

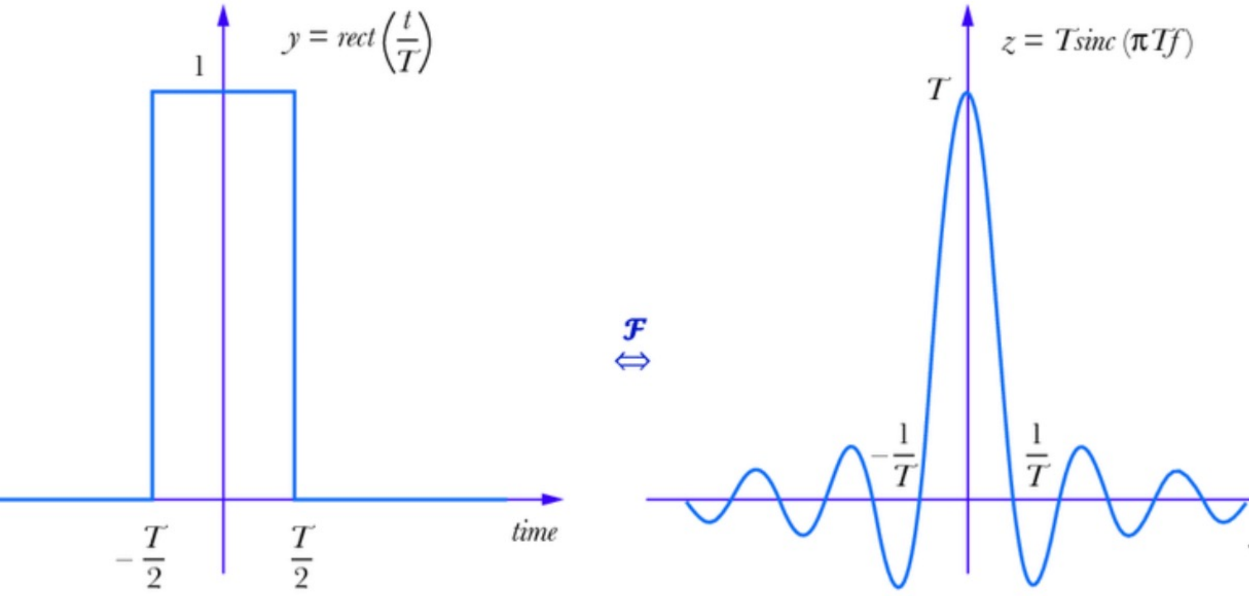


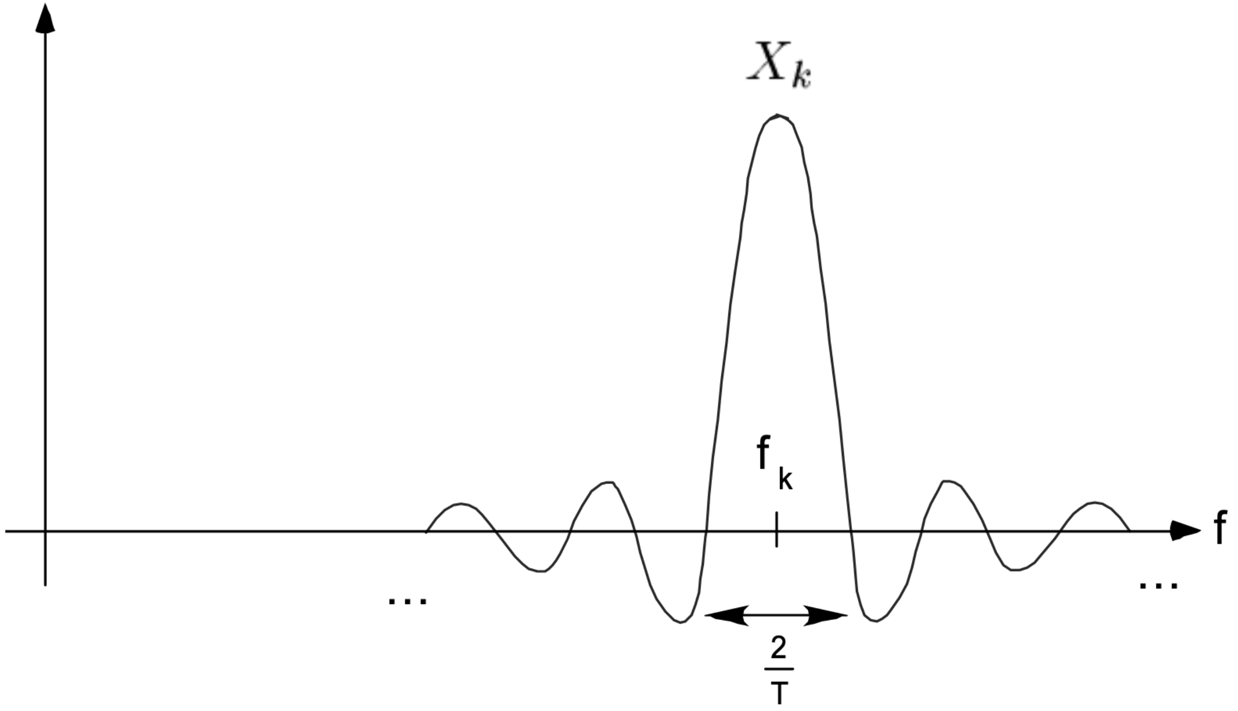


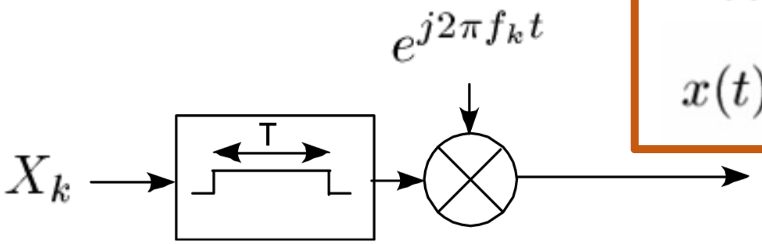


When carrier spacing Δf = 1/T each carrier become orthogonal to one another

Modulating with carrier frequency f_k shifts from 0-centered signal into a signal centered at f_k



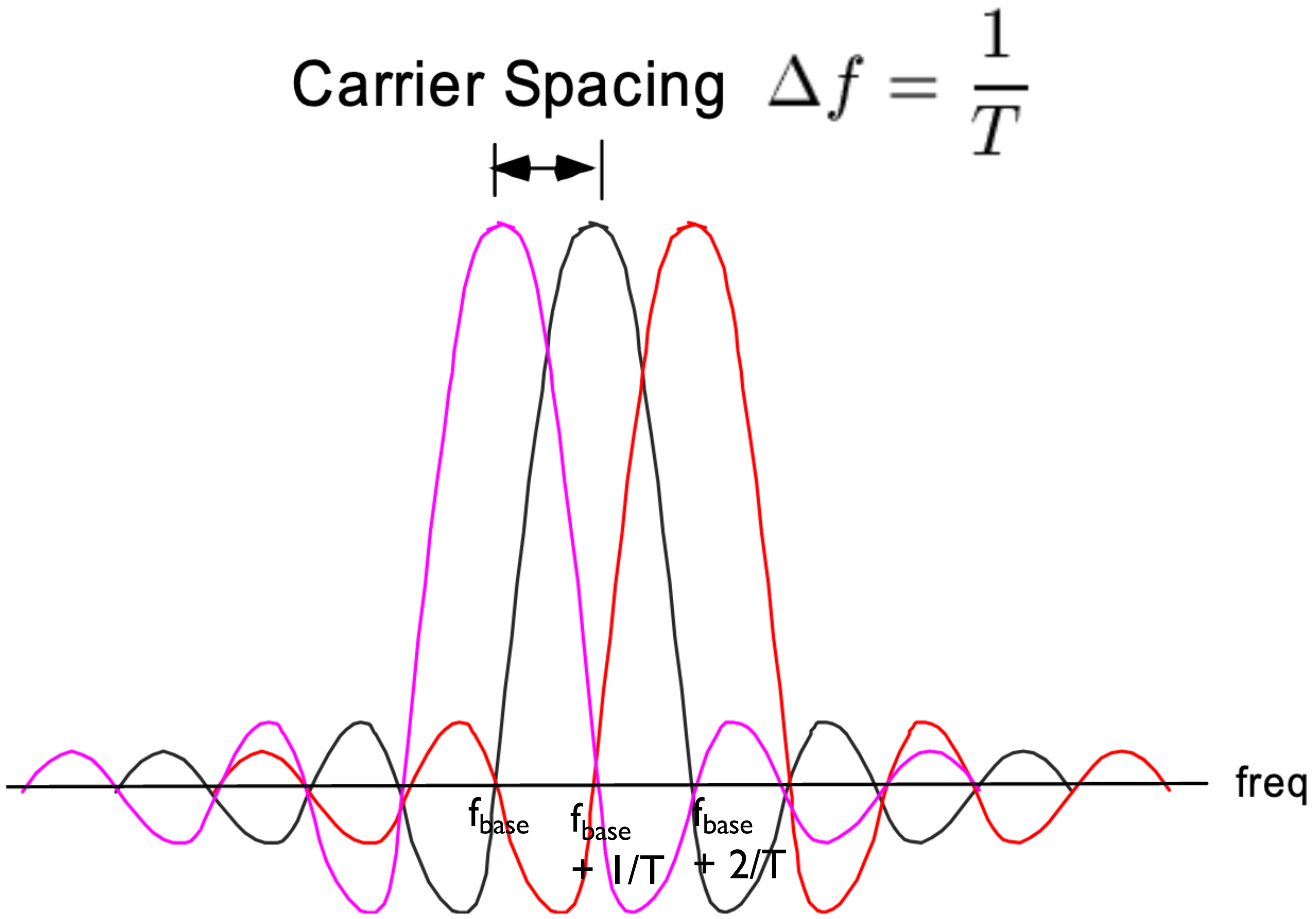




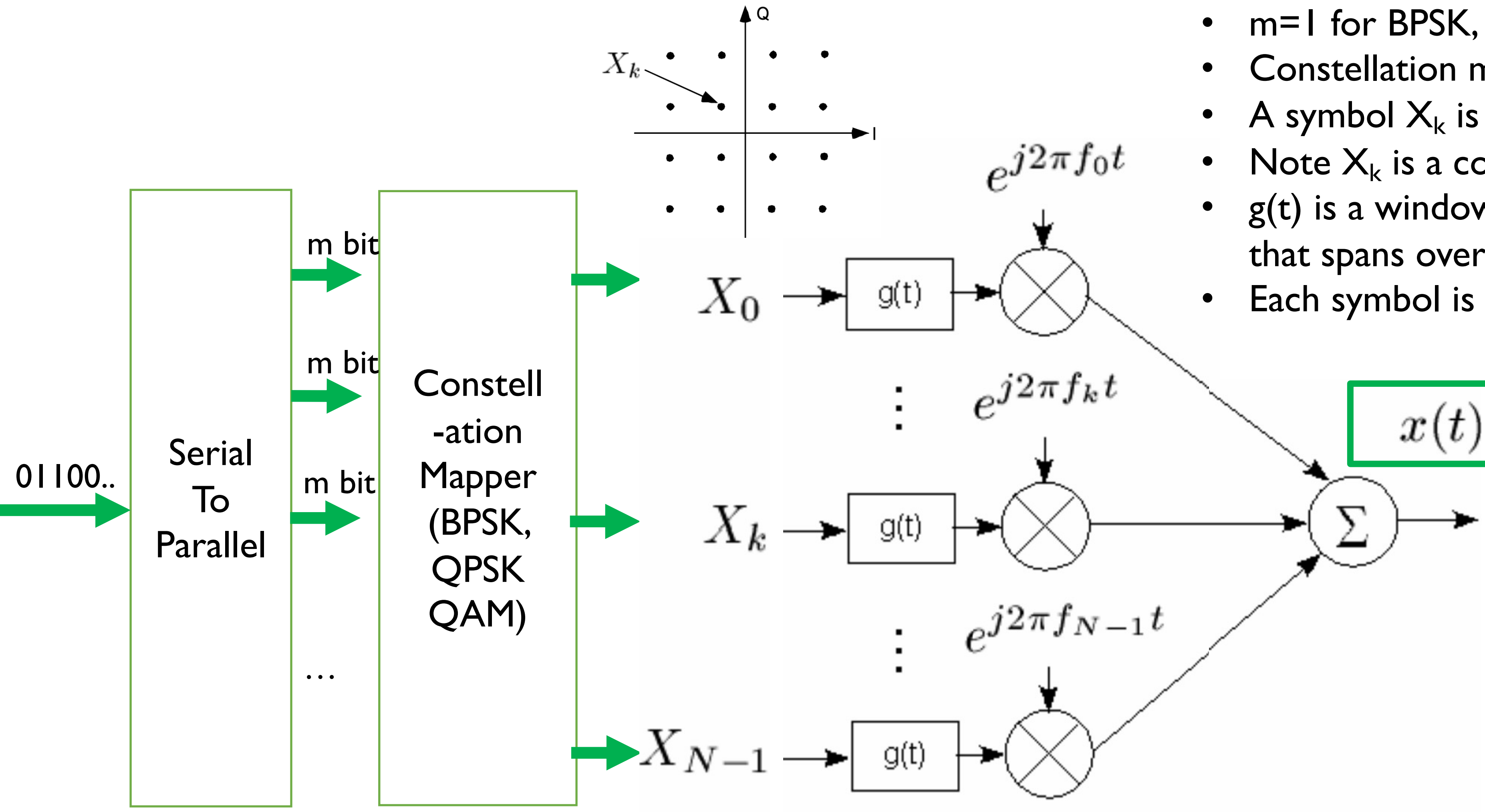
Modulation Property

$$x(t)e^{j2\pi at} \leftrightarrow X(f-a)$$

By setting carrier frequencies as $f_{base} + k/T$ we can make carriers orthogonal



Recall: Generalized Equation for OFDM



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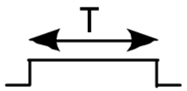


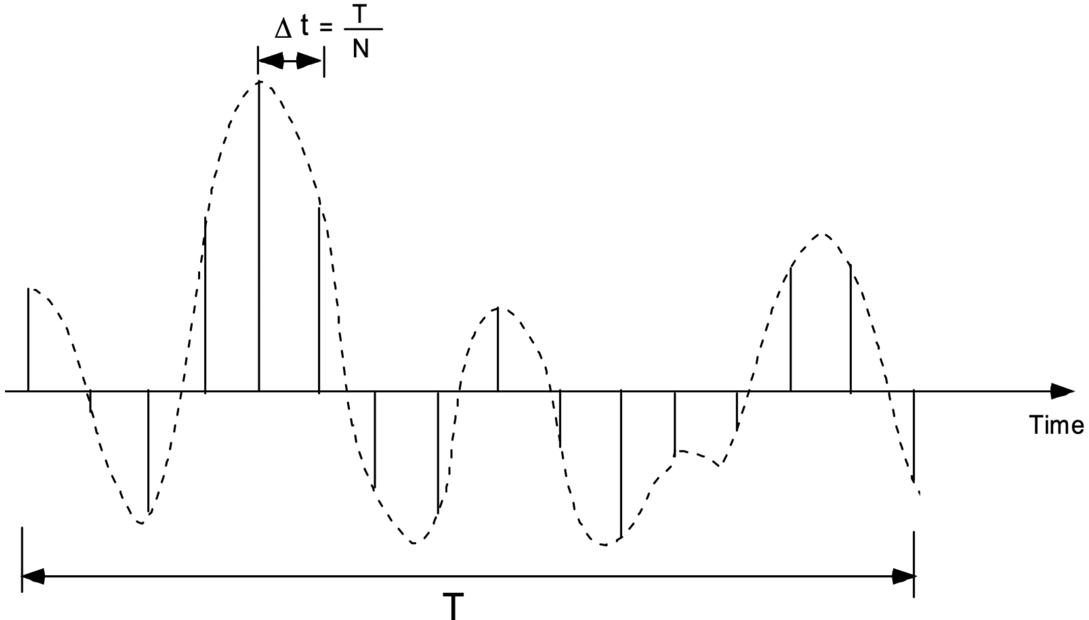
$$\boxed{x(t) = \sum_{k=0}^{n-1} X_k \, e^{i2\pi (k/T) t}}$$

Let's sample x(t) $= \sum_{k=0}^{n-1} X_k \, e^{i2\pi (k/T)\, t}$

- Let x_j be the j^{th} sample of x(t)

  $x_j = \sum_{k=0}^{n-1} X_k \, e^{i2\pi (k/T) t\_j}$ where t_j = $j\Delta t$ = j^{th} sample time

  - Given T is symbol time and n is the number of samples
  - Sampling interval $\Delta t = \frac{T}{n}$ and sampling rate $Fs = \frac{1}{\Delta t} = \frac{n}{T}$



OFDM time domain sample $x_j = \sum_{k=0}^{n-1} X_k\, e^{i2\pi\,(k/T)t\_j}$

- t_j/T = jΔt/T = jΔt/(n Δt) = j/n
- Thus, we can rewrite as $x_j = \sum_{k=0}^{n-1} X_k\, e^{i2\pi\, jk/n}$

DFT and IDFT

Note OFDM $x_j=\sum_{k=0}^{n-1} X_k\, e^{i2\pi\, jk/n}$

- x_j is j^{th} time domain sample out of n samples of period P
  - $x_0\ x_1\ x_2\ \ldots\ x_{n-1}$ are n time domain samples
- DFT of x_j's is

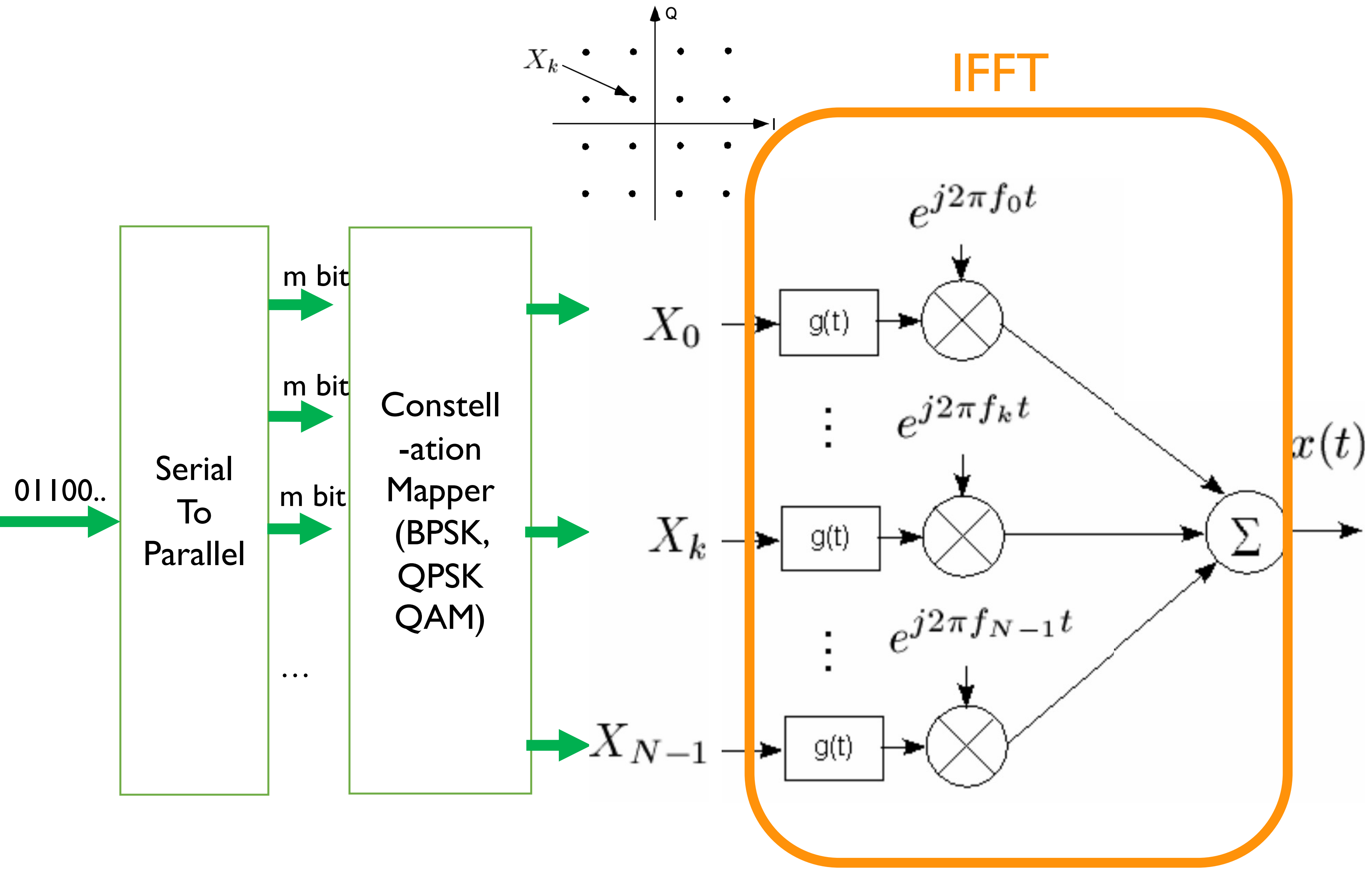
$$X_k = \sum_{j=0}^{n-1} x_j\, e^{-i\,(2\pi/n)jk}$$

- IDFT of X_k's is

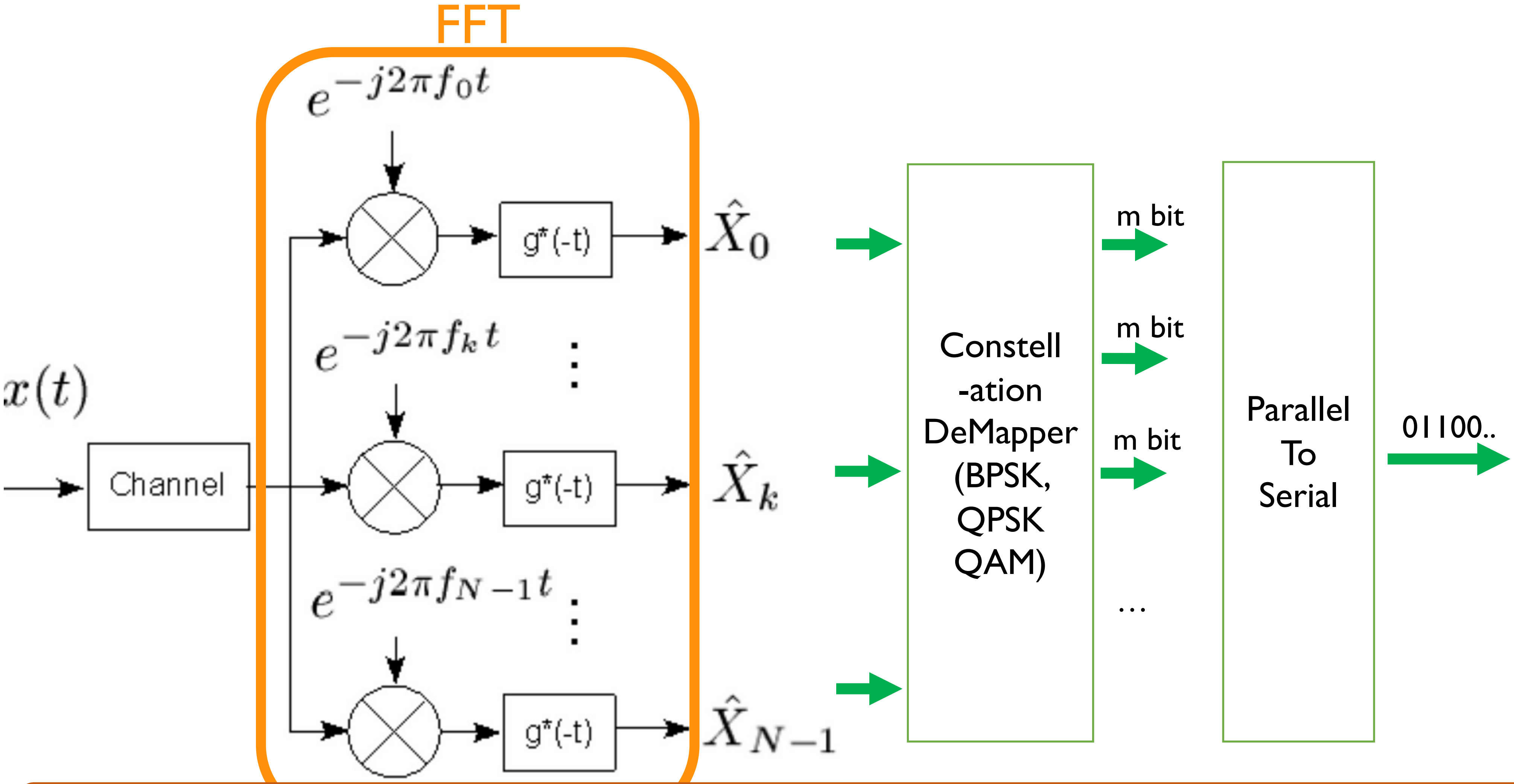
$$x_j = \frac{1}{n}\sum_{k=0}^{n-1} X_k\, e^{i\,(2\pi/n)jk}$$

Identical! Thus OFDM modulation can be implemented via IFFT where IFFT is a fast implementation of IDFT

OFDM sender can be implemented via IFFT



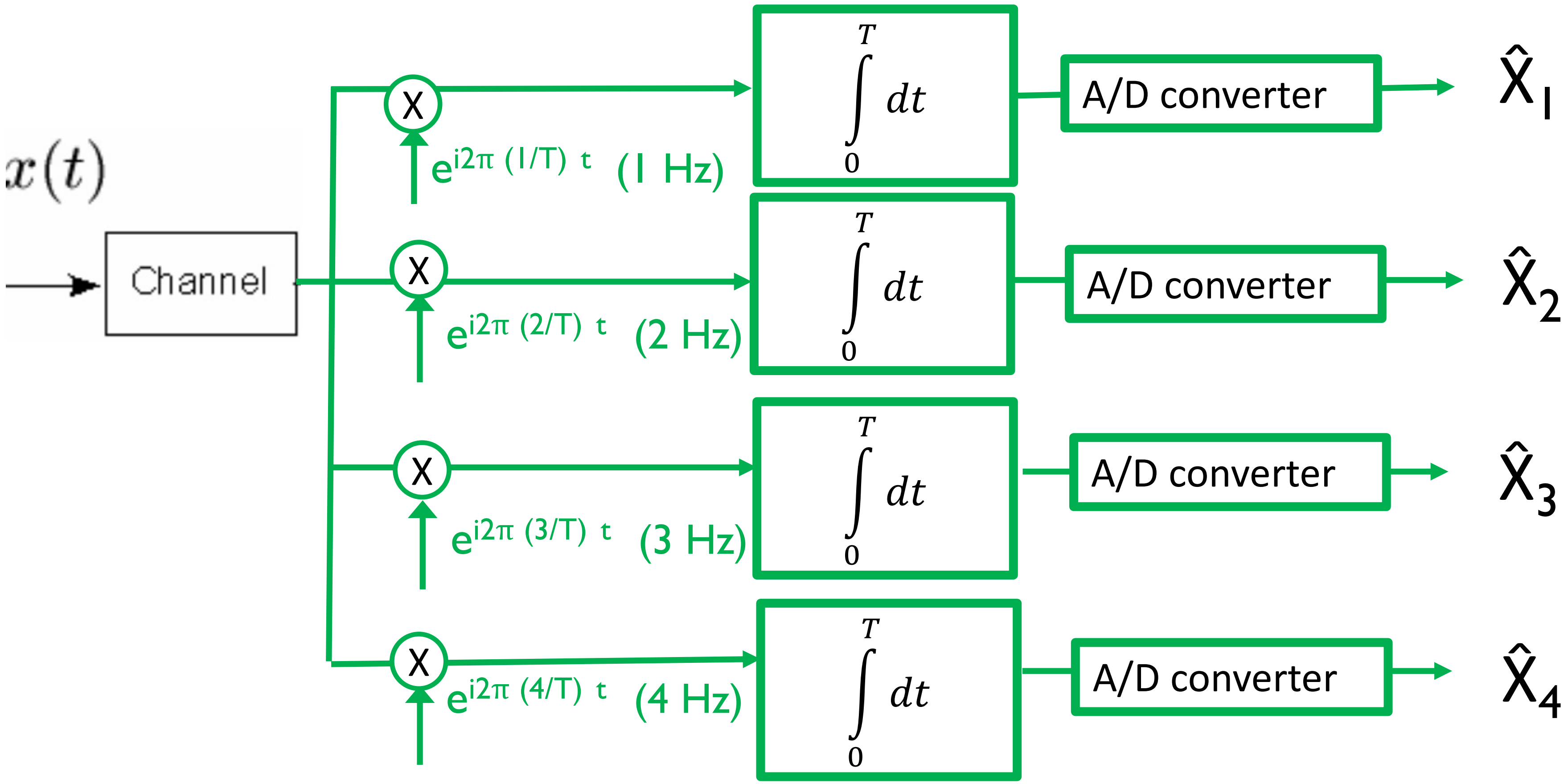
Receiver-side OFDM demodulation is implemented via FFT



$\hat{X}_k$ are the decomposed coefficients of each frequency components of x(t)

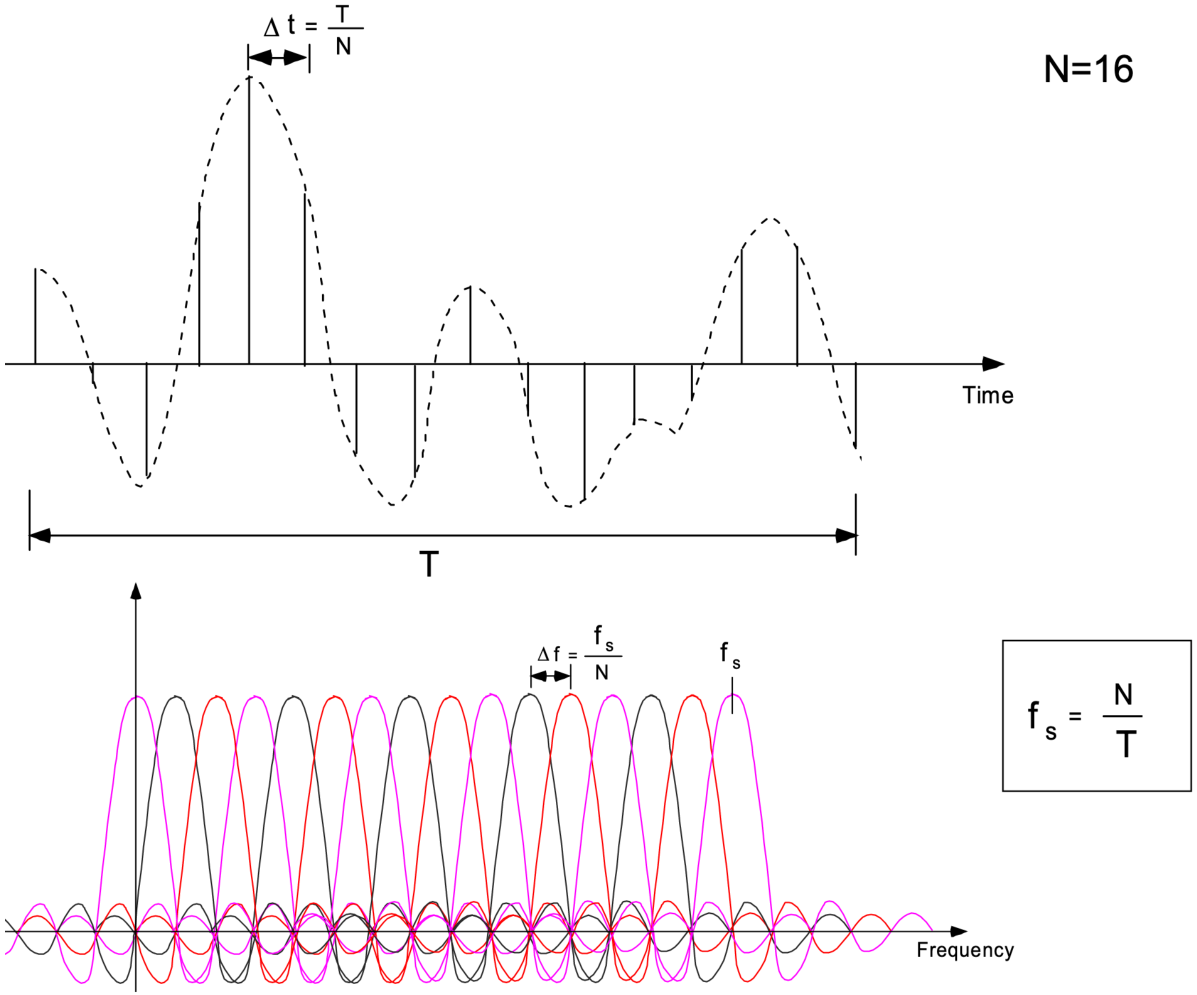
Comparing to "manual" implementation

- If we did NOT have FFT then… this is what we need in receiver side
  - BPSK example used

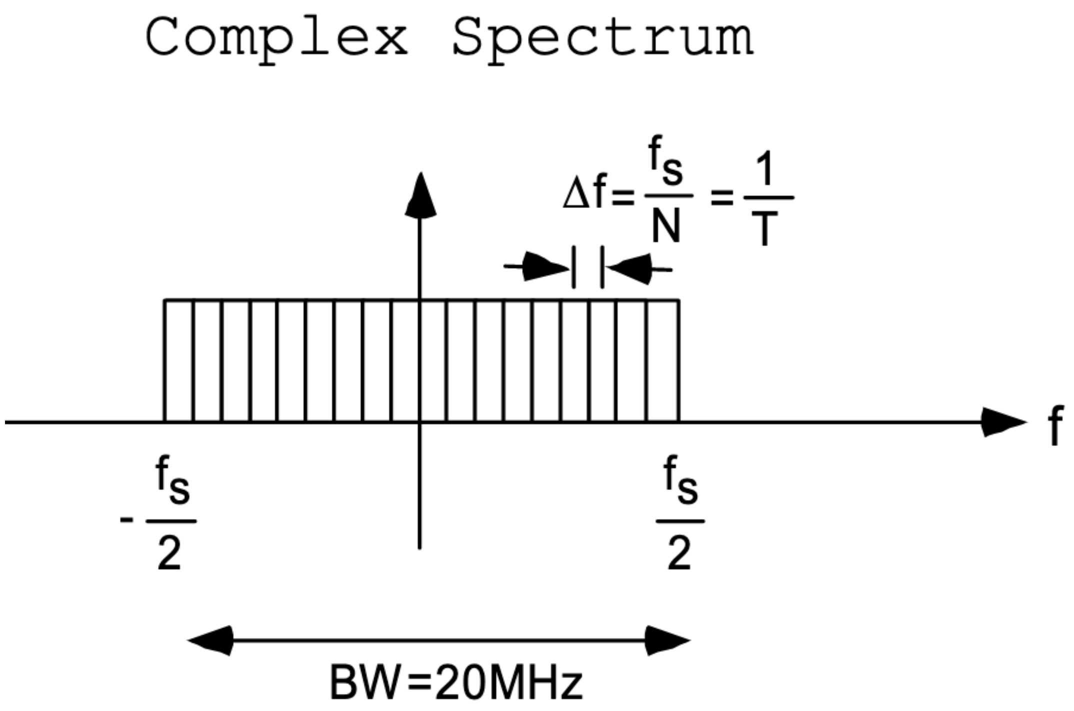


802.11a Wifi Example

- **Total bandwidth Bw == sampling rate Fs**
- **Sampling rate Fs = N/T**
  - T is the symbol period
  - N is the number of samples
- **Total bandwidth Bw = N Δf**
  - N is also the number of carriers
  - Δf is the bandwidth of each carrier



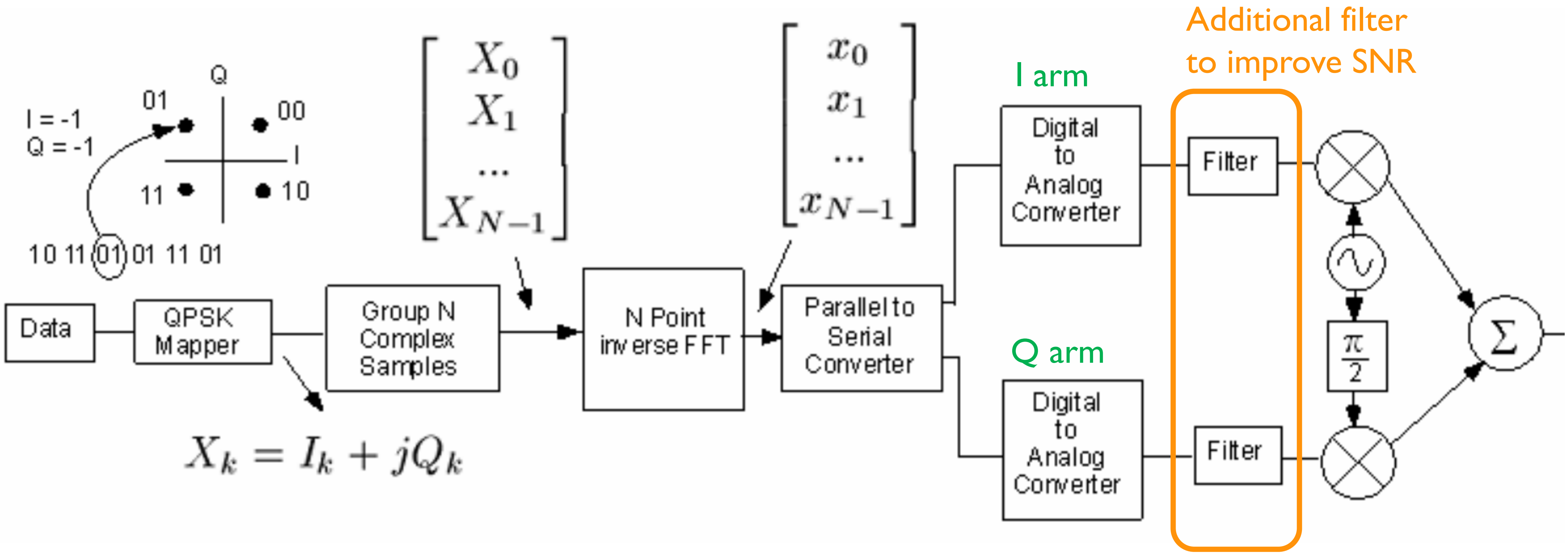
802.11a Wifi Example



- Total bandwidth Bw = 20 MHz
- Sampling rate is also 20 MHz
- If N = 64
  - Carrier spacing Δf = 20MHz/64
= 312.5 KHz
  - Symbol time T = 1/ Δf = 3.2 microsec

- Tolerable delay spread is rougly 1-2 microsec
  - More on delay spread later

Fuller Story: OFDM sender with I/Q modulation



Fuller Story: OFDM receiver with I/Q modulation