Lecture 03-14: Physical Layer Channel Basics

CS 356R

Intro to Wireless Networks

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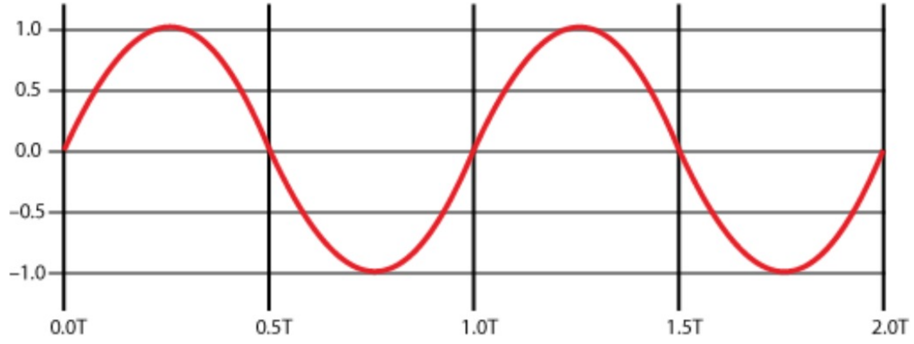
Outline

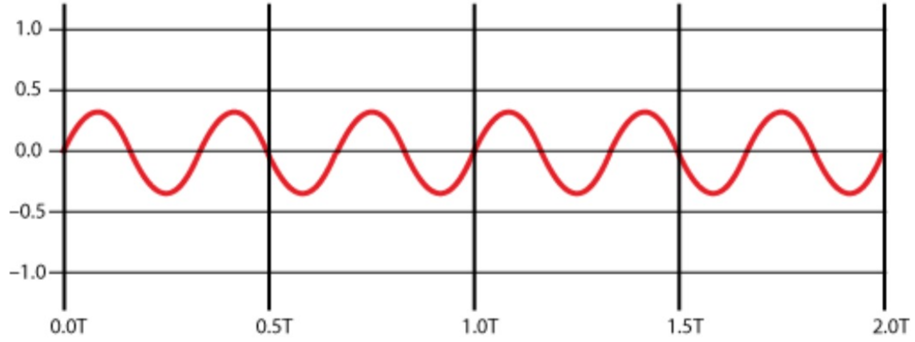
1. Relationship between Data Rate and Bandwidth

Definitions

- Spectrum of a signal: range of frequencies that the signal contains
- Bandwidth: the width of the spectrum
- Example (a)+(b)
  - Range [1f , 3f] is spectrum
  - Bandwidth 3f – 1f = 2f
  - T = 1/f

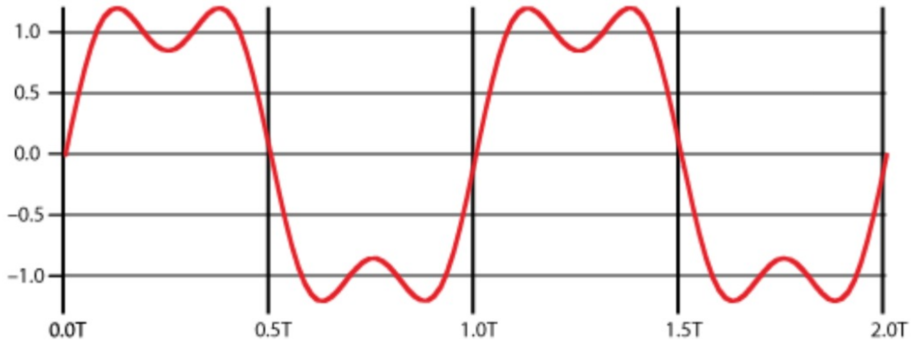


(a) *sin (2πft)*



(b) *(1/3) sin (2π(3f)t)*

Two signals added together



c) *(4/π) [sin (2πft) + (1/3) sin (2π(3f)t)]*

How are data rate and bandwidth related?

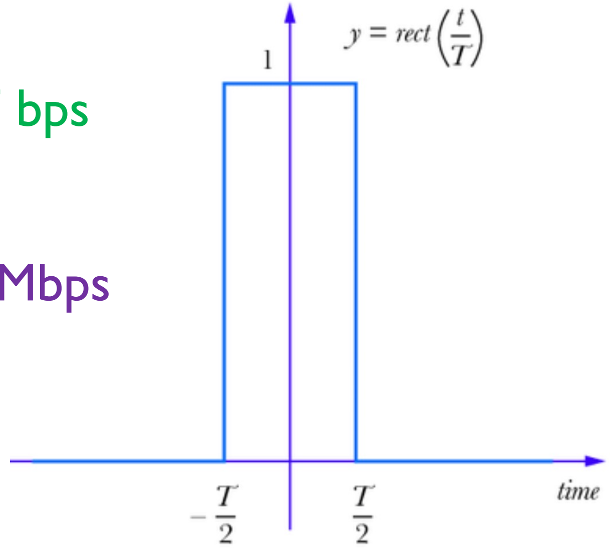
- Textbook (Beard and Stallings) Chap 2

Fourier transform of rectangular pulse is a sinc function

- Square wave s(t) = 1 when $-\frac{T}{2} <= t <= \frac{T}{2}$, 0 otherwise
- $F(w) = \frac{2\sin(w(\frac{T}{2}))}{w} = \frac{2\sin(w(\frac{T}{2}))}{w(\frac{T}{2})}\left(\frac{T}{2}\right) = T\,\mathrm{sinc}(w\,\frac{T}{2}) = T\,\mathrm{sinc}(\pi Tf)$
  - Since $w=2\pi f$ (w is rad/sec and f is cycle/sec (Hz) , 1 cycle = 2π rad)
  - 0 crossings whenever f is k/T (k=…-2,-1,0,1,2,…)

Data rate = 1/T bps

$T=10^{-6}$
Data rate = 10 Mbps



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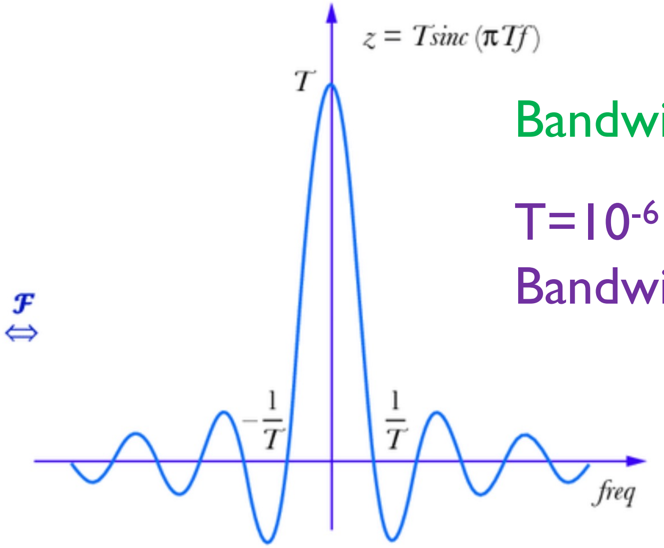


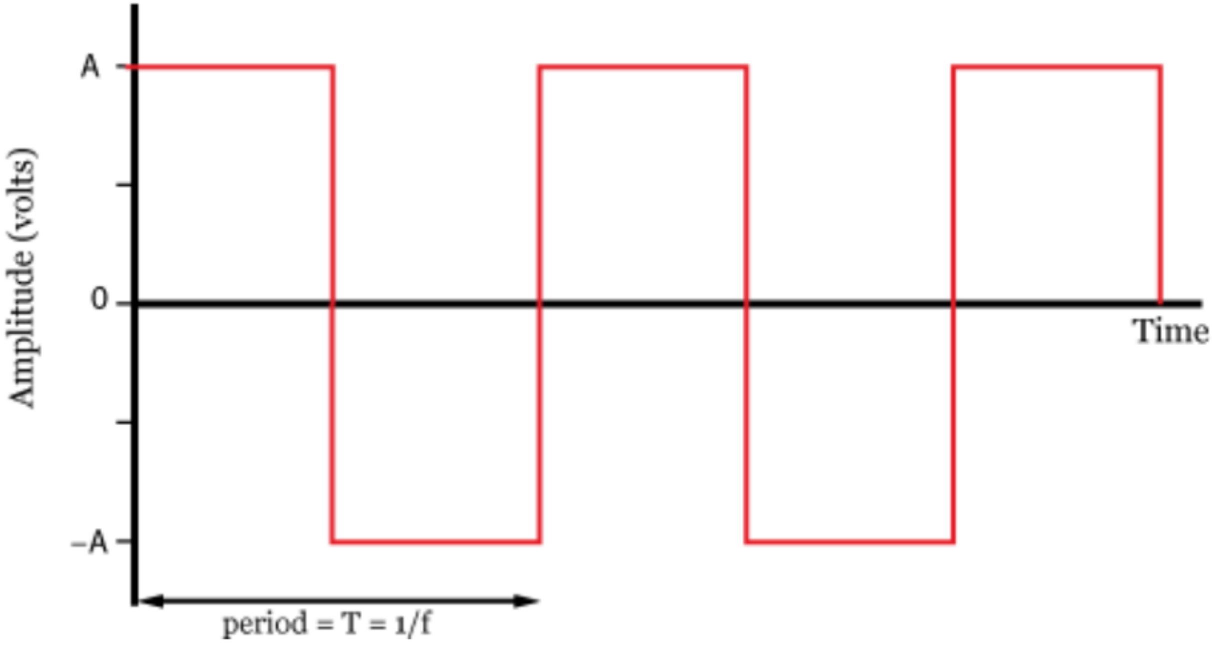
Bandwidth = 2/T Hz

$T=10^{-6}$
Bandwidth = 20 MHz ??

No! We need infinite frequencies

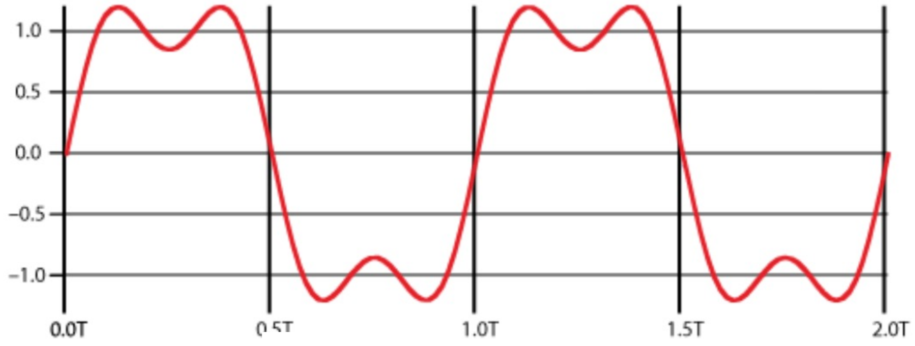
Square wave

- Let positive pulse represent binary bit 0, negative - binary bit 1
  - 010101 …
- Square wave with amplitudes A/-A can be expressed as

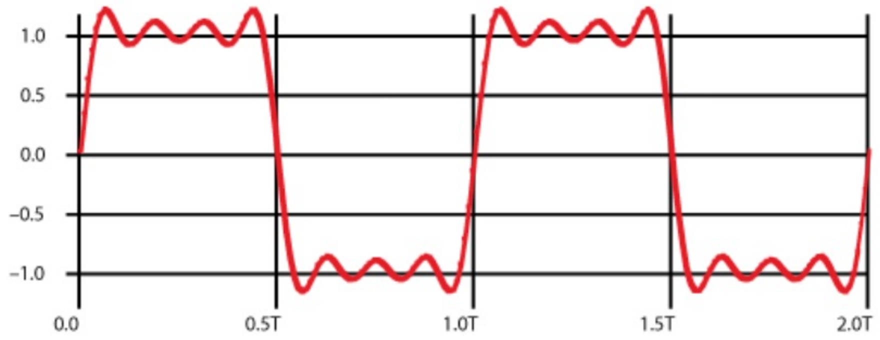


$$S(t) = A \times \frac{4}{\pi} \times \sum_{k=odd}^{\infty} \frac{\sin(2\pi k f t)}{k}$$

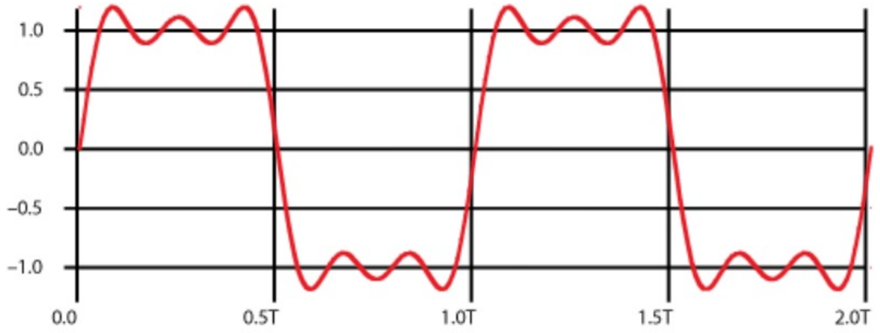
A: 2 signals B: 3 signals C: 4 signals D: inifinite



A $(4/\pi)$ $[\sin(2\pi ft) + (1/3)\sin(2\pi(3f)t)]$

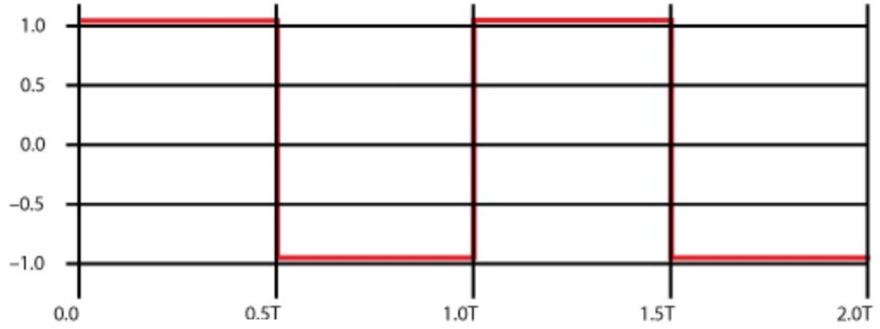


C $(4/\pi)$ $[\sin(2\pi ft) + (1/3)\sin(2\pi(3f)t) + (1/5)\sin(2\pi(5f)t) + (1/7)\sin(2\pi(7f)t)]$



B $4/\pi)$ $[\sin(2\pi ft) + (1/3)\sin(2\pi(3f)t) + (1/5)\sin(2\pi(5f)t)]$

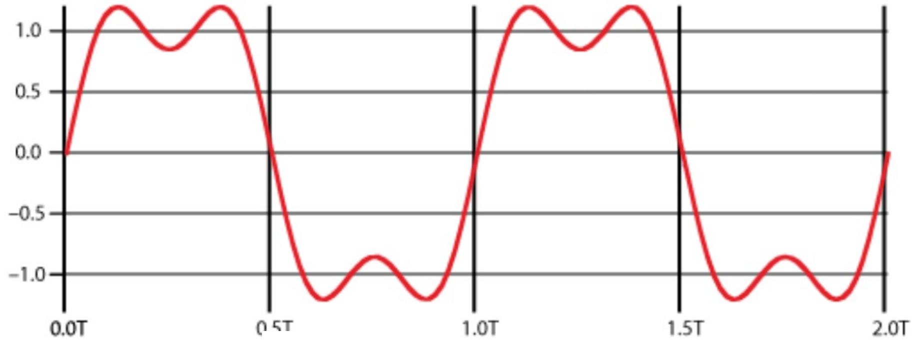
Wth infinte number of harmonics



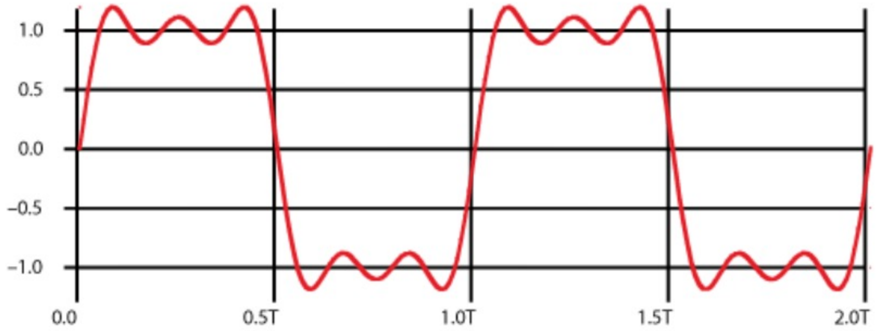
D $(4/\pi)$ $(1/k)\sin(2\pi(kf)t)$, for k odd

A: 2 signals B: 3 signals C: 4 signals D: infinite

- f = 1 MHz, BW = 3f-f = 2 MHz



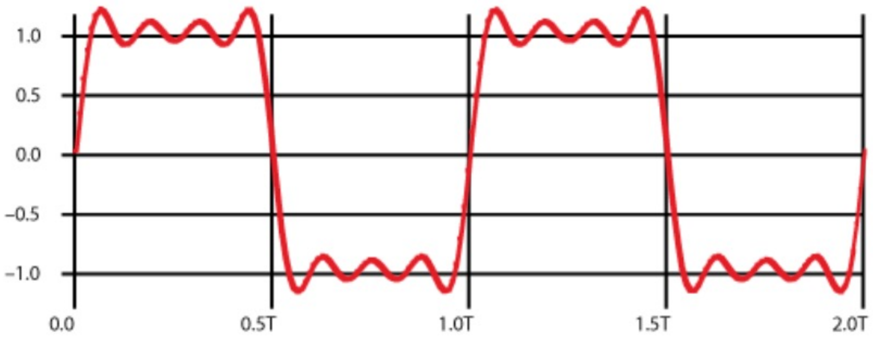
A $(4/\pi)\ [\sin(2\pi ft) + (1/3)\sin(2\pi(3f)t)]$



B $(4/\pi)\ [\sin(2\pi ft) + (1/3)\sin(2\pi(3f)t) + (1/5)\sin(2\pi(5f)t)]$

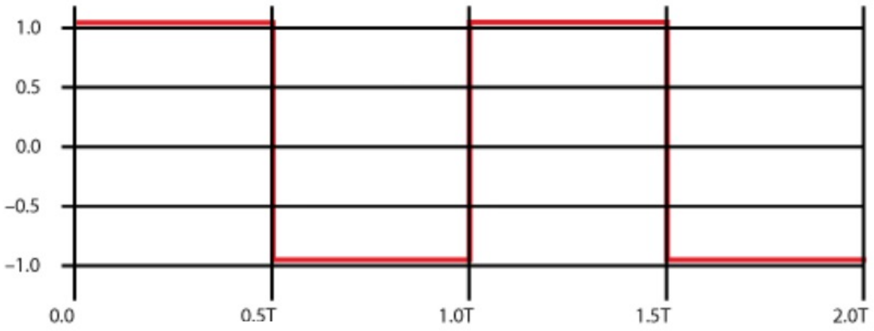
- f = 1 MHz, BW = 5f-f = 4 MHz

- BW = 7f-1f = 6 MHz



C $(4/\pi)\ [\sin(2\pi ft) + (1/3)\sin(2\pi(3f)t) + (1/5)\sin(2\pi(5f)t) + (1/7)\sin(2\pi(7f)t)]$

Wth infinte number of harmonics

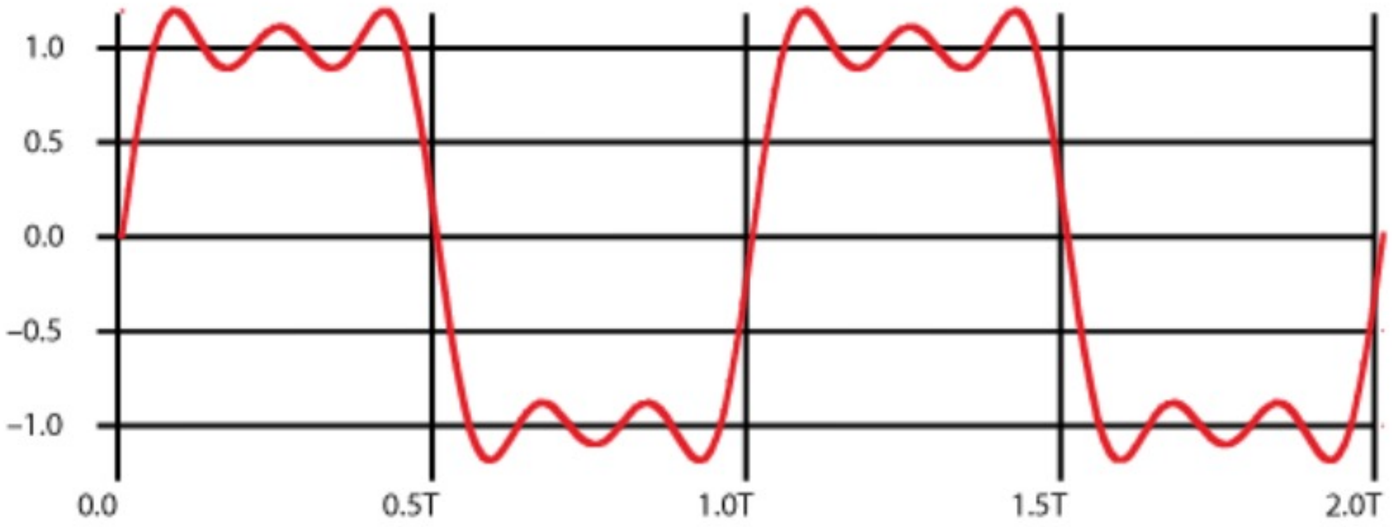


D $(4/\pi)\ (1/k)\sin(2\pi(kf)t)$, for k odd

- BW = infinite MHz

Varying f

- T=1/f Data rate = 2 bit/T sec = 2 f bps
- When f = 1MHz, Data rate = 2 Mbps, bandwidth = 4 MHz
- When f = 2 MHz, Data rate = 4 Mbps, bandwidth = 8 MHz



(a) $(4/\pi)\,[\sin(2\pi ft) + (1/3)\sin(2\pi(3f)t) + (1/5)\sin(2\pi(5f)t)]$

Insights

- We cannot transmit perfect rectangular waveform
  - Why?
- Limiting the bandwidth is necessary
  - Why?
- Limiting bandwidth creates distortion which could cause potential error on the receiver side

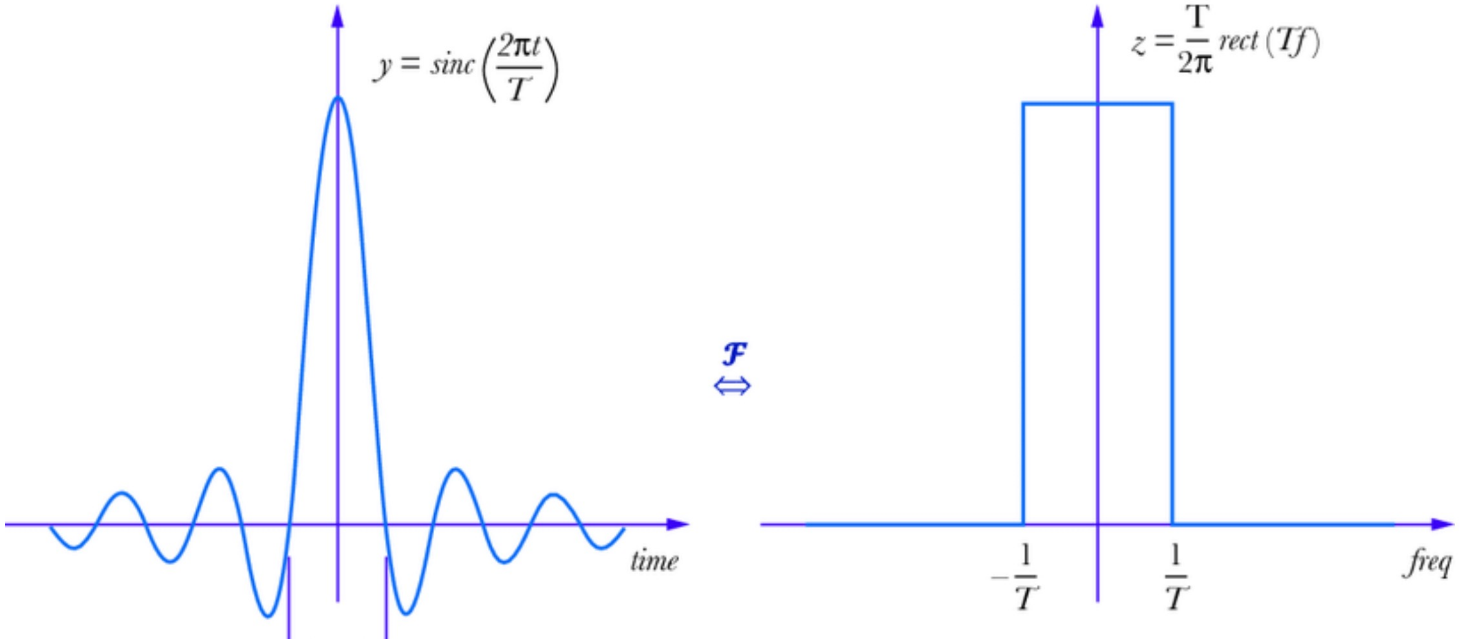
Outline

1. Relationship between Data Rate and Bandwidth
2. Bandpass filter (BPF)

Bandpass Filter (BPF)

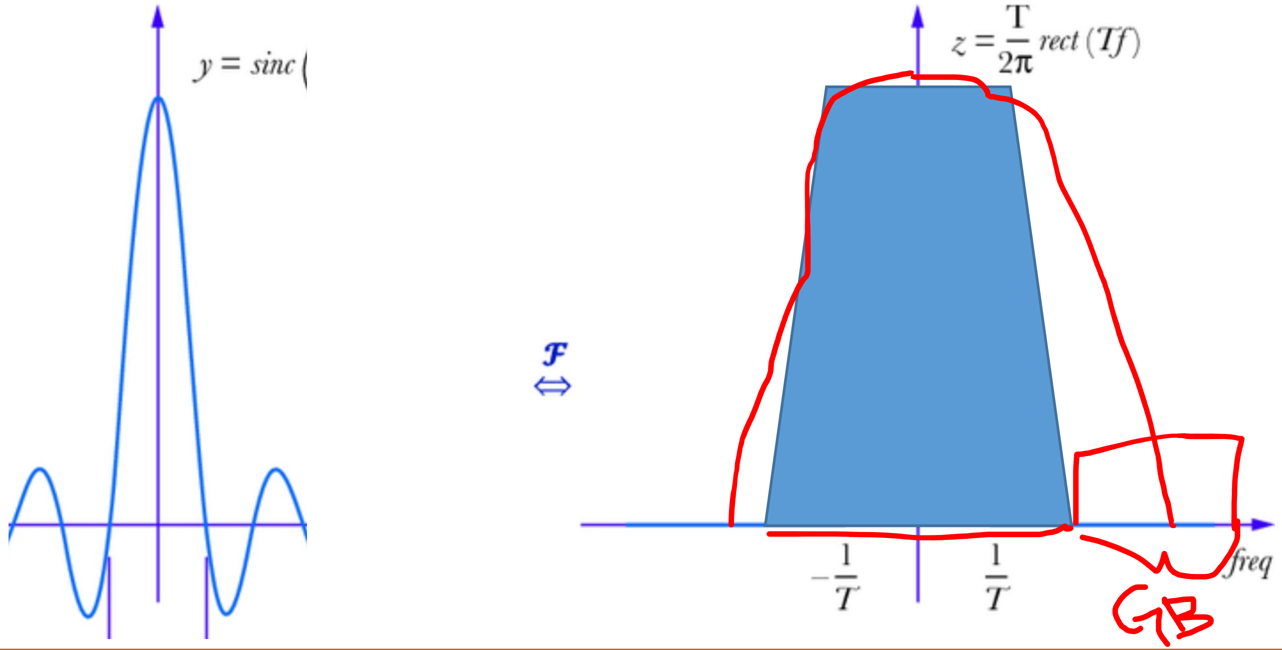
- BPF is a device that passes frequencies within a certain range and rejects (or attenuated) frequencies outside that range
- high-pass filter allows through components with frequencies above threshold
- low-pass filter allows through components with frequencies below threshold

IFFT of rectangular pulse is a sinc function



Only infinite time domain signal can fall into perfect square in freq domain

IFFT of rectangular pulse is a sinc function



Time limiting will result in spreading in bandwidth

We need additional guard band to ensure better filtering

Outline

1. Relationship between Data Rate and Bandwidth
2. Bandpass filter (BPF)
3. Nyquist Bandwidth

What is the maximum data rate given bandwidth?

- Assuming noise free channel
- What did Nyquist say?
  - If the rate of signal transmission is 2B then a signal with frequencies no greater than B is sufficient to carry the signal rate.
  - Given a bandwidth of B, the highest signal rate that can be carried is 2B.
- Signal rate: num symbols can be sent in a sec
- Let M be the number of possible symbols
  - How many bits this symbol can represent? $\log_2 M$ bits
  - Ex) BPSK 2 symbols (1 bit : 0 or 1), QPSK 4 symbols (2 bit: 00, 01, 10, 11), etc
- 2B symbols/sec (signal rate) is converted to 2B $\log_2 M$ bps (data rate)

The theoretic bound: Max possible data rate $C = 2B \log_2 M$

Trade off: $C = 2B \log_2 M$

- The more symbols (greater M) we have greater data rate we can achieve (greater C)

- **Greater burden on sending**
  - Possibly using greater amplitude
  - More variations of signal
- **Greater burden on receiving**
  - Distinguishing 2 symbols vs 64 symbols
  - With noise and multipath, fading, etc harder to distinguish

What are other factors that could affect C?

- Let’s add noise

Nyquist bandwidth assumes no noise which is unrealistic…

- What are other factors that could affect C?

Noise! Let's first assume thermal (white) noise

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1. Relationship between Data Rate and Bandwidth
2. Bandpass filter (BPF)
3. Nyquist Bandwidth
4. dB and dBm

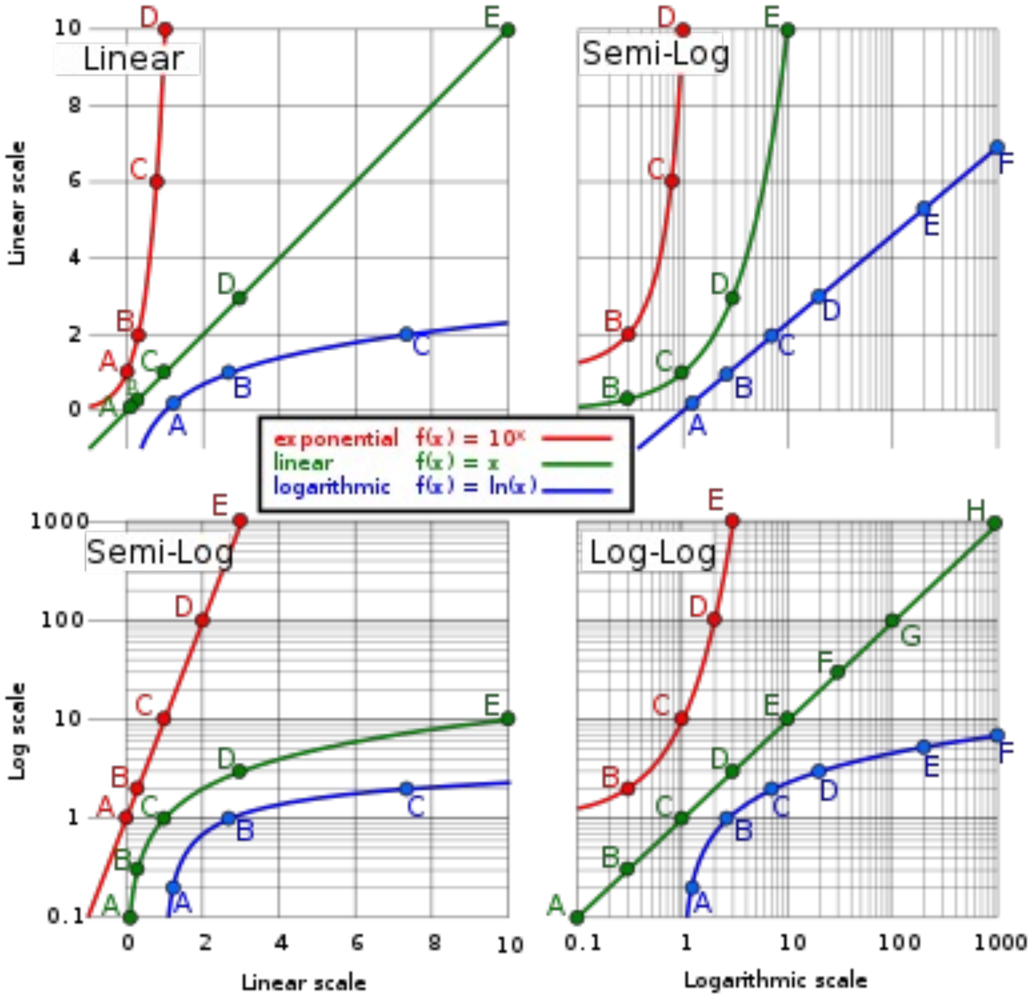
What is a dB (decibel)?

- $10 \times \log_{10} (\frac{P1}{P2})$
- The ratio between two power levels in logarithmic scale
  - P1 and P2 should be of the same unit that represents power

p1	p2	$\frac{P1}{P2}$	dB
1	1	1	0 dB
2	1	2	3 dB
10	1	10	10 dB
0.5	1	0.5	-3 dB
100	1	100	20 dB
1000	1	1000	300 dB

Why logarithmic scale?

Using log-scale enables to express data with large variations in one graph



dB is a ratio not an absolute quantity

- We can say this signal is 3 dB greater than the other signal
  - Meaning it has twice greater signal
- We cannot say this signal level is 3 dB
  - Makes no sense without a reference point

To convert dB into an absolute quantity
we must specify a reference point!

The suffix dB_ implies the reference

- **dBm: reference is 1 milliwatt (mW)**
  - 50 dBm means compared to 1 milliwatt, 10 $\log_{10}$ (ratio) is 50
  - To calculate the ratio two values must be in the same unit
  - Given 10 $\log_{10}$ (X W/0.001W) = 50, what is X?
  - X = 100 W = $10^{\wedge 50/10} \times 0.001$
- **dBμ: reference is 1 microwatt (μW)**
  - 50 dBμ means compared to 1 microwatt, 10 log10 (ratio) is 50
  - Given 10 $\log_{10}$ (Y W/0.000001W) = 50, what is Y?
  - Y = 0.1 W = $10^{\wedge 50/10} \times 0.000001$

**To convert dB into an absolute quantity
we must specify a reference point!**

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4. dB and dBm
5. SNR and Shannon Capacity

SNR also affects max data rate C

- Ratio between the signal and the noise
- Sometimes expressed in linear ratio SNR_{linear}
- Other times expressed in dB
  - $SNR_{dB} = 10 \log_{10} \frac{Signal\ Power}{Noise\ Power}$
  - Note signal power and noise power should be in the same linear unit

High SNR means high-quality signal

Shannon Capacity

- In the presence of white noise
- Max datarate $C = B \log_2 (1 + SNR_{linear})$
  - B is the bandwidth
- Note SNR is NOT in dB but just linear ratio

Again, this is a theoretical bound
(Does not account for impulse noise, attenuation, delay distortion, etc)

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1. Relationship between Data Rate and Bandwidth
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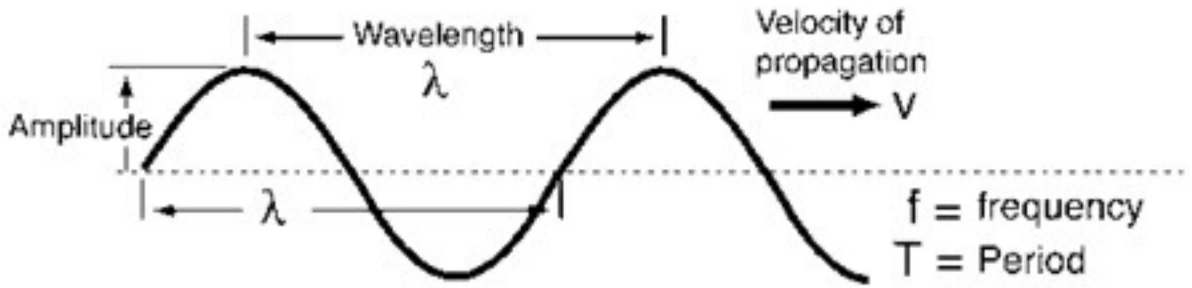
4. dB and dBm
5. SNR and Shannon Capacity
6. Channel attenuation

Any transmission of signal goes through attenuation

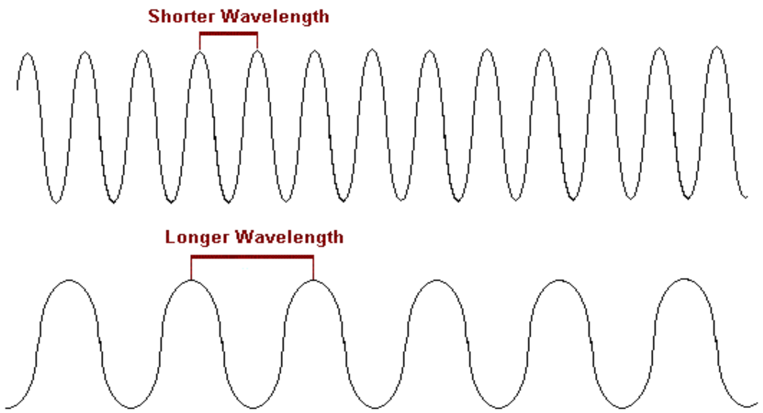
- Attenuation: the loss of signal strength over transmission medium
- Often expressed in dB
- In wireless, attenuation of radio frequencies is modeled with Free space path loss model
    - No obstacles but direct line of sight
- $L_{dB} = 10 \log_{10} (\frac{4\pi d}{\lambda})^2$ dB
- λ = wavelength
- Velocity $v = \frac{\lambda}{T} = \lambda f$



Radio wave travels at speed of light c

- Speed of light c = λf
- $\lambda = \frac{c}{f}$
  - C is constant 299,792,458 m / s
- Higher frequency has shorter wavelength
- $L_{dB} = 10 \log_{10} (\frac{4 \pi d}{\lambda})^2$ dB

  $= 10 \log_{10} (\frac{4 \pi d f}{c})^2$ dB



Longer wavelength suffer relatively less attenuation